Theory of the electron-spin susceptibility in antiferromagnetic superconductors

Hongguang Chi and A. D. S. Nagi
Guelph-Waterloo Program for Graduate Work in Physics, Department of Physics, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
(Received 17 March 1988)

A theory of the longitudinal electron spin susceptibility χ_s for an antiferromagnetic superconductor (AFS) has been given. For the AFS, we have assumed a homogeneous superconducting order parameter and a one-dimensional electron band that satisfies the nesting condition $\varepsilon_{\mathbf{k}}=-\varepsilon_{\mathbf{k}+\mathbf{Q}}$, where $\mathbf{Q}$ is the wave vector characterizing the antiferromagnetic (AF) order. First we have studied the dependence of χ_s on the scattering rates for the scattering of conduction electrons from the nonmagnetic, spin-orbit, and magnetic impurities by regarding H_Q as a parameter and neglecting the spin-fluctuation effects (H_Q is the AF field). The effect of impurities is found to be significant. Then we have investigated the temperature dependence of χ_s by taking $T_N<T_c$ and by including the spin-fluctuation effects and the temperature dependence of the AF field (T_N is the AF ordering temperature and T_c is the superconducting transition temperature). The aim has been to see if χ_s is enhanced or depressed by the AF ordering occurring below T_N. We find that (1) χ_s increases with the increase in scattering rate from spin-orbit impurities both above and below T_N; (2) keeping other parameters fixed, the enhancement or depression of χ_s below T_N depends on $H_Q(0)$—there is enhancement (depression) when $H_Q(0)$ is larger (smaller) [$H_Q(0)$ is the zero-temperature value of H_Q]; (3) nonmagnetic impurities have a dramatic effect on χ_s in the AF phase. For cleaner (dirtier) superconductors, χ_s is enhanced (depressed) below T_N; (4) in $SmRh_4B_4$, one expects a depression in χ_s by the AF ordering.

I. INTRODUCTION

In the past few years, the problem of the coexistence of superconductivity and antiferromagnetism has been extensively studied both experimentally and theoretically (for reviews see Refs. 1–6). On the experimental side the above coexistence has been established in the ternary compounds[7] RMo_6S_8 and RRh_4B_4 (where R denotes a rare-earth element) and in pseudoternaries[8] $Ho(Ir_xRh_{1-x})_4B_4$ and $Ho(Ru_xRh_{1-x})_4B_4$ and several other materials. On the theoretical side a few different models have been proposed for antiferromagnetic superconductors[9–14] (AFS), and the one given by Nass *et al.*[12] has been developed in most detail. In Ref. 12, a mean-field (MF) theory of AFS was given by introducing the antiferromagnetic (AF) molecular field into the Bardeen-Cooper-Schrieffer[15] (BCS) theory of superconductivity. Some impurity and spin-fluctuation effects in AFS using the above model were discussed in Ref. 13. The effect of homogeneous magnetic field on AFS was considered by Suzumura and Nagi.[16] The effect of nonmagnetic impurities was investigated by Okabe and Nagi.[17] A theory of upper critical field in AFS was given by Ro and Levin.[18] Some thermodynamic properties in the presence of nonmagnetic impurities were studied by Suzumura *et al.*[19] The MF model has been put in the Eliashberg formalism by Prohammer and Schachinger.[20] The tricritical curve in an AFS with nonmagnetic impurities was discussed by the present authors.[21] Finally, the present authors[22] used the physical model given in Ref. 12 and by including the impurity and spin-fluctuation effects explained the enhancement in the Josephson tunneling current and the superconducting order parameter by the AF ordering observed in $SmRh_4B_4$ by Vaglio *et al.*[23]

In the AFS, the antiferromagnetism is associated with the f electrons of the rare-earth atoms and the superconductivity is due to the d conduction electrons of Mo or Rh. The exchange interaction between the R spins and the conduction electrons is weak, but plays a crucial role in determining the coexistence of antiferromagnetism and superconductivity. The coexistence phenomenon is aided by the fact that the wave vector Q of the AF ordering is much larger than the inverse of the superconducting coherence length.

The study of the longitudinal electron-spin susceptibility χ_s for an AFS is important for understanding the coexistence of superconductivity and antiferromagnetism as χ_s is sensitive to above both kinds of long-range orders. χ_s for a BCS superconductor was calculated by Yosida.[24] He found that superconductivity has a drastic effect on χ_s: χ_s vanishes exponentially as the temperature T approaches zero. The important effect of spin-orbit scattering on χ_s was investigated by Ferrell,[25] Anderson,[26] and Abrikosov and Gor'kov.[27] It is found that in the presence of this scattering, χ_s remains finite at $T=0$. The vanishing of χ_s at $T=0$ for a pure BCS superconductor is connected with the fact that the states of such a superconductor are classified according to the eigenvalues of the total spin. However, in the presence of spin-orbit scattering centers, the state of the system can no longer be characterized by the eigenvalues of the spin, and this results in the nonvanishing of χ_s at $T=0$. The magnetic impurities break the time-reversal symmetry of the electron system and thus modify χ_s significantly. This problem was studied by Gor'kov and Rusinov[28] and by Maki and Fulde.[29] The nonmagnetic impurities have no effect on the χ_s of an ordinary superconductor.

For an AFS, the creation of the AF molecular field H_Q below the AF ordering temperature T_N would have a significant effect on the longitudinal electron-spin susceptibility. The χ_s would also be modified by the magnetic, spin-orbit and even the nonmagnetic impurity scattering. Thus, it is of interest to give a theory of χ_s for an AFS including the effect of various kinds of impurities. Such a study has been carried out in the present work. When discussing the temperature dependence of χ_s, we have also included the effect of the elastic spin-fluctuations and the temperature dependence of H_Q. χ_s for a pure AFS has been studied in Refs. 30 and 31.

The plan of the paper is as follows: Section II gives the formalism. In Sec. III, the general expression for χ_s is derived and its limiting cases are given. Section IV describes a model for including the effect of elastic spin fluctuations. Numerical results are given in Sec. V. Section VI is a summary.

II. FORMALISM

The Hamiltonian of the system in the absence of impurities is given by

$$H = H_{\rm BCS} + H_{\rm ex} , \tag{2.1}$$

with

$$H_{\rm BCS} = \sum_{\mathbf{k},\alpha} \varepsilon_{\mathbf{k}} C^\dagger_{\mathbf{k},\alpha} C_{\mathbf{k},\alpha} - \Delta \sum_{\mathbf{k}} (C^\dagger_{\mathbf{k},\uparrow} C^\dagger_{-\mathbf{k},\downarrow} + \text{H.c.}) , \tag{2.2}$$

$$H_{\rm ex} = -\frac{I}{N} |g_J - 1| \sum_{\substack{\mathbf{k},\mathbf{k}' \\ i,\mu,\nu}} \mathbf{J}_i \cdot \sigma_{\mu,\nu} C^\dagger_{\mathbf{k},\mu} C_{\mathbf{k}',\nu} \times \exp[i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{R}_i] . \tag{2.3}$$

Here $H_{\rm BCS}$ is the BCS Hamiltonian, which is responsible for the occurrence of superconductivity, and $H_{\rm ex}$ describes the exchange interaction between the conduction electrons and the rare-earth ions. Further, $\varepsilon_{\mathbf{k}}$ is the single-particle energy measured from the Fermi surface, $C^\dagger_{\mathbf{k},\alpha}$ is the creation operator for the conduction electron, α, μ, and ν are spin indices, Δ is the superconducting order parameter, $\mathbf{J}_i$ is the total angular momentum operator of a rare-earth ion at site $\mathbf{R}_i$, I is the exchange interaction constant, g_J is the Lande's g factor, σ is the conduction electron spin operator.

