Interpretation of Shubnikov–de Haas oscillations and the quantum Hall effect in a heterojunction with two populated subbands

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Quantum Hall effect and Shubnikov–de Haas measurements have been presented by Guldner *et al.* [Phys. Rev. B **33**, 3990 (1986)] for $In_xGa_{1-x}As$-InP heterojunctions with two populated subbands. We calculate for this system the transverse magnetoresistance, taking into account the broadening of the Landau levels and the supposition that a background density of localized states is present. Remarkable agreement with the experimental curve is obtained. The results allow us to explain quite naturally both the occurrence of two different periods of the Shubnikov–de Haas oscillations and the observed anomalous behavior of the quantum Hall effect. Contradictions in the original discussion of Guldner *et al.* are resolved.

Recently, Guldner *et al.*[1] presented measurements of the quantum Hall effect (QHE) and of Shubnikov–de Haas (SdH) oscillations in modulation-doped $In_{0.53}Ga_{0.47}As$-InP heterojunctions. In this structure electrons are transferred from InP to $In_xGa_{1-x}As$ and an accumulation layer is formed as a two-dimensional electron gas (2D EG). In spite of the low electron density ($n_s \leq 5\times10^{11}$ cm^{-2}), the SdH oscillations show two different periods which clearly reveal the occupation of two electric subbands in the 2D EG. Further on, an anomalous behavior of the QHE is observed, particularly some plateaus are missing and others are enhanced. It should be noted that a similar anomalous behavior was found also in other systems with more than one occupied electric subband.[2] Guldner *et al.*[1] tried to explain their results with the help of the unbroadened Landau-level (LL) diagram and the variation of the Fermi energy, which jumps from one level to the next for integral values of the filling factor. Although Guldner *et al.* explained their observation, there remains a fundamental contradiction. In their picture there can principally occur only one oscillation period determined by the total electron density n_s. This situation is only slightly modified by the fact that some of the jumps can disappear when two levels are crossing near an integral filling factor. Thus, Guldner *et al.* concluded correctly that it would be necessary to take into account the broadening of the LL's and the oscillating dependence of the effective g factor on B and the LL index (our results show that the latter is less important).

Our successful experience in analyzing magnetotransport measurements in inversion layers adjacent to grain boundaries in InSb bicrystals[3] with four occupied electric subbands and the above-mentioned problems stimulated us to perform model calculations of the Fermi energy E_F and the transverse magnetoresistance $\rho_{xx}(B)$ for the $In_xGa_{1-x}As$-InP system. Basic assumptions are the broadening of the LL's and the occurrence of a background of localized states. The calculated ρ_{xx} agrees well with the measured dependence and allows us to discuss the observed peculiarities without artificial assumptions.

For the calculation the following model is used. An accumulation layer forms the 2D EG on the $In_xGa_{1-x}As$ side of the heterojunction. The bulk conduction band is assumed to be a parabolic one.[1] Consequently, the LL's without scattering broadening E_{ils} ($i=1,2$ is the electric subband index, $l=0,1,2,\ldots$ is the Landau quantum number, and $s=\pm\frac{1}{2}$ is the spin quantum number) depend linearly on the magnetic field B which is applied perpendicular to the interface. For $B=0$ the LL's converge at the subband energies E_i. The latter ones connect the Fermi level $E_F=E_F(B=0)$ with the density n_{si} in the ith electric subband by $E_F-E_i=n_{si}\pi\hbar^2/m^*$.[4] m^* is the 2D EG effective mass.[1] A self-consistent calculation of the electric subbands[3-5] is avoided here by taking the two values n_{s1} and n_{s2} from the experiment.[1] Then the magnetic-field-dependent Fermi energy $E_F(B)$ is determined by the condition of a constant total electron density $n_s=n_{s1}+n_{s2}$. The resulting $E_F(B)$ oscillates sawtooth-like, as shown in Fig. 1, with vertical jumps from one level to the next for integral filling factors[4] $\nu=n_sh/eB$. This behavior of $E_F(B)$ is characterized by only one oscillation period on the $1/B$ scale $\Delta(1/B)=n_sh/e$. However contrary to this the experiments clearly reveal two oscillation periods. Therefore, as a first step towards a more consistent description, the broadening of the LL's due to scattering processes will be included. Calculations based on short-range potential scattering show a rapid decay of the density of states (DOS) in the band tails of the LL's.[4] Higher-order approximations do not change the main characteristic of this picture.[4] The DOS in the midgap between two LL's should decrease drastically with increasing magnetic field. This is in contrast to the interpretation of recent magnetotransport measurements on several heterostructures based on GaAs compounds.[6] There the results are consistent with a background density of localized states between the LL's. Accordingly, we model the DOS by a superposition of Gaussian peaks and a constant background,

$$D(E)=\frac{eB}{h}\sum_{i}\sum_{l,s}(1-x_i)\frac{1}{\pi^{1/2}\Gamma_i}\times\exp\left[-\left[\frac{E-E_{ils}}{\Gamma_i}\right]^2\right]+xD_0 \qquad (1)$$

(for the delocalized states this model resembles that of Ono,[7] who used Gaussian LL's with a Gaussian distribution of localized states in the band tails of the LL's). $\Gamma_i \propto (B/\mu_i)^{1/2}$ (Ref. 4) is the broadening parameter for a Gaussian Landau peak centered at E_{ils} with the electron mobility μ_i in the electric subband i. x is the total percentage of localized states, the DOS, D_0, of which is assumed to be constant.

