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Hall effect and ballistic conduction in one-dimensional quantum wires

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The nature of the Hall effect in one-dimensional ballistically conducting quantum channels is clarified by considering the current-induced transverse polarization of the channel, and by generalizing the concept of a Hall voltage to one dimension. The one-dimensional Hall effect can be observed *noninvasively*, is quantized, and is *not* quenched at low magnetic fields. A surprising conclusion is that the quantized resistance of narrow ballistic channels at $B=0$ recently reported by Wharam *et al.* is a limiting case of the quantum Hall effect.

In the past few years there have been many exciting developments in the study of the Hall effect in two-dimensional systems.[1] In the case of one-dimensional (1D) systems, however, it has been unclear whether a Hall effect could be observed experimentally or is even conceptually reasonable. This fundamental problem is of particular interest today because it has recently become possible to fabricate narrow conducting channels in semiconductor heterostructures, with only a few of the transverse quantum states (and associated 1D subbands) populated with electrons, making these channels a precise 1D analog of the familiar 2D electron systems.[2-4] Furthermore, the channels are of such high quality that the electron mean free path can exceed the channel length, making electron transport through the channel ballistic.

Roukes *et al.*[3] have recently measured a Hall resistance by connecting Hall voltage leads directly to the sides of such a 1D conduction channel, and observed Hall plateaus as well as complete quenching of the Hall effect at low magnetic fields. However, Peeters[5] has shown that in this arrangement the physics is strongly influenced by electron scattering at the junction with the Hall leads. Thus, while this phenomenon is very interesting in its own right, its implications for the Hall effect *intrinsic to a 1D conductor* are not obvious. As an alternative to disturbing the channel by contact with Hall probes, Störmer has suggested studying the 1D Hall effect by measuring the transverse polarization of the current-carrying channel in a magnetic field, possibly by a capacitive technique. But, in the absence of any theory, it has been unclear how to interpret such a measurement.

In this article I examine the question of an *intrinsic* 1D Hall effect in ballistic channels theoretically. The transverse polarization of a channel associated with an electric current in a magnetic field is calculated and its relationship to the Hall effect is clarified. In particular, the intrinsic Hall effect is *not* quenched in narrow channels at low magnetic fields. The proper generalization of the concept of a Hall voltage to the case of 1D ballistic systems is introduced by making an analogy with the theory of the Hall effect in 2D systems. This *intrinsic* 1D Hall voltage can be measured experimentally without perturbing the 1D channel in any way, under certain conditions. It turns out that in a 1D ballistic channel, the intrinsic Hall plateaus not only are not quenched at low fields, but extend to *zero* magnetic field where they manifest themselves as the zero-field quantization of the channel resistance recently observed by Wharam *et al.*[4]

Consider a heterostructure in the x-y plane with a narrow ballistically conducting channel running in the y direction formed by means of electrostatic confinement.[2-4] There is an in-plane potential $V(x)$ confining electrons to the channel, and a magnetic field B in the z direction. For convenience I assume that in addition to the confining potential $V(x)$, there is an electric field E_x in the x direction. Strong confinement in the z direction will be implicit throughout. The electron Hamiltonian in the Landau gauge is

$$H=[p_x^2+(p_y-q_eBx)^2]/2m+V(x)-xq_eE_x\ ,$$

where q_e is the electron charge, m the effective mass, and the effect of the magnetic field on the spin variables is not shown explicitly. Eigenstates of H are of the form $\psi_{kn}=e^{iky}U_{kn}(x)$ with eigenvalues ε_{kn} satisfying $H\psi_{kn}=\varepsilon_{kn}\psi_{kn}$. The current operator is

$$j_y=q_e[y,H]/i\hbar L=-q_e(i\hbar\partial/\partial y+Bq_ex)/Lm\ ,$$

where L is the channel length. The current carried by the channel is

$$J_y = \sum_{kn} \langle \psi_{kn} | j_y | \psi_{kn} \rangle f_{kn} = \sum_{kn} (\hbar k q_e - B q_e^2 \langle \psi_{kn} | x | \psi_{kn} \rangle) f_{kn} / Lm \,, \quad (1)$$

where f_{kn} is the number of electrons in state ψ_{kn} and a suitable summation over spin indices is understood here and throughout this paper.