The $H_{\rm ex}$ is rewritten as

$$H_{\rm ex} = H^{\rm MF}_{\rm ex} + (H_{\rm ex} - H^{\rm MF}_{\rm ex}) = H^{\rm MF}_{\rm ex} + H^{\rm fluc} . \tag{2.4}$$

Here $H^{\rm MF}_{\rm ex}$ represents the mean-field approximation to $H_{\rm ex}$:

$$H^{\rm MF}_{\rm ex} = -H_Q \sum_{\mathbf{k},\sigma} \sigma (C^\dagger_{\mathbf{k}+\mathbf{Q},\sigma} C_{\mathbf{k},\sigma} + \text{H.c.}) , \tag{2.5}$$

where $\sigma = \pm 1$ corresponds to the electron spin up and down, H_Q is the AF molecular field, $\mathbf{Q}$ is the wave vector characterizing the AF order. The quantity $H^{\rm fluc}$ describes the scattering of a conduction electron from the spin fluctuations and will be considered later. The Hamiltonian describing the interaction of conduction electrons with impurities is

$$H_{\rm imp} = \frac{1}{N} \sum_{\mathbf{k},\mathbf{k}'} \sum_{\eta,\mu,\nu} \hat{U}^{\eta}(\mathbf{k},\mathbf{k}')_{\mu,\nu} C^\dagger_{\mathbf{k},\mu} C_{\mathbf{k}',\nu} \times \exp[i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{R}^{\eta}_j] . \tag{2.6}$$

The index η refers to the nature of the impurity. For $\eta = 1,2,3$, one has nonmagnetic, magnetic, and spin-orbit impurity, respectively. That is

$$\hat{U}^{1}(\mathbf{k},\mathbf{k}') = U_1(\mathbf{k}-\mathbf{k}') , \tag{2.7a}$$

$$\hat{U}^{2}(\mathbf{k},\mathbf{k}') = U_2(\mathbf{k}-\mathbf{k}')(g^i_J - 1)\mathbf{J}^i \cdot \sigma , \tag{2.7b}$$

$$\hat{U}^{3}(\mathbf{k},\mathbf{k}') = U_{\rm so}(\mathbf{k}-\mathbf{k}') i \frac{(\mathbf{k}\times\mathbf{k}')}{k_F^2} \cdot \sigma . \tag{2.7c}$$

Here g^i_J and $\mathbf{J}^i$ refer to the case of impurity and k_F is the Fermi wave vector.

The superconducting order parameter Δ coming in Eq. (2.2) is determined self-consistently by

$$\Delta = g \sum_{\mathbf{k}} \langle C_{-\mathbf{k},\downarrow} C_{\mathbf{k},\uparrow} \rangle , \tag{2.8}$$

where g is the BCS coupling constant and the angular brackets denote the thermal average. We will take Δ as real.

We introduce the finite-temperature Green's function

$$G_{\mathbf{k},\mathbf{k}'}(\tau) = -\langle T_\tau [\psi_{\mathbf{k}}(\tau) \psi^\dagger_{\mathbf{k}'}(0)] \rangle \tag{2.9}$$

having an eight-dimensional base with

$$\psi^\dagger_{\mathbf{k}} = (C^\dagger_{\mathbf{k},\uparrow}, C^\dagger_{-\mathbf{k},\downarrow}, C_{\mathbf{k},\uparrow}, C_{-\mathbf{k},\downarrow} , C^\dagger_{\mathbf{k}+\mathbf{Q},\uparrow}, C^\dagger_{-\mathbf{k}-\mathbf{Q},\downarrow}, C_{\mathbf{k}+\mathbf{Q},\uparrow}, C_{-\mathbf{k}-\mathbf{Q},\downarrow}) \tag{2.10}$$

and T_τ as the ordering operator for the imaginary time τ.

In the absence of impurities and within the mean-field approximation, the Green's function is given by

$$G^0_{\mathbf{k}}(i\omega_n) = (i\omega_n - \varepsilon_s \rho_3 - \varepsilon_a \tau_3 \rho_3 - \Delta \rho_2 \sigma_2 + H_Q \tau_1 \rho_3 \sigma_3)^{-1} , \tag{2.11}$$

with

$$\varepsilon_s = \tfrac{1}{2}(\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+\mathbf{Q}}) , \tag{2.12}$$

$$\varepsilon_a = \tfrac{1}{2}(\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{Q}}) , \tag{2.13}$$

where ω_n is the Matsubara frequency [i.e., $\omega_n = \pi T(2n+1)$, with T as temperature and n as an integer]. Further σ_i, ρ_i, and τ_i ($i=1,2,3$) are Pauli matrices operating on the ordinary spin states, the electron-hole states, and the positive- and negative-momentum states, respectively.

In order to write the Green's function in the presence of impurities, we assume that the impurities are randomly distributed and that their concentration is low enough so that the impurity-impurity interaction is negligible. If a general three-dimensional electron band is taken, the self-energy (averaged over the impurity positions and their spin directions) can be evaluated only numerically. However, the calculations can be done analytically if one takes a one-dimensional electron band that satisfies the nesting condition $\varepsilon_{\mathbf{k}}=-\varepsilon_{\mathbf{k}+\mathbf{Q}}$, that is, $\varepsilon_s=0$ and $\varepsilon_a=\varepsilon_{\mathbf{k}}$. We make this assumption in order to bring out the essential new results of the present study. Then, within the self-consistent Born approximation, we have

$$G_k(i\omega_n)=(i\tilde{\omega}_n-\varepsilon_k\tau_3\rho_3-\tilde{\Delta}_n\rho_2\sigma_2+i\tilde{\Omega}_n\tau_1\rho_1\sigma_1+\tilde{H}_{Qn}\tau_1\rho_3\sigma_3)^{-1}$$

$$=-\frac{1}{2}\left\{\frac{1}{K_{n+}}[i\tilde{\omega}_{n+}(1+\tau_1\rho_1\sigma_1)+\varepsilon_k(\tau_3\rho_3-\tau_2\rho_2\sigma_1)+\tilde{\Delta}_{n+}(\rho_2\sigma_2-\tau_1\rho_3\sigma_3)]\right.$$
$$\left.+\frac{1}{K_{n-}}[i\tilde{\omega}_{n-}(1-\tau_1\rho_1\sigma_1)+\varepsilon_k(\tau_3\rho_3+\tau_2\rho_2\sigma_1)+\tilde{\Delta}_{n-}(\rho_2\sigma_2+\tau_1\rho_3\sigma_3)]\right\}, \tag{2.14}$$

$$K_{n\pm}=\tilde{\omega}^2_{n\pm}+\varepsilon_k^2+\tilde{\Delta}^2_{n\pm}, \tag{2.15}$$

$$\tilde{\omega}_{n\pm}=\tilde{\omega}_n\pm\tilde{\Omega}_n, \tag{2.16}$$

$$\tilde{\Delta}_{n\pm}=\tilde{\Delta}_n\pm\tilde{H}_{Qn}. \tag{2.17}$$

The quantities $\tilde{\omega}_{n\pm}$ and $\tilde{\Delta}_{n\pm}$ are determined by

$$\tilde{\omega}_{n\pm}=\omega_n+Y_{\mp}\frac{\tilde{\omega}_{n+}}{2\lambda_+}+Y_{\pm}\frac{\tilde{\omega}_{n-}}{2\lambda_-}, \tag{2.18}$$

$$\tilde{\Delta}_{n\pm}=\Delta\pm H_Q+X_{\mp}\frac{\tilde{\Delta}_{n+}}{2\lambda_+}+X_{\pm}\frac{\tilde{\Delta}_{n-}}{2\lambda_-}, \tag{2.19}$$

$$\lambda_{\pm}=(\tilde{\omega}^2_{n\pm}+\tilde{\Delta}^2_{n\pm})^{1/2}, \tag{2.20}$$