The portion of localized states in the electric subband i is x_i with $x=(x_1n_{s1}+x_2n_{s2})/n_s$. D_0 begins a few Γ_1 below the lowest LL. Thus we arbitrarily choose $D_0(E)=(m^*/\pi\hbar^2)\Theta(E-E_{10+}+3\Gamma_1)$. Then for $T=0$ the Fermi energy $E_F(B)$ is determined implicitly by

$$n_s=\int_{-\infty}^{E_F(B)}D(E)dE \; . \qquad (2)$$

The transverse magnetoconductivity is calculated by an approximation valid for $T=0$ and high magnetic fields $\omega_c\tau \gg 1$,

$$\sigma_{xx}(B)=\frac{e^2}{h}\sum_{i}\sum_{l,s}(l+\tfrac{1}{2})\times\exp\left[-2\left[\frac{E_F(B)-E_{ils}(B)}{\Gamma_i(B)}\right]^2\right] \; . \qquad (3)$$

Equation (3) was derived by several authors[8] by a systematic evaluation of Kubo's formula. Neglecting the intersubband scattering, one obtains the total conductivity from the parallel contribution of all electric subbands to the current.[4] This leads to the sum over i in (3). It should be noted that actually only the delocalized states in the Gaussian peaks contribute to ρ_{xx}. It is obtained from the 2D tensor relation,

$$\rho_{xx}=\frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2} \; . \qquad (4)$$

For the Hall conductivity we use as a simple estimate the classical expression $\sigma_{xy}=-en_s/B$. In order to compare directly with the experimental results of Guldner *et al.*,[1] we used in our calculation the parameters given in their paper.

The LL diagram is shown in Fig. 1. The dependence of the Fermi energy on the magnetic field calculated without LL broadening shows one oscillation period which is determined by the total electron density according to $\Delta(1/B)=n_sh/e$. For integral filling factors $\nu=n_sh/eB$, the Fermi energy shows vertical jumps from one LL to the next, passing the region of localized states. From this argument Guldner *et al.*[1] concluded that σ_{xx} should be small for the corresponding B values. Actually, this means that $\sigma_{xx}(B)$ could show only one oscillation period. Even if certain jumps of $E_F(B)$ are missing or small due to crossing of LL's no second period will appear. But, indeed, just the experiments of Guldner *et al.* revealed two oscillation periods for which $\Delta_i(1/B)=n_{si}h/e$ was used to obtain n_{s1} and n_{s2}. Consequently, the picture with δ-shaped LL's, in general, cannot describe the 2D EG with more than one occupied electric subband. The situation is completely changed in our model with broadened LL's and a background of localized states. The best agreement with the experimental ρ_{xx} curve was obtained with the following parameters: $\Gamma_1=(0.8\text{ meV})B^{1/2}$, $\Gamma_2=(0.24\text{ meV})B^{1/2}$ (B in tesla) and $x=0.26$, $x_1=0.2$, and $x_2=0.5$. The order of magnitude of x is in agreement with values determined for GaAs-$Al_xGa_{1-x}As$ structures.[6] By using $\Gamma_i=(1/\pi)^{1/2}(\hbar e/m^*)(B/\mu_i)^{1/2}$,[8] one gets from the broadening parameters Γ_1 and Γ_2 the mobilities in the subband $i=1$ and 2, respectively, with $\mu_1=3\times10^4$ cm^2/V s and $\mu_2=32.7\times10^4$ cm^2/V s. With a suitable weighted mean $\mu=\sum_i(1-x_i)\mu_in_{si}/(1-x)n_s$, the total mobility is $\mu=6.0\times10^4$ cm^2/V s, which is in agreement with the experimental value[1] $\mu\sim6\times10^4$ cm^2/V s. It should be noted that it was not possible to obtain a reasonable fit of the experimental results without a background DOS even when the Gaussian band tails were treated as localized ones. Likewise a modeling of the broadened LL's by elliptic peaks was not successful. Figure 1 shows that due to the broadening of the levels the vertical jumps of the Fermi energy are smeared out strongly; in particular, only near the original jumps at $\nu=2,5,8,10,12$ does the Fermi energy pass through localized regions of the DOS.

