Low-field magnetoresistance of n-type GaAs in the variable-range hopping regime

M. Benzaquen, D. Walsh, and K. Mazuruk
Department of Physics, McGill University, Rutherford Physics Building, 3600 University Street, Montréal, Québec, Canada H3A 2T8
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Low-T magnetotransport data on epitaxial n-type GaAs, in the variable-range hopping regime, are presented with emphasis on negative-magnetoresistance effects at low magnetic fields. A minimum in the variation of the magnetoresistance as a function of the magnetic field is observed. The negative part of the magnetoresistance is in good agreement with a model which accounts for the Zeeman splitting of the localized energy levels. A weak dependence of the characteristic temperature of Mott's law on the magnetic field is observed.

Anomalous magnetoresistance in III-V compounds has been observed previously at very low temperatures (T). The general features depend on impurity concentrations and are as follows.

(a) For pure enough material, falling below the Mott transition,[1] the low-T magnetoresistance is negative at low magnetic induction (B), reaches a minimum as B increases, and eventually becomes positive.[2,3] The negative part reaches in such cases 30%. The positive part increases rapidly with B, indicating several distinct transport regimes.[4–6]

(b) For barely metallic samples, the negative magnetoresistance is quite weak and may remain negative at high B. Those features are well described on the basis of a theory of localized spins due to Toyozawa,[7] another possible interpretation being based on the suppression of phase coherence by B.[8]

(c) The negative magnetoresistance increases in absolute value with decreasing impurity concentration, and eventually becomes weaker again for very pure samples.[3,9] Recent studies are generally devoted to the positive magnetoresistance at large B,[6] the negative contribution being neglected.

We report on the magnetoresistance of two n-type GaAs epitaxial layers grown by metal organic vapor-phase epitaxy (MOVPE). Both samples were shaped to a bridge configuration and all Au-Ge contacts were tested for linearity of the I-V characteristics. The samples were a few micrometers thick and were not intentionally doped. Good uniformity was confirmed by resistivity readings across the contacts of the multiarm bridge. The measurements were performed under low electric field conditions with a high-resolution, high-impedance data acquisition system.[10]

The donor (N_D) and acceptor (N_A) impurity concentrations of both samples are given in Table I. The donor binding energies E_D were extracted from the slopes of $\ln(n_c)$ as a function of $1/T$ between 30 and 60 K, where the condition $N_D > N_A > n_c$ was verified. N_D and N_A were calculated from the Falicov-Cuevas drift-mobility expression[11] for ionized impurity scattering at 50 K, keeping consistency with the observed n_c at 300 K.[12]

Both samples, as shown in Table I, fall well below the critical electron concentration $n_{\rm cr}$ corresponding to the Mott criterion[1] for the metal-insulator (MI) transition:

$$n_{\rm cr}^{1/3} a_0 = 0.25 \ , \tag{1}$$

where a_0 is the effective Bohr radius of GaAs ($a_0 = 98$ Å). Fritszche[13] proposed a similar condition, where $n_{\rm cr}$ in Eq. (1) is replaced by the critical donor concentration $N_{\rm DC}$, pointing out the importance of compensation. As shown in Table I, according to the Fritszche criterion, our samples fall again in the insulating side of the MI transition, and the strong localization regime is thus expected at low T.[14]

Figure 1 shows the variations of $\ln(\sigma)$ versus $T^{-1/4}$ for sample 1 between 1.4 and 5.5 K. The conductivity follows Mott's law:[15]

$$\sigma = \sigma_0 \exp[-(T_0/T)^s] \ \text{ with } s = \tfrac{1}{4} \ . \tag{2}$$

σ_0, T_0, and s are parameters determined from our data with a least-squares technique. The solid lines of Figs. 1 and 2 are the corresponding fits, with the parameters quoted in Table I.

Figure 3 shows the variations of the transverse magnetoresistance of sample 1 as a function of B for 1.4 and 4.2 K. The corresponding data for sample 2 are shown in

TABLE I. Donor (N_A) and acceptor (N_D) impurity concentrations and electronic concentration (n_c) at 300 K. K_c is the compensation ratio, a_0 the Bohr radius of a shallow impurity in GaAs, and E_D the binding energy as extracted from freeze-out statistics. σ_0, T_0, and s are the parameters of Mott's law as obtained experimentally.