In order to obtain a physical picture of the nature of the current-induced transverse polarization P_x of the 1D channel, let us consider the exactly soluble case of a harmonic confining potential $V(x) = cx^2$. The self-consistent numerical calculations of Laux, Frank, and Stern[6] suggest that this should be a good approximation for very strong electrostatic confinement, i.e., for the case of interest here. (For wider channels the confining potential is no longer harmonic, and P_x can be calculated perturbatively or numerically, by methods analogous to those used by Stern[7] to calculate the transverse polarization of 2D systems.) The electron Schrödinger equation for harmonic confining potentials takes the form

$$\left[\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} - \phi (x - \mu_k)^2 + \varepsilon_{kn} - \gamma_k \right] U_{kn}(x) = 0 \,, \quad (2)$$

where

$$\phi = c\tilde{m}/m \,,$$

$$\mu_k = (B\hbar q_e k + E_x m q_e)/2c\tilde{m} \,,$$

$$\gamma_k = \hbar^2 (k - k_0)^2 / 2\tilde{m} - E_x^2 q_e^2 / 4c \,,$$

$$\tilde{m} = m + B^2 q_e^2 / 2c \,,$$

$$k_0 = B E_x q_e^2 / 2c\hbar \,.$$

Since the *effective* confining potential in (2) is harmonic, $\mu_k = \langle \psi_{kn} | x | \psi_{kn} \rangle$. Using the two expressions for μ_k, one can eliminate k from the form (1) for J_y in favor of $\langle \psi_{kn} | x | \psi_{kn} \rangle$, yielding

$$J_y = \frac{2c}{Bq_e} P_x - \frac{E_x q_e}{B} N \,, \quad (3)$$

which holds for *any* population $\{f_{kn}\}$ of the current-carrying states. In (3),

$$P_x = \sum_{k,n} q_e f_{kn} \langle \psi_{kn} | x | \psi_{kn} \rangle / L$$

is the transverse polarization and $N = \sum_{k,n} f_{kn}/L$ is the 1D electron density.

Notice that (3) is exactly the result which would be obtained classically by balancing the Lorentz force on an electron moving along the channel against the restoring force of the confining potential. The electric field E_{x0} which nulls the polarization in (3) satisfies the usual Hall form $J_y/E_{x0} = -q_e N/B$. Thus the transverse polarization described by (3) is clearly a manifestation of the Hall effect. Equation (3) is an exact result (within one-electron theory) and shows *no* quenching of the Hall effect at low B.

The transverse polarization could, at least in principle, be measured noninvasively. For example, a polarization P_x in the conducting channel would induce a potential difference $\sim P_x/\pi\varepsilon d$ between two parallel conductors lying in the x-y plane on opposite sides of the channel, each separated from it by a distance d. If d is sufficiently large, the influence of the two "probe" conductors on the 1D channel will be negligible and the measurement noninvasive.

Having shown above that an intrinsic 1D Hall effect exists in narrow ballistic channels and is not quenched at low magnetic fields, I will now address the question whether there is an analog of the 2D *quantum* Hall effect which is intrinsic to a 1D channel. Observing the 2D quantum Hall effect involves measuring a Hall voltage V_H in order to obtain a Hall resistance. However, attaching conventional Hall probes to a 1D channel to measure the Hall voltage destroys the 1D character of the channel at the point where the measurement is made. Thus a 1D analog of the 2D Hall voltage, which can be measured noninvasively, needs to be identified.

The ratio $J_y/E_{x0} = -q_e N/B$ is not quantized in one dimension. For $E_x = 0$, $J_y/P_x = 2c/Bq_e$ is independent of the number of populated subbands for harmonic confinement. This means that neither P_x nor E_{x0} can play the role of the Hall voltage in an experiment attempting to observe an intrinsic 1D quantized Hall effect. We thus turn to the theory of the 2D quantum Hall effect for guidance.