The calculated transverse magnetoresistance $\rho_{xx}(B)$ is compared with the experimental curve in Fig. 2. Remarkable agreement is achieved by fitting only Γ_i and x_i. This allows one, in connection with the behavior of the Fermi energy, to interpret the anomalous features of the experimental results. At first, it is clearly seen that mini-

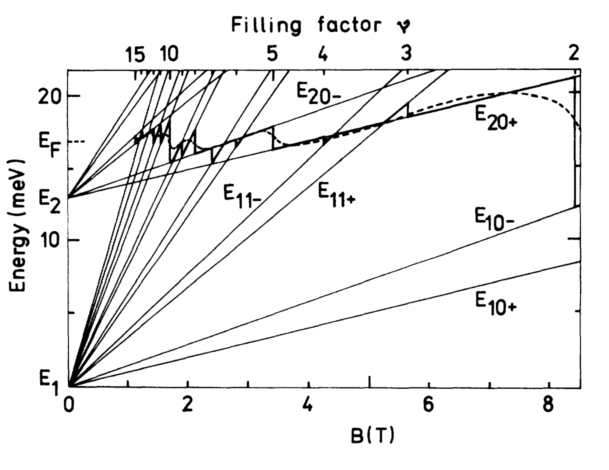

FIG. 1. Landau levels E_{ils} as a function of the magnetic field B and the filling factor $\nu=n_sh/eB$ (+ and − refer to spin up and down, respectively). E_1 and E_2 are the electric subband energies. The variation of the Fermi energy $E_F(B)$ calculated for the case of infinitely sharp levels (solid line) and that calculated within the broadened Landau level scheme using the DOS of Eq. (1) are also shown (dashed curve). E_F is the Fermi energy for $B=0$.

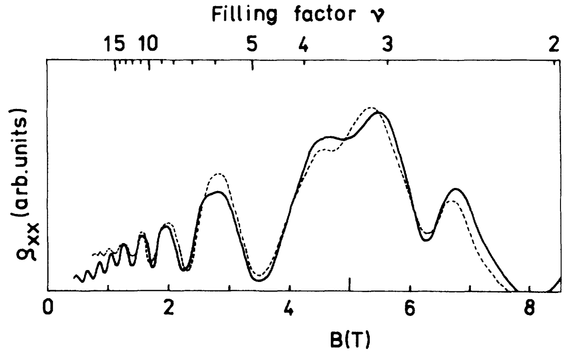

FIG. 2. Transverse magnetoresistance ρ_{xx} as a function of B and the filling factor ν. The solid curve represents the theoretical result. For comparison the experimental result obtained by Guldner *et al.* (Ref. 1) is also shown (dashed curve).

ma of ρ_{xx} occur when the Fermi energy is in the region of localized states far from the center of any LL. For low magnetic fields, $E_F(B)$ approaches the horizontal line $E_F(B=0)$. Then, of course, the above-mentioned two oscillation periods determined by n_{s1} and n_{s2} occur. The structure for higher magnetic fields will be discussed in detail. The maximum at $B \sim 6.8$ T arises from the lowest LL of the subband $i=2$ (E_{20+}). The humped structure around $B=5$ T is essentially caused by the spin splitting between the levels E_{11+} and E_{11-}. The maximum at $B \sim 2.8$ T is, above all, determined by E_{20-} and $E_{12\pm}$. The spin splitting is no longer resolved. The minima for higher fields, with the exception of that at 6.3 T, coincide with the remaining steps in the smooth $E_F(B)$ curve, but modifications due to the occupation of the broadened levels do occur. So the minimum which in the unbroadened picture should occur at $\nu=3$ is shifted to a higher B value according to the occupation of the E_{20+} level. Figure 1 shows that $E_F(B)$ lies above the center of this level. The distance between $E_F(B)$ and E_{20+} is largest for $B=6.3$ T, resulting in the ρ_{xx} minimum. In general, it is clearly seen that a correspondence between the E_F jumps in the unbroadened picture with the minima of ρ_{xx} is accidental, particularly if LL's from several electric subbands are crossing. Then a discussion is only possible using a broadened level scheme.

It is well known that the plateaus of the Hall resistance ρ_{xy} in the QHE are attributed to a position of the Fermi energy within a mobility gap.[4] As mentioned above (Fig. 1), in the broadened picture this is the case for filling factors $\nu=2,5,8,10,12,\ldots$. Actually, the experimental Hall resistance[1] shows well-developed plateaus just for these values with $\rho_{xy}=h/\nu e.^2$ The well-developed "anomalous"[1] plateau at $\nu=5$ is caused by the spin splitting of the lowest ($i=2$) LL. According to the position of the Fermi energy in regions with extended states, the plateaus for $\nu=4,6,7,\ldots$, are missing, in accordance with the experiment. The plateau occurring at $B \sim 6$ T is significantly larger than $h/3e^2$. This should be caused by the perturbation of the E_{11+} level by E_{20+} mentioned above.

We conclude as follows. If only one electric subband is occupied [or if there are several occupied ones, but $E_F(B)$ lies only on the LL's of one electric subband as in some p-inversion layers[9]], the Fermi energy passes regions with localized states at the same B values for both the unbroadened and broadened LL pictures. This situation is completely changed when $E_F(B)$ lies on the LL's of different subbands. Then it is impossible to explain the experimental results with unbroadened δ-shaped LL's. Although we have still used a constant effective g factor, we were able within our simple model with broadened LL's and a localized background DOS to explain the magnetotransport properties observed by Guldner *et al.*[1] Our scheme has been applied successfully already to inversion layers in InSb bicrystals with four occupied electric subbands.[3]

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