Sample no.	N_D ($\rm cm^{-3}$)	N_A ($\rm cm^{-3}$)	n_c ($\rm cm^{-3}$)	K_c	$n_c^{1/3}a_0$	$N_D^{1/3}a_0$	E_D (meV)		σ_0 ($\Omega^{-1}\,\rm cm^{-1}$)	T_0 (K)	s
1	8.68×10^{15}	4.95×10^{15}	3.55×10^{15}	0.571	0.15	0.20	2.09	1	1.53	91.5	0.320
2	1.02×10^{16}	9.30×10^{15}	8.34×10^{14}	0.911	0.09	0.21	3.46	2	0.607	3180	0.257

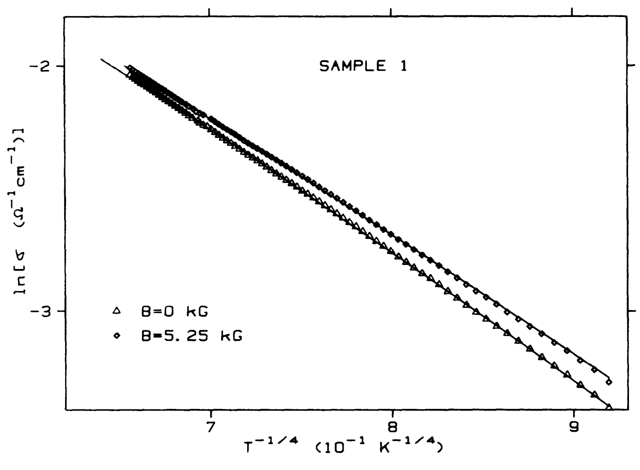

FIG. 1. $\ln(\sigma)$ as a function of $T^{-1/4}$ for sample 1. For $B=0$ kG, the solid line corresponds to the fit of Mott's law to the experimental data by using the parameters of Table I. For $B=5.25$ kG, the solid line was obtained by using Eq. (6) and T_1 as given in Table III. The temperature range is from 1.4 to 5.5 K approximately.

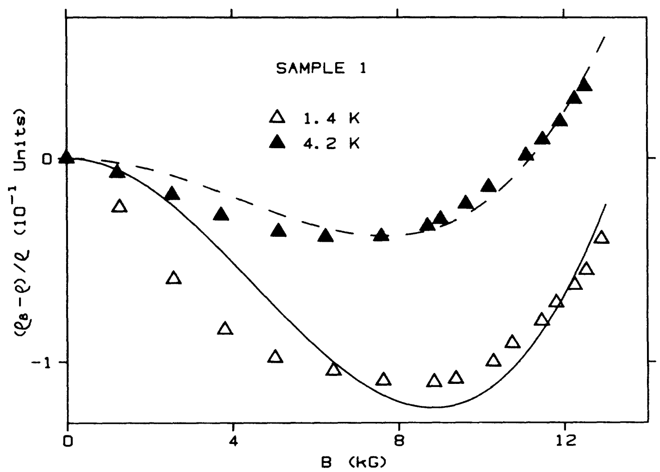

FIG. 3. Magnetoresistance as a function of the magnetic field of sample 1 for two values of the temperature. The lines correspond to the fits of the expression $\exp(K_s B^2)-2+[\cosh(a_1 B)]^{-1}$ to the data with K_s and a_1 given in Table II.

Fig. 4. Such a behavior can be described by an expression of the form

$$(\rho_B-\rho)/\rho=N(B)+P(B)\ , \tag{3}$$

where $N(B)$ and $P(B)$ are, respectively, the negative and the positive contribution to the magnetoresistance, generally assumed to be due to distinct mechanisms.[16]

Although several models accounting for magnetoresistance effects in isotropic semiconductors are available in the literature, they either apply to the weak localization regime[7,8] or are empirical.[16] Only two theories, to our knowledge, have been developed for Mott's variable-range hopping regime.