$$X_{\pm}=g_2\pm g_3, \tag{2.21}$$

$$Y_{\pm}=g_1\pm g_4, \tag{2.22}$$

$$g_1=\frac{1}{2}\left[\frac{1}{\tau_1}+\frac{1}{\tau_{so}}+\frac{1}{\tau_2^i}\right], \tag{2.23}$$

$$g_2=\frac{1}{2}\left[\frac{1}{\tau_1}+\frac{1}{\tau_{so}}-\frac{1}{\tau_2^i}\right], \tag{2.24}$$

$$g_3=\frac{1}{2}\left[\frac{1}{\tau_1}+\frac{1}{3\tau_{so}}-\frac{1}{3\tau_2^i}\right], \tag{2.25}$$

$$g_4=\frac{1}{2}\left[\frac{1}{\tau_1}+\frac{1}{3\tau_{so}}+\frac{1}{3\tau_2^i}\right], \tag{2.26}$$

$$\frac{1}{\tau_1}=2\pi n_1N(0)\int\frac{d\Omega}{4\pi}|U_1(\mathbf{k}-\mathbf{k}')|^2, \tag{2.27}$$

$$\frac{1}{\tau_2^i}=2\pi n_2N(0)J^i(J^i+1)(g_J^i-1)^2\times\int\frac{d\Omega}{4\pi}|U_2(\mathbf{k}-\mathbf{k}')|^2, \tag{2.28}$$

$$\frac{1}{\tau_{so}}=2\pi n_3N(0)\int\frac{d\Omega}{4\pi}|U_{so}(\theta)|^2\sin^2(\theta), \tag{2.29}$$

where n_1, n_2, and n_3 are the concentration of nonmagnetic, magnetic, and spin-orbit impurities, respectively, and $1/\tau_1$, $1/\tau_2^i$, and $1/\tau_{so}$ are the scattering rates for scattering of conduction electrons from these impurities. Further, $N(0)$ is the density of single-particle states at the Fermi level in normal metal for single spin.

Defining $U_{n\pm}=\tilde{\omega}_{n\pm}/\tilde{\Delta}_{n\pm}$, one can combine Eqs. (2.18) and (2.19) to give

$$\omega_n=(\Delta\pm H_Q)U_{n\pm}+(X_{\mp}U_{n\pm}-Y_{\mp}U_{n+})\frac{1}{2(U^2_{n+}+1)^{1/2}}+\operatorname{sgn}(\tilde{\Delta}_{n-})(X_{\pm}U_{n\pm}-Y_{\pm}U_{n-})\frac{1}{2(U^2_{n-}+1)^{1/2}}, \tag{2.30}$$

with

$$\operatorname{sgn}(\tilde{\Delta}_{n-})=\begin{cases}\operatorname{sgn}(U_{n-}) & \text{for } \omega_n\geq 0,\\ -\operatorname{sgn}(U_{n-}) & \text{for } \omega_n<0,\end{cases} \tag{2.31a}$$

$$U_{n\pm}(-\omega_n)=-U_{n\pm}(\omega_n). \tag{2.31b}$$

The order-parameter equation is written by following standard procedure.[21] We have

$$\ln\frac{T}{T_{c0}}=\pi T\sum_{n=0}^{\infty}\left[\frac{1}{\Delta}\left\{\frac{1}{(U^2_{n+}+1)^{1/2}}+\frac{\operatorname{sgn}(U_{n-})}{(U^2_{n-}+1)^{1/2}}\right\}-\frac{2}{\omega_n}\right], \tag{2.32}$$

with $T_{c0}=(2\gamma\omega_D/\pi)\exp[-1/gN(0)]$. Here ω_D is the Debye cutoff frequency, $\ln\gamma$ is Euler's constant (=0.57721 . . .), and T_{c0} is the transition temperature of a superconductor with $H_Q=0$ and having no impurities.

III. THE ELECTRON-SPIN SUSCEPTIBILITY

A. General expression

Here we calculate the general expression for the longitudinal electron-spin susceptibility for an antiferromagnetic superconductor. We have

$$\chi_s(\omega) \equiv \chi_s^{zz}(\omega) = \chi_s^{zz}(i\omega_n)\,|_{i\omega_n=\omega+i\delta} , \tag{3.1}$$

$$\chi_s^{zz}(i\omega_n) = \int_0^{1/T} d\tau \langle T_\tau m_z(\tau) m_z(0)\rangle e^{i\omega_n \tau} , \tag{3.2}$$

with $\delta=0^+$ and m_z as the magnetization given by

$$m_z = \mu_B \sum_{\mathbf{k},\sigma} \sigma C_{\mathbf{k},\sigma}^\dagger C_{\mathbf{k},\sigma} , \tag{3.3}$$

where μ_B is the Bohr magnetron. Using Eqs. (3.2) and (3.3) in Eq. (3.1) we obtain

$$\chi_s = \chi_s(0) = \mu_B^2 \int_0^{1/T} d\tau \sum_{\mathbf{k},\mathbf{k}',\sigma,\sigma'} \sigma\sigma' \langle T_\tau C_{\mathbf{k},\sigma}^\dagger(\tau) C_{\mathbf{k},\sigma}(\tau) C_{\mathbf{k}',\sigma'}^\dagger(0) C_{\mathbf{k}',\sigma'}(0)\rangle . \tag{3.4}$$

In order to proceed further, it is convenient to introduce a 4×4 Green's function matrix

$$g_{\mathbf{k},\mathbf{k}'}(\tau) = -\langle T_\tau[\phi_{\mathbf{k}}(\tau)\phi_{\mathbf{k}'}^\dagger(0)]\rangle , \tag{3.5}$$

with

$$\phi_{\mathbf{k}}^\dagger = (C_{\mathbf{k},\uparrow}^\dagger, C_{-\mathbf{k},\downarrow}^\dagger C_{\mathbf{k},\uparrow} C_{-\mathbf{k},\downarrow}) . \tag{3.6}$$

Now χ_s involves the thermal average of the product of four electron operators. This average can be decomposed in terms of the product of above Green's function. In the presence of impurity scattering, one obtains

$$\chi_s = -2\mu_B^2 T \sum_{\mathbf{k}} \sum_{\omega_n} \tfrac{1}{4}\mathrm{Tr}[\rho_3\sigma_3 g_{\mathbf{k}}(i\omega_n)(\rho_3\sigma_3)_r g_{\mathbf{k}}(i\omega_n)] , \tag{3.7}$$

where $g_{\mathbf{k}}(i\omega_n)$ is now the impurity averaged Green's function and $(\rho_3\sigma_3)_r$ represents the renormalized vertex function given by

$$(\rho_3\sigma_3)_r - \rho_3\sigma_3 = \sum_\eta n_\eta \sum_{\mathbf{k}'} \hat{U}^\eta(\mathbf{k},\mathbf{k}') g_{\mathbf{k}'}(i\omega_n)(\rho_3\sigma_3)_r \times g_{\mathbf{k}'}(i\omega_n)\hat{U}^\eta(\mathbf{k}',\mathbf{k}) , \tag{3.8}$$

where η refers to the nature of the impurity and n_η is the corresponding impurity concentration. For $\eta=1,2,3$, one has nonmagnetic, magnetic and spin-orbit impurity, respectively. In 4×4 notation

$$\begin{aligned}
&\hat{U}^1 = U_1(\mathbf{k}-\mathbf{k}')\rho_3 , \\
&\hat{U}^2 = U_2(\mathbf{k}-\mathbf{k}')(g_J^i - 1)\mathbf{J}^i\cdot\boldsymbol{\alpha} , \\
&\hat{U}^3 = U_{\mathrm{so}}(\mathbf{k}-\mathbf{k}') i \frac{(\mathbf{k}\times\mathbf{k}')}{k_F^2}\cdot\boldsymbol{\alpha}\rho_3 , \\
&\boldsymbol{\alpha} = \tfrac{1}{2}(1+\rho_3)\boldsymbol{\sigma} + \tfrac{1}{2}(1-\rho_3)\sigma_2\boldsymbol{\sigma}\sigma_2 .
\end{aligned} \tag{3.9}$$