In a 2D system, $q_e V_H$ is equal to the difference in the chemical potential between electrons at the two edges of the sample which run parallel to the direction of the overall flow of the electric current. This was most recently used by Streda, Kucera, and MacDonald[8] and Jain and Kivelson[9] in their work which demonstrated the fundamental relationship between the quantum Hall effect and the Landauer[10] formulation of transport theory. Although the Landauer theory is one dimensional, both of these treatments considered the system to be essentially two dimensional in that the two sample edges were assumed to be well separated spatially. Thus the electron states at the opposite edges did not overlap with each other, and each edge could have its own quasiequilibrium state with its own well-defined (and measurable) chemical potential.

Here we are considering a true 1D quantum channel, where there is no clear spatial separation between any of the electron states, although $\langle \psi_{kn} | x | \psi_{kn} \rangle$, the "guiding center" of the wave function ($\langle \psi_{kn} | x | \psi_{kn} \rangle = \mu_k$ in the case of harmonic confinement), moves across the channel with changing k. Now suppose that (i) electrons are injected into the channel at its two ends [labeled left (L) and right (R)] up to energies ε_L and ε_R respectively, (ii) the channel is perfectly ballistic so that the electrons pass through without any scattering, and (iii) all leftward- (rightward-) moving states in the channel are filled up to the energy ε_R (ε_L) and are empty above that energy.

The guiding centers of the occupied states with energies ε_L and ε_R occur at opposite sides of the channel so that by analogy with the 2D case we can in one dimension *define* $V_H = |(\varepsilon_L - \varepsilon_R)/q_e|$, even though there is a spatial overlap between the states at opposite sides of the channel, and thus ε_L and ε_R cannot be thought of as local values of a chemical potential. Notice that for a wide ballistic channel which is effectively two dimensional, this definition of the intrinsic Hall voltage V_H reduces to the usual Hall voltage.

The current J_y is now easy to calculate by standard techniques[1] *without* assuming that the confinement is harmonic

$$J_y = \sum_{k,n} f_{kn} \langle \psi_{kn} | j_y | \psi_{kn} \rangle$$

$$= \sum_{k,n} f_{kn} q_e (\partial \varepsilon_{kn} / \partial k)/\hbar L = \sum_n q_e h^{-1} \int_{\varepsilon_R}^{\varepsilon_L} d\varepsilon .$$

This yields the *quantized* 1D Hall resistance

$$R_H = | V_H / J_y | = h / v q_e^2 , \quad (4)$$

where v is the number of 1D subbands (counting spin) which contain electrons.

This shows that the concept of a quantized Hall resistance can, in principle, be extended to the case of a 1D ballistic quantum channel, but this would be moot if the effect were not observable. To demonstrate that it is observable, it is sufficient to show that conditions (i) and (iii) can be satisfied and V_H, as defined above, can be measured in the same system.

A suitable system is one such as that studied by Wharam *et al.*[4] This consists of a narrow 1D ballistic channel connecting two wide regions of the usual 2D electron gas. The resistance of the 2D regions is negligible[4] compared to that of the channel so that the two-terminal potential difference across the device is equal to the potential difference V between the two ends of the channel. Thus condition (i) is satisfied with $V = | (\varepsilon_L - \varepsilon_R)/q_e |$, at least for $T = 0$ and weak magnetic fields for which the Hall voltage V_{H2D} across the 2D regions is small, satisfying $V_{H2D} \ll V$, so that the potential difference between the two ends of the channel is uniquely defined. Condition (iii) is usually *assumed* to be satisfied if condition (i) is in Landauer-type theories of 1D transport. An examination of its accuracy for the present system has been carried out but is quite involved and will be presented elsewhere. The main result is that while condition (iii) is not met *exactly* under any conditions, it is fulfilled approximately to a high degree of precision for physically reasonable confining potentials provided that V is small and ε_L and ε_R are well separated in energy from the bottom of every 1D subband of the quantum channel. That is, condition (iii) is met provided that one is not too close to the transition regions between the different Hall plateaus described by (4).