(a) Shklovskii and Efros[17] considered the positive contribution on the basis of the Miller-Abrahams[18] resistor network model. When the conductivity follows Mott's law,[15] they find an exponential increase of the resistivity with B due to the shrinking of the wave functions of the localized electrons. The low-B magnetoresistance has the form

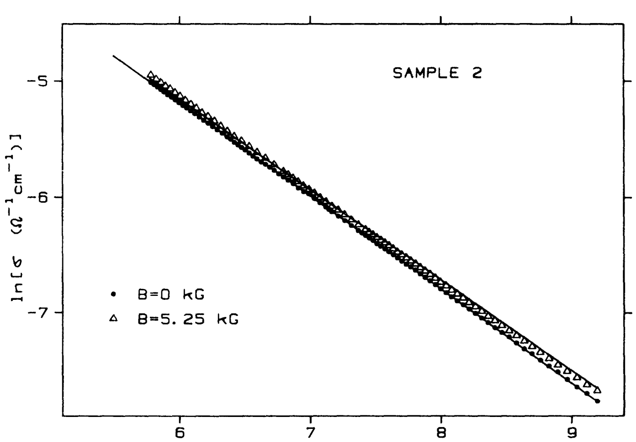

FIG. 2. $\ln(\sigma)$ as a function of $T^{-1/4}$ for sample 2. For $B=0$ kG, the solid line corresponds to the fit of Mott's law to the experimental data by using the parameters of Table I. For $B=5.25$ kG, the solid line was obtained by using Eq. (6) and T_1 as given in Table III. The temperature range is from 1.4 to 9 K.

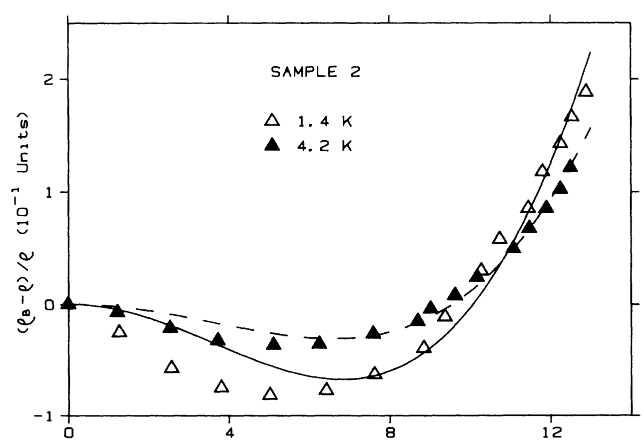

FIG. 4. Magnetoresistance as a function of the magnetic field for sample 2 for two values of the temperature. The lines correspond to the fit of the expression $(\rho_B-\rho)/\rho=\exp(K_s B^2)-2+[\cosh(a_1 B)]^{-1}$ to the data with K_s and a_1 given in Table II. The solid line corresponds to 1.4 K and the dashed line to 4.2 K.

$$P(B)=\exp(K_s B^2)-1 \ , \tag{4}$$

with

$$K_s = t(q^2 a_0^4/c^2\hbar^2)(T_0/T)^{3/4} \ ,$$

where $t=\frac{5}{2016}$. This expression is valid in Gaussian units, q being the electronic charge and c the speed of light. Low-B conditions apply when a_0 is small compared both to $d=N_D^{-1/3}$ and the magnetic length $\lambda=(c\hbar/qB)^{1/2}$. This is the case for lightly doped samples at relatively low B and corresponds to our experimental conditions: $d_1=487$ Å (sample 1) and $d_2=461$ Å (sample 2). For $B=13$ kG, $\lambda=225$ Å. With $a_0=98$ Å, we thus have $d>\lambda>a_0$.