The Green's function $g_{\mathbf{k}}(i\omega_n)$ is a subset of the Green's function introduced in Sec. II. It is easily obtained from Eq. (2.14) and we have

$$g_{\mathbf{k}}(i\omega_n) = -\frac{1}{2}\left[\frac{1}{K_{n+}}(i\tilde{\omega}_{n+} + \varepsilon_k\rho_3 + \tilde{\Delta}_{n+}\rho_2\sigma_2) + \frac{1}{K_{n-}}(i\tilde{\omega}_{n-} + \varepsilon_k\rho_3 + \tilde{\Delta}_{n-}\rho_2\sigma_2)\right] . \tag{3.10}$$

Now we proceed to calculate $(\rho_3\sigma_3)_r$ from Eq. (3.8). Evaluating the right-hand side by putting $(\rho_3\sigma_3)_r = \rho_3\sigma_3$ and using Eqs. (3.9) and (3.10) we find that $(\rho_3\sigma_3)_r$ has the matrix form

$$(\rho_3\sigma_3)_r = F\rho_3\sigma_3 + iE\rho_1\sigma_1 . \tag{3.11}$$

Substituting this form in Eq. (3.8), we obtain coupled equations for F and E as

$$F = 1 + B_1 F + B_2 E , \tag{3.12}$$

$$E = B_3 E + B_4 F , \tag{3.13}$$

where

$$B_1 = \tfrac{1}{4}(H_1 - H_2)\left[\frac{1}{2\varepsilon_1}\eta_+ + \frac{1}{2\varepsilon_2}\eta_- + \frac{1}{\varepsilon_1+\varepsilon_2}[1 + (1 - U_{n+}U_{n-})\eta_+^{1/2}\eta_-^{1/2} I_{\mathrm{sn}}]\right] , \tag{3.14}$$

$$B_2 = \tfrac{1}{4}(H_1 - H_2)\left[\frac{U_{n+}\eta_+}{2\varepsilon_1} + \frac{U_{n-}\eta_-}{2\varepsilon_2} + \frac{(U_{n+} + U_{n-})}{\varepsilon_1+\varepsilon_2}\eta_+^{1/2}\eta_-^{1/2} I_{\mathrm{sn}}\right] , \tag{3.15}$$

$$B_3 = \tfrac{1}{4}(H_1 + H_2)\left[\frac{U_{n+}^2\eta_+}{2\varepsilon_1} + \frac{U_{n-}^2\eta_-}{2\varepsilon_2} + \frac{1}{\varepsilon_1+\varepsilon_2}[1 - (1 - U_{n+}U_{n-})\eta_+^{1/2}\eta_-^{1/2} I_{\mathrm{sn}}]\right] , \tag{3.16}$$

$$B_4 = \frac{H_1 + H_2}{H_1 - H_2} B_2 . \tag{3.17}$$

In the above equations

$$H_1=\frac{1}{\tau_1}-\frac{1}{3\tau_{so}}\ , \qquad (3.18a)$$

$$H_2=\frac{1}{3\tau_2^i}\ , \qquad (3.18b)$$

$$\eta_\pm=(1+U_{n\pm}^2)^{-1}\ , \qquad (3.18c)$$

$$I_{sn}=\mathrm{sgn}(\tilde{\Delta}_{n-})\ , \qquad (3.18d)$$

$$\varepsilon_1=|(\tilde{\omega}_{n+}^2+\tilde{\Delta}_{n+}^2)^{1/2}|\ , \qquad (3.18e)$$

$$\varepsilon_2=|(\tilde{\omega}_{n-}^2+\tilde{\Delta}_{n-}^2)^{1/2}|\ . \qquad (3.18f)$$

Using Eqs. (3.12)–(3.18) we obtain

$$F=\left[1-\tfrac{1}{4}(H_1+H_2)\left[\frac{U_{n+}^2\eta_+}{2\varepsilon_1}+\frac{U_{n-}^2\eta_-}{2\varepsilon_2}+\frac{1}{\varepsilon_1+\varepsilon_2}[1-(1-U_{n+}U_{n-})\times\eta_+^{1/2}\eta_-^{1/2}I_{sn}]\right]\right]P\ , \qquad (3.19)$$

$$E=\tfrac{1}{4}(H_1+H_2)\left[\frac{U_{n+}\eta_+}{2\varepsilon_1}+\frac{U_{n-}\eta_-}{2\varepsilon_2}+\frac{U_{n+}+U_{n-}}{\varepsilon_1+\varepsilon_2}\eta_+^{1/2}\eta_-^{1/2}I_{sn}\right]P\ , \qquad (3.20)$$

where

$$P=[1-\tfrac{1}{4}(H_1D_1+H_2D_2)+(H_1^2-H_2^2)D_3]^{-1}\ , \qquad (3.21)$$

$$D_1=\frac{1}{2\varepsilon_1}+\frac{1}{2\varepsilon_2}+\frac{2}{\varepsilon_1+\varepsilon_2}\ , \qquad (3.22a)$$

$$D_2=\frac{\eta_+}{2\varepsilon_1}(U_{n+}^2-1)+\frac{\eta_-}{2\varepsilon_2}(U_{n-}^2-1)-\frac{2(1-U_{n+}U_{n-})}{\varepsilon_1+\varepsilon_2}\eta_+^{1/2}\eta_-^{1/2}I_{sn}\ , \qquad (3.22b)$$

$$D_3=\frac{1}{64\epsilon_1\epsilon_2}\{(U_{n+}-U_{n-})^2\eta_+\eta_-+2[1-(1+U_{n+}U_{n-})\eta_+^{1/2}\eta_-^{1/2}I_{sn}]\}\ . \qquad (3.22c)$$

Using Eqs. (3.10) and (3.11) in Eq. (3.7) and performing the momentum summation using the standard procedure, we obtain

$$\frac{\chi_s}{\chi_n^0}=1-\frac{\pi T}{2}\sum_{\omega_n}[D_4-4(H_1+H_2)D_3]P\ , \qquad (3.23)$$

where $\chi_n^0=2\mu_B^2N(0)$ and

$$D_4=\frac{\eta_+}{2\varepsilon_1}+\frac{\eta_-}{2\varepsilon_2}+\frac{1}{\varepsilon_1+\varepsilon_2}[1+(1-U_{n+}U_{n-})\eta_+^{1/2}\eta_-^{1/2}I_{sn}]\ . \qquad (3.24)$$

The quantities ε_1 and ε_2 are given by Eqs. (3.18e) and (3.18f) and can be rewritten by using Eq. (2.19). We have

$$\varepsilon_1=\left|(\Delta+H_Q)\eta_+^{-1/2}+\frac{X_-}{2}+\frac{I_{sn}}{2}X_+\eta_+^{-1/2}\eta_-^{1/2}\right|\ , \qquad (3.25)$$

$$\varepsilon_2=\left|(\Delta-H_Q)\eta_-^{-1/2}+\frac{I_{sn}}{2}X_-+\frac{X_+}{2}\eta_+^{1/2}\eta_-^{-1/2}\right|\ . \qquad (3.26)$$

Using Eqs. (2.31), one sees that ε_1, ε_2, D_1, D_2, D_3, D_4, and P are symmetric under the interchange $\omega_n\to-\omega_n$. Then Eq. (3.23) is rewritten as

$$\frac{\chi_s}{\chi_n^0}=1-\pi T\sum_{n\geq 0}[D_4-4(H_1+H_2)D_3]P\ . \qquad (3.27)$$