Thus the conditions for observing the intrinsic 1D analog of the 2D quantum Hall effect should be satisfied by ballistic systems such as that of Wharam *et al.*[4] near the centers of the Hall plateaus, at low temperatures, for weak magnetic fields. Note that it is precisely for *weak* magnetic fields that the system is effectively one dimensional since at high fields the electron states at the opposite sides of the channel become well separated spatially and the accepted theoretical and experimental ways of handling the 2D quantum Hall effect apply.[8,9]

The above argument shows that the intrinsic 1D Hall voltage which was introduced, somewhat abstractly, through the definition $V_H = | (\varepsilon_L - \varepsilon_R)/q_e |$, is measurable as an actual physical voltage, the two-terminal voltage V across the device. To understand this intuitively, recall that the guiding centers of the wave functions of the electrons injected into the channel by the two 2D reservoirs at *their respective Fermi levels* travel along opposite sides of the channel. Thus the 2D reservoirs themselves act as the Hall probes measuring the Hall voltage in the channel. The measurement is clearly noninvasive. Alternatively, one can think of the Hall voltage in terms of the net work involved in moving an electron from the highest occupied state whose guiding center is on one side of the channel to the lowest unoccupied state on the other side. Under the above conditions, this is the same as the net work involved in moving an electron between the two reservoirs. It follows that $V_H = V$. Interestingly, the two-terminal resistance of 2D electron systems at *high* magnetic fields is also equal to the Hall resistance, as was demonstrated experimentally by Fang and Stiles,[11] who used two-terminal measurements to observe the quantized Hall effect.

Another important conclusion is that the quantized intrinsic 1D Hall effect is not quenched at low magnetic fields but in fact persists down to $B = 0$. This is because the result (4) is valid for weak magnetic fields, and the quantization index v remains finite as $B \rightarrow 0$ due to the finite 1D subband splitting resulting from the confining potential of the channel.

Since $V_H = V$, the quantized resistance $R = h / v q_e^2$ of narrow ballistic channels observed by Wharam *et al.*[4] *at zero magnetic field* is thus seen to be a limiting case of the quantum Hall effect, and provides striking experimental evidence in support of the above ideas.

In common with the earlier work of Streda, Kucera, and MacDonald[8] and Jain and Kivelson[9] on the quantum Hall effect associated with 2D conduction, the present theory of the intrinsic 1D Hall resistance is based on the Landauer approach to transport problems. However, some differences are apparent between the present 1D results and those obtained for the 2D case.[8,9] For example, the results of Streda *et al.*[8] reduce to the familiar 2D form $R_H = h/2q_e^2$ and $R = 0$ for dissipationless transport with one Landau level occupied and ignoring spin splittings, whereas the corresponding 1D result obtained above for weak magnetic fields is $R = R_H = h/2q_e^2$. These results are in fact not inconsistent with each other because the 2D formulas apply to the usual four-terminal Hall measurements, whereas the 1D results are for two-terminal measurements.

Very recently, Beenakker and van Houten[12] have argued that the Hall effect should be completely quenched and the Hall resistance should vanish as soon as the magnetic field is small enough for the electron edge states at the opposite sides of the channel to begin to overlap spatially. It should be emphasized, however, that the experiments to which their theory applies are similar to those of Roukes *et al.*,[3] with Hall probes attached to the sides of the channel. Here I have shown that in experiments which use *noninvasive* techniques, the results are very different.

So far I have considered only ballistic channels where there is no electron scattering. In a channel with isolated scattering centers (but no localized states) the derivation of expression (3) relating the transverse polarization to the current for harmonic confinement still holds, so that the intrinsic 1D Hall effect still exists and is not quenched. However the scattering breaks the symmetry between V and V_H. Jain and Kivelson[9] have recently shown that for quasi-one-dimensional channels (which are, however, wide enough to have well-separated edge states so that a 2D-like Hall effect can be observed), resonant impurity scattering can lead to fluctuations in the channel resistance R while the Hall resistance R_H is relatively unaffected. Thus, although scattering does not destroy the quantization of the 2D Hall effect, its influence on the quantization of the intrinsic 1D Hall effect cannot be determined by a two-terminal experiment measuring V/J_y. It also follows that the accuracy of the quantization of the $B=0$ channel resistance depends on the degree of perfection of the ballistic channel.

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