(b) Fukuyama and Yosida[19] proposed a simple mechanism that explains a significant negative magnetoresistance in the variable-range hopping regime. Their approach is as follows. In a band of Anderson localized states, the wave function of an electron with energy E decays exponentially with $(E_c-E)^{\beta}R$, E_c being the mobility edge, R the distance from the impurity, and β a parameter close to 1. Spherical symmetry is assumed, and T_0 in Eq. (2) is proportional to $(E_c-E_F^0)^{\beta d'}$, E_F^0 being the Fermi level in the absence of magnetic field and d' the dimension of the system. The application of a magnetic field leads, for a given eigenstate, to a Zeeman splitting between the two states with opposite spin directions and induces a repopulation among the Anderson localized states. The higher-energy Zeeman state gets closer to E_c and the corresponding wave function extends spatially. On the contrary, the lower Zeeman energy level is more separated from E_c and the corresponding wave function decays more rapidly. Although E_c-E_F is expected to have a weak B^2 dependence, the corresponding energy variation is generally less than the Zeeman splitting. This effect corresponds to a situation where the majority carriers are in the upper Zeeman state, which offers a better overlap of the wave functions and leads to negative magnetoresistance. If we define

$$a=(\beta d'/2)g[\mu_B/(E_c-E_F)] \ , \tag{5}$$

where μ_B is the effective Bohr magneton of the medium and g is the Landé factor of the considered impurity, with $aB \ll 1$, the conductivity has the form

$$\sigma_B=\sigma_0\exp[-(T_1/T)^s]\cosh[saB(T_1/T)^s] \ , \tag{6}$$

where T_1 is the characteristic temperature of the variable-range hopping regime in a field B. With $T_0 \sim T_1$, the corresponding magnetoresistance is

$$N(B)=[\cosh(a_1B)]^{-1}-1 \ , \tag{7}$$

with

$$a_1=(s\beta d'/2)g[\mu_B/(E_c-E_F)](T_0/T)^s \ . \tag{8}$$

Equation (3), with $P(B)$ given by Eq. (4) and $N(B)$ given by Eq. (7), was fitted to our experimental results with K_s [Eq. (4)] and a_1 [Eq. (8)] as free parameters. The best fit corresponded to the values quoted in Table II. The solid lines of Figs. 3 and 4 are the fits for 1.4 K. The dashed lines correspond to 4.2 K. Although the fits are not very accurate, the general features of the data are reproduced. The values of T_0 quoted in Table I allow the evaluation of K_s in Eq. (4). For sample 1 at 1.4 K, we obtain $K_s^{1.4}=1.32\times10^{-5}$ kG^{-2}. The corresponding value at 4.2 K is $K_s^{4.2}=5.8\times10^{-6}$ kG^{-2}. The values of K_s obtained experimentally are, as can be seen in Table II, about 2 orders of magnitude larger than the theoretical predictions. Moreover, only a quite weak T dependence of K_s is observed for sample 1. For sample 2, we obtain $K_s^{1.4}=1.9\times10^{-4}$ kG^{-2} and $K_s^{4.2}=8.3\times10^{-5}$ kG^{-2}. These values are again lower than the experimental results by about 1 order of magnitude, and the T dependence of the magnetoresistance of Eq. (4) is definitely not observed. Moreover, K_s, as given in Table II, is almost constant, at a fixed T, for both our samples, despite very different values of T_0. This is inconsistent with Eq. (4). According to the standard model, the values of T_0 quoted in this work are unusually low, which appears to be a characteristic of both GaAs (Refs. 14 and 20) and InF (Refs. 21 and 22) in the variable-range hopping regime for reasonably pure samples. However, consistency has been achieved recently[23] by assuming an appropriate shape for the density of localized states, while keeping consistency with the form of Eq. (2).

The temperature dependence of a_1, as given by Eq. (8) can be tested from the values quoted in Table II. For sample 1, we obtain $a_1^{1.4}/a_1^{4.2}=1.3$. The theoretical prediction is, as given by Eq. (8), $3^s=1.4$, in excellent agreement. The corresponding ratios for sample 2 are 1.2 and 1.3, again in excellent agreement. The product βg can be extracted from the experimental values of a_1 by assuming that E_c-E_F is approximately equal to the corresponding binding energy. For sample 1, we obtain $\beta g=1.5$ at 1.4 K and 1.6 at 4.2 K. The corresponding values for sample 2 are 1.7 and 1.8 approximately. βg appears, as expected, approximately constant. As g falls usually between 1 and 2 and β is close to 1, the experimental results are in agreement with the Fukuyama and Yosida model.[19] In addition, the values of a_1 quoted in Table II for both samples

TABLE II. Parameters of the expression $(\rho_B-\rho)/\rho=\exp(K_sB^2)-2+[\cosh(a_1B)]^{-1}$ as given by the best fit to the experimental data at fixed temperature. The superscripts indicate the temperature and E_r is the average relative error.