B. Limiting case: $H_Q=0$

In this case, $U_{n+}=U_{n-}=\tilde{\omega}_n/\tilde{\Delta}_n$, $\eta_+=\eta_-=(U_n^2+1)^{-1}=\eta$, $I_{sn}=1$, $D_1=2/\varepsilon$, $D_2=2\eta(U_n^2-1)/\varepsilon$, $D_3=0$, $D_4=2\eta/\varepsilon$, and

$$\varepsilon_1=\varepsilon_2=\varepsilon=\Delta\eta^{-1/2}+\frac{1}{2}\left[\frac{1}{\tau_1}+\frac{1}{\tau_{so}}-\frac{1}{\tau_2^i}\right]\ . \qquad (3.28)$$

Further,

$$P=\left[1-\frac{1}{2\varepsilon}\left[\frac{1}{\tau_1}-\frac{1}{3\tau_{so}}+\frac{1}{3\tau_2^i}(U_n^2-1)\eta\right]\right]^{-1}\ . \qquad (3.29)$$

Using these values in Eq. (3.27) we obtain

$$\frac{\chi_s}{\chi_n^0}=1-2\pi T\sum_{n\geq 0}\frac{1}{1+U_n^2}\left[\Delta\left[(U_n^2+1)^{1/2}-\frac{1}{3\tau_2^i\Delta}\frac{2U_n^2+1}{U_n^2+1}\right]+\frac{2}{3\tau_{so}}\right]^{-1}\ , \qquad (3.30)$$

which agrees with Eq. (132) of Ref. 32. For $1/\tau_2^i=0$ and $1/\tau_{so}=0$, respectively, above equation was derived in Refs. 27 and 28.

IV. MODEL FOR SPIN-FLUCTUATION EFFECTS

In Ref. 22 we explained the enhancement in the superconducting order parameter and the maximum Josephson current by the AF ordering observed in $SmRh_4B_4$ by including the spin fluctuations as described below. The same model will be used in the present study. We take $T_c>T_N$ and our purpose is to see if the electron spin susceptibility is enhanced or depressed by the AF ordering occurring below T_N. As the inelastic scattering from spin fluctuations is relevant only for T far below T_N, we include only the elastic scattering with the scattering rate obtained as

$$\frac{1}{\tau_2(T)}=\begin{cases}\dfrac{1}{\tau_2^R}, & T>T_N\ ,\\[2ex] \dfrac{1}{\tau_2^R}\dfrac{(\langle \mathbf{J}^2\rangle-\langle \mathbf{J}\rangle^2)}{J(J+1)}=\dfrac{1}{\tau_2^R}[1-F^2(T)], & T\le T_N\ .\end{cases} \tag{4.1}$$

In writing the above, we have used $|\langle \mathbf{J}\rangle| = |\langle \mathbf{J}_z\rangle| = [J(J-1)]^{1/2}F(T)$. Further,

$$\frac{1}{\tau_2^R}=2\pi n_R N(0)(g_J-1)^2I^2J(J+1)\ . \tag{4.2}$$

The function $F(T)$ also describes the temperature dependence of the staggered field with

$$H_Q(T)=H_Q(0)F(T)\ , \tag{4.3}$$

$$H_Q(0)=n_R I\,|g_J-1|\,[J(J+1)]^{1/2}\ . \tag{4.4}$$

$F(T)$ may be modeled as

$$F(T)=1-\left[\frac{T}{T_N}\right]^{\nu}\ , \tag{4.5}$$

and the parameter ν can be obtained from the experimental data. In above the quantity n_R is the concentration of rare-earth ions in the AFS and $1/\tau_2^R$ is the scattering rate of the conduction electrons from these ions for $T>T_N$. For $T<T_N$, the scattering rate becomes temperature dependent and its value decreases with the decrease in temperature (as the magnetic moments become more and more frozen).

In order to also include the effect of magnetic impurities in the system we introduce the effective magnetic scattering rate

$$\frac{1}{\tau_2^{\text{eff}}}=\frac{1}{\tau_2^i}+\frac{1}{\tau_2(T)}\ . \tag{4.6}$$

The effect of spin fluctuations in the calculation of the order parameter and the spin susceptibility can be easily included by replacing $1/\tau_2^i$ by $1/\tau_2^{\text{eff}}$ in the various equations derived in Secs. II and III.

V. NUMERICAL RESULTS

A. General

First we give our results by regarding H_Q as a parameter and neglecting the spin-fluctuation effects. Our aim is to study the dependence of the longitudinal electron-spin susceptibility χ_s on the scattering rate for scattering of conduction electrons from the nonmagnetic, spin-orbit and magnetic impurities. The dependence of χ_s on H_Q will also be given. As our purpose is to investigate the superconducting state, the dependence of normal state spin susceptibility on H_Q is not considered. The numerical procedure consists of two steps. First we calculate the order parameter Δ by solving Eqs. (2.30) and (2.32) self-consistently. Newton's method was used to solve Eq. (2.30). Then we find $U_{n\pm}$ again from Eq. (2.30) and calculate χ_s/χ_n^0 by using Eqs. (3.27), (3.18), (3.21), (3.22), and (3.24)–(3.26). Our results are shown in Figs. 1–4.

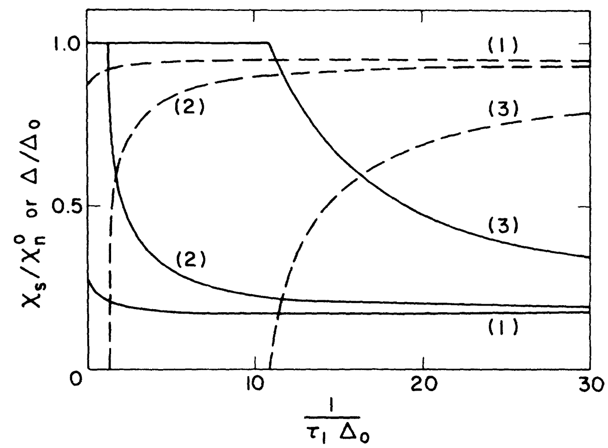

FIG. 1. The normalized longitudinal electron spin susceptibility χ_s/χ_n^0 (solid curve) and the normalized order parameter Δ/Δ_0 (dashed curve) as a function of $1/\tau_1\Delta_0$ for $T/T_{c0}=0.5$ and $H_Q/\Delta_0=0.3$ (1), 0.8 (2), and 2.0 (3) and with $1/\tau_{so}=1/\tau_2^i=0$. Here χ_n^0 and Δ_0, respectively, are the normal state spin susceptibility and zero-temperature order parameter for a BCS superconductor in the absence of impurities.

Figure 1 shows the dependence of χ_s (solid curve) and Δ (dashed curve) on $1/\tau_1\Delta_0$ for $T/T_{c0}=0.5$ and $H_Q/\Delta_0=0.3$, 0.8, and 2.0, and with $1/\tau_{so}=1/\tau_2^i=0$. Here T_{c0} and Δ_0, respectively, are the values of the transition temperature and the zero-temperature order parameter in the absence of H_Q and the impurities. We note that for $H_Q/\Delta_0=0.3$, one has superconductivity when $1/\tau_1=0$. With the increase in $1/\tau_1$, $\Delta(\chi_s)$ increases (decreases) initially and then saturates. For $H_Q/\Delta_0=0.8$, there is no superconductivity up to $1/\tau_1\Delta_0\sim 1.4$. Increas-