Sample no.	$K_s^{1.4}$ (kG^{-2})	$a_1^{1.4}$ (kG^{-1})	$E_r^{1.4}$	$K_s^{4.2}$ (kG^{-2})	$a_1^{4.2}$ (kG^{-1})	$E_r^{4.2}$
1	2.5×10^{-3}	1.10×10^{-1}	2.2×10^{-1}	2.3×10^{-3}	8.47×10^{-2}	4.5×10^{-1}
2	3.4×10^{-3}	1.13×10^{-1}	3.5×10^{-1}	2.8×10^{-3}	9.16×10^{-2}	3.4×10^{-1}

are almost identical despite very dissimilar values of T_0 (see Table I). From Eq. (8) at fixed T, it can be seen that this is due to the fact that the small value of T_0 obtained for sample 1 corresponds to both a smaller binding energy and a larger power s than for sample 2. An average binding energy decreasing with decreasing T_0 is consistent with the Fukuyama and Yosida approach: it corresponds to a situation where, due to compensation, the Fermi level gets closer to the mobility edge, where the corresponding energy states are less localized, leading to stronger hopping conduction.

In order to check the validity of Eq. (6), the conductivity of our samples was measured as a function of T with an applied magnetic field. To minimize the contribution of the positive part of the magnetoresistance, B was kept low and only three values, namely 2, 3.5, and 5.25 kG, could be considered with well-differentiated data. For clarity reasons, the conductivity is only plotted in Figs. 1 and 2 for $B=5.25$ kG. In Eq. (6), T_1 is proportional to $(E_c-E_F)^{\beta d'}$, E_c-E_F having a weak B^2 dependence. It is then clear that T_1 carriers a stronger B dependence than E_c-E_F. Moreover, E_c-E_F only appears through a [given by Eq. (5)] in the cosh factor of Eq. (6). The T dependence of this factor being weaker than the exponential term, it is reasonable to take E_c-E_F as a constant and leave T_1 as the only free parameter for the fit. For sample 1, by using $\beta g=1.6$ and taking E_c-E_F as E_D, we obtain $a=9.78$ kG^{-1}. For sample 2, we obtained $a=6.68\times10^{-2}$ kG^{-2} with $\beta g=1.7$. Equation (6) was then fitted to our data for different B values and the results are given in Table III, with which the solid lines of Figs. 1 and 2 were generated.

TABLE III. T_1 is the characteristic temperature of Eq. (6) as obtained from the best fit to the experimental data at fixed magnetic field. The superscripts indicate the sample and E_r is the average relative error of the fit.

B (kG)	$(T_1)^1$ (K)	$(E_r)^1$	$(T_1)^2$ (K)	$(E_r)^2$
2.00	91.39	1.7×10^{-3}	3159	4.9×10^{-3}
3.50	92.43	3.1×10^{-3}	3195	1.7×10^{-2}
5.25	97.37	3.5×10^{-3}	3331	1.9×10^{-2}

The function $T_1=T_0(1+\alpha B^2)$, where T_0 is given in Table I, was used to determine α from the variation of T_1 with B, given in Table III. We obtain $\alpha_1=1.65\times10^{-3}$ kG^{-2} for sample 1 and $\alpha_2=1.17\times10^{-3}$ kG^{-2} for sample 2, with an error of 25%. The weak B^2 dependence of T_1, expected from the Fukuyama and Yosida model, is consequently roughly obtained.

In conclusion we provided data on the low-B magnetoresistance of n-type GaAs in the variable-range hopping regime. The B dependence of the magnetoresistance was described with two components. The positive contribution followed the general form predicted by a model based on the shrinking of the electronic wave functions by the B field, but was not fully consistent with it. The negative contribution was well described by a model based on the Zeeman splitting of the localized energy states. Consistency between theory and experiment was obtained for both the T and B dependence of the negative magnetoresistance. The B dependence of the characteristic temperature of Mott's law was also adequately described by this model.

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