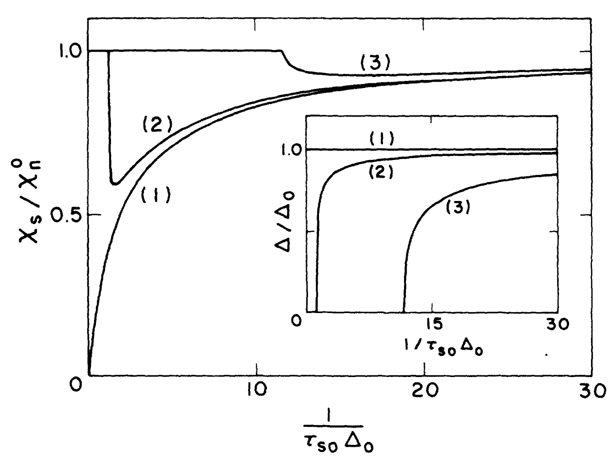

FIG. 2. χ_s/χ_n^0 as a function of $1/\tau_{so}\Delta_0$ with $T/T_{c0}=0.1$, $1/\tau_2^i=1/\tau_1=0$, and H_Q/Δ_0 fixed at 0 (1), 0.8 (2), and 2.0 (3). In the insert, Δ/Δ_0 is plotted vs $1/\tau_{so}\Delta_0$ for the same set of parameters.

ing the scattering rate, the quantity $\Delta(\chi_s)$ increases (decreases) sharply and then saturates. For $H_Q/\Delta_0=2.0$, the superconductivity starts near $1/\tau_1\Delta_0 \sim 10.8$. Increasing the scattering rate the behavior of Δ and χ_s is similar to that for the case of $H_Q/\Delta_0=0.8$ except now the initial rise (fall) of $\Delta(\chi_s)$ is much slower. For a large fixed value of $1/\tau_1\Delta_0$, χ_s is more when H_Q is more. This can be understood by the fact that in the short mean-free-path limit, there is an effective pair breaking parameter given by[17] $H_Q^2\tau_1/\Delta_0$.

The effect of spin-orbit impurity scattering on χ_s is quite interesting and is shown in Fig. 2. We have taken $T/T_{c0}=0.1$, $1/\tau_1=1/\tau_2^i=0$, and $H_Q/\Delta_0=0$, 0.8, and 2.0. The dependence of Δ on $1/\tau_{so}\Delta_0$ for the same set of parameters is shown in the insert. The dependence of χ_s on $1/\tau_{so}\Delta_0$ for the case of $H_Q/\Delta_0=0$ (BCS superconductor) is already known in literature.[25] When $H_Q/\Delta_0=0.8$, χ_s curve exhibits a sharp minimum. The spin-orbit impurity scattering affects χ_s in two opposite ways. Firstly, it increases Δ from zero and this is responsible for the initial sharp drop in χ_s. Secondly, it increases χ_s by the process of spin flipping as it does in a BCS superconductor. The minimum is a result of the interplay of these two mechanisms. For $H_Q/\Delta_0=2.0$, the χ_s curve shows only a shallow minimum.

In Fig. 3, we show the dependence of χ_s and Δ on $1/\tau_2^i\Delta_0$. We have taken $1/\tau_{so}=0$, $T/T_{c0}=0.1$, $1/\tau_1\Delta_0=15$, and $H_Q/\Delta_0=0$, 0.8, and 2.0. One notes that χ_s increases as $1/\tau_2^i\Delta_0$ is increased from zero. This behavior is due to the pair breaking nature of the magnetic scattering. The sharp increase in χ_s is related to the sharp decrease in Δ.

In Fig. 4 we have shown the dependence of χ_s and Δ on H_Q/Δ_0 by taking $1/\tau_2^i=1/\tau_{so}=0$, $1/\tau_1\Delta_0=15$, and $T/T_{c0}=0.5$. At $H_Q=0$, χ_s has the BCS value. χ_s increases with an increase in H_Q until the superconductivity is destroyed by the molecular field. Comparing the two curves, one notes that the increase of χ_s results from the decrease of Δ. The increase (decrease) of $\chi_s(\Delta)$ is due to the fact that now H_Q and $1/\tau_1$ both contribute to pair breaking.

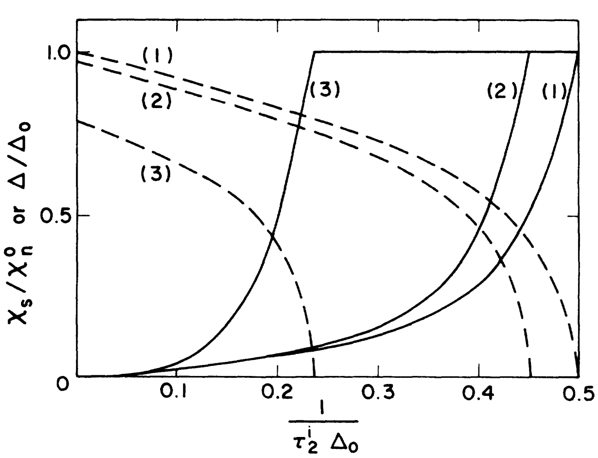

FIG. 3. χ_s/χ_n^0 (solid curve) and Δ/Δ_0 (dashed curve) as a function of $1/\tau_2^i\Delta_0$ with $T/T_{c0}=0.1$, $1/\tau_1\Delta_0=15$, $1/\tau_{so}=0$, and H_Q/Δ_0 fixed at 0 (1), 0.8 (2), and 2.0 (3).

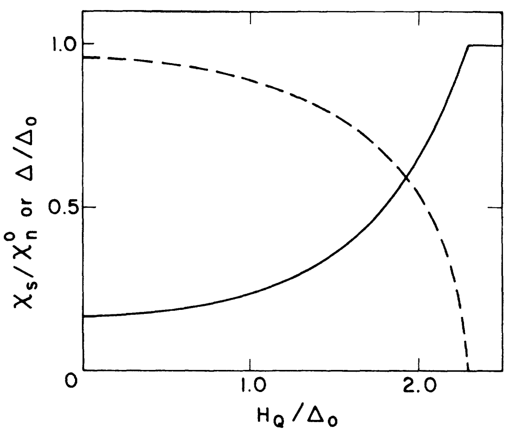

FIG. 4. χ_s/χ_n^0 (solid curve) and Δ/Δ_0 (dashed curve) as a function of H_Q/Δ_0 with $T/T_{c0}=0.5$, $1/\tau_1\Delta_0=15$, $1/\tau_2^i=1/\tau_{so}=0$.

B. Temperature dependence of χ_s

Here we give our results by including the spin-fluctuation effects and also by including the temperature dependence of the staggered field. In Fig. 5, χ_s is plotted as a function of temperature. We have taken $1/\tau_2^{\text{eff}}\Delta_0=0.45$ and $T_N/T_{c0}=0.1$. For temperatures between T_N and T_c (paramagnetic phase: $H_Q=0$), Eqs. (2.30) and (2.32) reduce to the AG equations[32]

$$\frac{\omega_n}{\Delta}=U_n\left[1-\frac{1}{\tau_2^{\text{eff}}\Delta}\frac{1}{(U_n^2+1)^{1/2}}\right], \tag{5.1}$$

$$\ln\left[\frac{T}{T_{c0}}\right]=2\pi T\sum_{n=0}^{\infty}\left[\frac{1}{\Delta}\frac{1}{(U_n^2+1)^{1/2}}-\frac{1}{\omega_n}\right]. \tag{5.2}$$

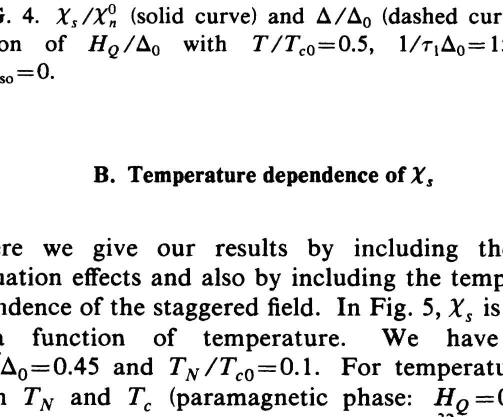

FIG. 5. χ_s/χ_n^0 as a function of T/T_{c0}. We have taken $1/\tau_2^{\text{eff}}\Delta_0=0.45$ and $T_N/T_{c0}=0.1$. Above value of $1/\tau_2^{\text{eff}}\Delta_0$ corresponds to $T_c/T_{c0}=0.24$. For curves (1)–(3), $1/\tau_2^i=0$, $1/\tau_1\Delta_0=10$, $H_Q(0)/\Delta_0=2.0$, and $1/\tau_{so}\Delta_0=0.5$, 2.0, and 5.0, respectively. Curve (4) has same set of parameters as curve (2) except that now $H_Q(0)/\Delta_0=2.4$. Curve (5) has $1/\tau_2^R\Delta_0=0.35$ and $1/\tau_2^i\Delta_0=0.10$ and the other parameters are same as for curve (2). Here $1/\tau_2^{\text{eff}}$ is effective magnetic scattering rate defined in Eq. (4.6) and $H_Q(0)$ is the value of the staggered field at $T=0$.

When $\Delta \to 0$, the above equations lead to AG T_c equation

$$\ln\left[\frac{T_c}{T_{c0}}\right]=\psi(\tfrac{1}{2})-\psi\left[\frac{1}{2}+\frac{1}{2\pi T_c \tau_2^{\text{eff}}}\right], \tag{5.3}$$

where $\psi(z)$ is the digamma function and $1/\tau_2^{\text{eff}}$ is the effective magnetic scattering rate defined in Eq. (4.6). In this temperature range, Δ and T_c do not depend on $1/\tau_1$ and $1/\tau_{\text{so}}$. Equation (5.3) gives $T_c/T_{c0}=0.24$ when $1/\tau_2^{\text{eff}}\Delta_0=0.45$. We take $1/\tau_2^i=0$ so that $1/\tau_2^R\Delta_0=0.45$. The susceptibility χ_s/χ_n^0 is obtained from Eq. (3.30) with $1/\tau_2^i$ replaced by $1/\tau_2^{\text{eff}}$. Curves (1)–(3) correspond to above value of $1/\tau_2^{\text{eff}}\Delta_0$ and $1/\tau_{\text{so}}\Delta_0=0.5$, 2.0, and 5.0, respectively. For the antiferromagnetic phase ($T \le T_N$), Eqs. (2.30)–(2.32), (3.18), (3.21), (3.22), and (3.24)–(3.27) must be used with $1/\tau_2^i$ replaced by $1/\tau_2^{\text{eff}}$. Now $1/\tau_2^{\text{eff}}$ and the staggered field are temperature dependent as given in Sec. IV and the values of $1/\tau_1$ and $H_Q(0)$ are also relevant. We have taken $1/\tau_1\Delta_0=10$, $H_Q(0)/\Delta_0=2.0$, and $\nu=4$ for curves (1)–(3). From these curves, we see that χ_s increases with the increase of $1/\tau_{\text{so}}$ both above and below T_N. Curve (4) has the same set of parameters as curve (2) except that now $H_Q(0)/\Delta_0=2.4$. Comparing these two curves, one notes that whether χ_s is enhanced or depressed by the AF ordering depends on $H_Q(0)$. χ_s is enhanced (depressed) when $H_Q(0)$ is larger (smaller). For curve (5) we have $1/\tau_2^R\Delta_0=0.35$ and $1/\tau_2^i\Delta_0=0.10$ and the other parameters are the same as for curve (2). One observes that in the AF phase, the χ_s values given by curve (5) are larger than given by curve (2). This happens because now a part of the effective magnetic scattering is coming from the magnetic impurities whose pair breaking effect is not suppressed in the AF phase.

In Fig. 6, we show the dramatic influence of nonmagnetic impurity scattering rate on the temperature dependence of χ_s in the AF phase. We have taken $1/\tau_2^i=0$, $1/\tau_2^R\Delta_0=0.4$ which gives $T_c/T_{c0}=0.355$. Further, $T_N/T_c=0.5$, $H_Q(0)/\Delta_0=2.0$, $\nu=4.0$, $1/\tau_{\text{so}}\Delta_0=0.2$. Defining $1/\tau=1/\tau_1+2/3\tau_{\text{so}}$, the quantity $1/\tau\Delta_0=7.5$, 8.0, 12.0, and 30.0 for curves (1)–(4), respectively. The cleanest superconductor corresponds to curve (1) and the dirtiest corresponds to curve (4). The curve marked AG is the extension of the paramagnetic phase curve into the AF region. We note that for the cleaner superconductors [curves (1) and (2)] the susceptibility χ_s is enhanced by the AF ordering (with respect to the AG curve), whereas for the dirtier ones [curves (3) and (4)] there is a depression in χ_s below T_N. The order parameter Δ used to calculate χ_s was evaluated by us in Ref. 22, where it is shown as Fig. 1. It may be observed that the enhancement (depression) in χ_s below T_N happens in those cases where Δ is depressed (enhanced). In $SmRh_4B_4$, Δ is enhanced by the AF ordering. Thus, for this material, one expects a depression in χ_s below T_N.

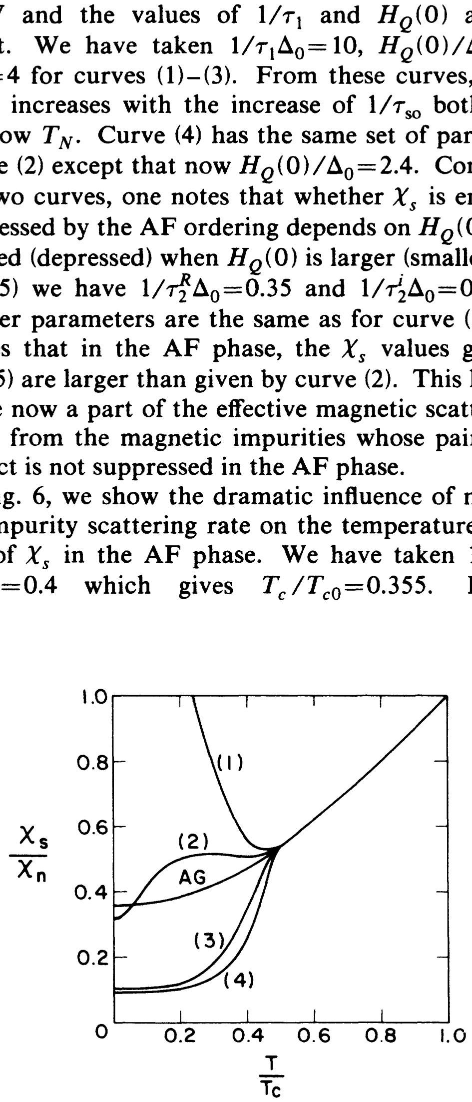

FIG. 6. χ_s/χ_n^0 as a function of T/T_c. We have taken $1/\tau_2^i=0$, $1/\tau_2^R\Delta_0=0.4$ (then $T_c/T_{c0}=0.355$), $T_N/T_c=0.5$, $1/\tau_{\text{so}}\Delta_0=0.2$, $H_Q(0)/\Delta_0=2.0$, $\nu=4$. Defining $1/\tau=1/\tau_1+2/3\tau_{\text{so}}$, the quantity $1/\tau\Delta_0=7.5$, 8.0, 12.0, and 30.0 for curves (1)–(4), respectively. The curve marked AG is the extension of the paramagnetic phase curve into the AF region.

VI. SUMMARY

We have presented a theory of the longitudinal electron-spin susceptibility χ_s for an antiferromagnetic superconductor. In Sec. II, the single-particle Green's function for the conduction electrons has been written by treating the exchange interaction between the conduction electrons and the rare-earth ions within the mean-field approximation and by assuming a one-dimensional electron band that satisfies the nesting condition $\varepsilon_{\mathbf{k}}=-\varepsilon_{\mathbf{k}+\mathbf{Q}}$. The order-parameter equation is also given. The effect of impurities have been included in the formalism. In Sec. III, the general expression for χ_s has been derived and the limiting case when $H_Q=0$ is considered. Section IV described a model for including the spin-fluctuation effects. Numerical results have been given in Sec. V and are shown in Figs. 1–6.

Figures 1–4 give our results by regarding H_Q as a parameter and neglecting the spin-fluctuation effects. The effect of nonmagnetic impurities is shown in Fig. 1. The general result is that with the addition of nonmagnetic impurities, the value of χ_s (for $H_Q\neq 0$) is initially depressed and then saturates. Figure 2 shows the dependence of χ_s on the spin-orbit scattering rate $1/\tau_{\text{so}}$. The spin-orbit scattering influences the AFS in two ways. When $H_Q\neq 0$, it weakens the effect of the AF field which is responsible for the enhancement of Δ and the depression in χ_s. Secondly, it increases χ_s by the process of spin flipping. The interplay of these two mechanisms results in a minimum in the χ_s versus $1/\tau_{\text{so}}$ curve (when $H_Q>0.49\Delta_0$). Figure 3 shows the effect of magnetic impurities on χ_s. Here the pair breaking nature of the magnetic scattering dominates. The dependence of χ_s on H_Q is shown in Fig. 4. χ_s increases with the increase of H_Q. This increase is due to the fact that now H_Q and $1/\tau_1$ both contribute to pair breaking.

The temperature dependence of χ_s is shown in Figs. 5 and 6. Here the spin-fluctuation effects and the temperature dependence of the staggered field are included. We have taken $T_N<T_c$, and our aim is to see if χ_s is enhanced or depressed by the AF ordering occurring below T_N. In the paramagnetic phase ($T_N<T<T_c$), χ_s

depends on the scattering rates for scattering of conduction electrons from the spin-orbit and magnetic impurities and also from the magnetic rare-earth ions. Now U_n, Δ, and T_c are determined from Eqs. (5.1)–(5.3) and χ_s is obtained from Eq. (3.30) with $1/\tau_2^i$ replaced by $1/\tau_2^{\rm eff}$ defined in Eqs. (4.6). In the antiferromagnetic phase ($T \le T_N$), Eqs. (2.30)–(2.32), (3.18), (3.21), (3.22), and (3.24)–(3.27) must be used with $1/\tau_2^i$ replaced by $1/\tau_2^{\rm eff}$. Now $1/\tau_2^{\rm eff}$ and the staggered field are temperature dependent and the values of $1/\tau_1$ and $H_Q(0)$ are also relevant. The main results of Fig. 5 are that: (1) χ_s increases with the increase in $1/\tau_{\rm so}$ both above and below T_N; (2) The enhancement or depression of χ_s by the AF ordering depends on $H_Q(0)$—there is enhancement (depression) when $H_Q(0)$ is larger (smaller). Figure 6 shows that the nonmagnetic impurities have a dramatic effect on the temperature dependence of χ_s in the AF phase. For cleaner (dirtier) superconductors, χ_s is enhanced (depressed) by the AF ordering occurring below T_N. In $SmRh_4B_4$, one expects a depression in χ_s below T_N as Δ is enhanced by the AF ordering.

Our results regarding the dependence of χ_s on the concentration of nonmagnetic, spin-orbit, and magnetic impurities at a fixed temperature (Figs. 1–3), on the temperature dependence of χ_s (Figs. 5 and 6), and on the expected depression in χ_s below T_N for $SmRh_4B_4$ need verification in future experiments.

ACKNOWLEDGMENT

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1] *Ternary Superconductors,* edited by G. K. Shenoy, B. D. Dunlap, and F. Y. Fradin (North-Holland, Amsterdam, 1981).

[2] *Superconductivity in Ternary Compounds,* edited by Ø. Fischer and M. B. Maple (Springer, Berlin, 1982), Vol. II.

[3] K. Machida, Appl. Phys. A **35**, 193 (1984).

[4] K. N. Shrivastava and K. P. Sinha, Phys. Rep. **115**, 93 (1984).

[5] *Superconductivity in Magnetic and Exotic Materials,* edited by T. Matsubara and A. Kotani (Springer, Berlin, 1984), pp. 104-134.

[6] K. Levin, M. J. Nass, C. Ro, and G. S. Grest, in *Superconductivity in Magnetic and Exotic Materials,* edited by T. Matsubara and A. Kotani (Springer, Berlin, 1984), pp. 104–111.

[7] M. Ishikawa and Ø. Fisher, Solid State Commun. **24**, 747 (1977); W. A. Fertig, D. C. Johnston, L. E. DeLong, R. W. McCallum, M. B. Maple, and B. T. Matthias, Phys. Rev. Lett. **38**, 987 (1977).

[8] H. C. Ku, B. T. Matthias, and H. Bartz, Solid State Commun. **32**, 937 (1978); A. Thoma, H. Adrian, and A. Meinelt, J. Low Temp. Phys. **64**, 329 (1986).

[9] K. Machida, K. Nokura, and T. Matsubara, Phys. Rev. B **22**, 2307 (1980).

[10] G. Zwicknagl and P. Fulde, Z. Phys. B **43**, 23 (1981); J. Ashkenazi, D. G. Kuper, and A. Ron, Phys. Rev. B **28**, 468 (1983); J. Keller, J. Magn. Magn. Mater. **28**, 193 (1982).

[11] T. V. Ramakrishnan and C. M. Varma, Phys. Rev. B **24**, 137 (1981).

[12] M. J. Nass, K. Levin, and G. S. Grest, Phys. Rev. Lett. **46**, 614 (1981).

[13] M. J. Nass, K. Levin, and G. S. Grest, Phys. Rev. B **25**, 4541 (1982).

[14] E. W. Fenton, Solid State Commun. **54**, 633 (1985); **57**, 241 (1986).

[15] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[16] Y. Suzumura and A. D. S. Nagi, Phys. Rev. B **24**, 5103 (1981).

[17] Y. Okabe and A. D. S. Nagi, Phys. Rev. B **28**, 6290 (1983).

[18] C. Ro and K. Levin, Phys. Rev. B **29**, 6155 (1984).

[19] Y. Suzumura, Y. Okabe, and K. Ishino, Prog. Theor. Phys. **74**, 211 (1985).

[20] M. Prohammer and E. Schachinger, Solid State Commun. **58**, 491 (1986).

[21] H. Chi and A. D. S. Nagi, J. Low Temp. Phys. **67**, 475 (1987).

[22] H. Chi and A. D. S. Nagi, Solid State Commun. **65**, 885 (1988).

[23] R. Vaglio, B. D. Terris, J. F. Zasadzinski, and K. E. Gray, Phys. Rev. Lett. **53**, 1489 (1984).

[24] K. Yosida, Phys. Rev. **110**, 769 (1958).

[25] R. A. Ferrell, Phys. Rev. Lett. **3**, 262 (1959).

[26] P. W. Anderson, Phys. Rev. Lett. **3**, 325 (1959).

[27] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **42**, 1088 (1962) [Sov. Phys.—JETP **15**, 752 (1962)].

[28] L. P. Gor'kov and A. I. Rusinov, Zh. Eksp. Teor. Fiz. **46**, 1363 (1964) [Sov. Phys.—JETP **19**, 922 (1964)].

[29] K. Maki and P. Fulde, Phys. Rev. **140**, A1586 (1965).

[30] K. Ishino and Y. Suzumura, Prog. Theor. Phys. **68**, 1776 (1982).

[31] E. W. Fenton, Solid State Commun. **56**, 1033 (1985).

[32] K. Maki, in *Superconductivity,* edited by R. D. Parks (Marcel Dekker, New York, 1969), Vol. 2, pp. 1035–1102.