# Quantum corrections to the conductivity in Mg-based metallic glasses

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We present measurements of the low-temperature magnetoresistance of high-purity Mg-Cu and Mg-Zn metallic glasses where the spin-orbit scattering rate has been varied by the addition of Ag and Au. Because these alloys are well-characterized, simple metals, the results provide a strict test of the validity of the theories of quantum corrections to the conductivity in bulk disordered metals. The effect of superconductivity may be viewed directly by comparing the behavior of Mg-Zn and Mg-Cu. We find that the quantum corrections work well at low magnetic fields, but fail at higher fields, irrespective of the level of spin-orbit scattering.

## I. INTRODUCTION

The recognition of quantum corrections to the conductivity (QCC), namely weak localization[1] and enhanced electron-electron interaction,[2] has led to a considerable advance in our understanding of electrical transport in disordered conductors, particularly in two-dimensional systems such as metal-oxide-semiconductor field-effect transistor (MOSFET) inversion layers and cryoquenched thin films, where good quantitative agreement between theory and experiment has been found.[3,4] In three-dimensional systems, particularly metallic glasses, the success of the quantum correction theories is less complete. Although they give a good semiquantitative description of the data[5,6] there are significant discrepancies when experiment and theory are compared in detail.[7,8] However, all three-dimensional systems studied to date have suffered from complications such as $d$-band conduction,[5] superconductivity[8] or a significant level of magnetic impurities,[9] complications that introduce sufficient uncertainties to the analysis that it becomes difficult to assess how and where the theories fall short. It was to provide a more stringent test of the quantum correction theories that we undertook the present research. We have examined with high resolution the low-temperature magnetoresistance of high-purity Mg-Cu and Mg-Zn, doped with varying levels of Ag and Au. Mg-Cu is a free-electron[10] nonsuperconducting metallic glass, so well characterized that only the electron phase coherence time $\tau_\phi$ and the spin-orbit scattering time $\tau_{s.o.}$ are unknown. Mg-Zn is essentially identical to Mg-Cu, except that it is superconducting[11] below 0.12 K, thus allowing us, in principle, to isolate experimentally the influence of superconductivity. By adding known amounts of Ag and Au to Mg-Cu and Mg-Zn we can change the spin-orbit scattering, and so examine QCC theories at low and high spin-orbit scattering strengths in otherwise unaltered systems.

## II. EXPERIMENTAL METHODS

The alloys were prepared by induction melting elemental constituents in high-purity graphite crucibles under a high-purity argon atmosphere. The purity of the starting materials was Cu 99.9999%, Zn 99.9999%, Ag 99.999%, Au 99.9999%. The magnesium was prepared by distillation of 99.95% pure Mg onto a carefully cleaned quartz substrate at an ambient pressure of 10 Pa. This technique reduces the level of magnetic impurities (Mn and Fe) by up to 2 orders of magnitude.[12] The Mg distillate as well as the Zn and Cu ingots were etched in a mixture of hydrochloric and nitric acid before alloying. The metallic glass ribbons were prepared by melt spinning onto a copper wheel at tangential velocity of 50 m/s under 30 kPa He atmosphere. Great care was taken during all of above preparations to minimize the introduction of magnetic impurities. After fabrication the samples were stored in liquid nitrogen.

The resulting ribbons were typically 0.5–1.5 mm wide, 20–30 $\mu$m thick, and up to 3 m long. The quality of the glassy structure was examined by x-ray diffraction (using Cu $K\alpha$ radiation) and by differential scanning calorimetry in a Perkin-Elmer DSC 2C. On all the samples measured the x-ray diffraction scans did not show any signs of crystallinity, and the crystallization data were consistent with those found by Altounian, Guo-Hua, and Strom-Olsen[13] and Mizutani and Yoshino.[14] The composition and homogeneity of the samples were checked by electron-beam microprobe. In all cases the samples were found to be homogeneous over the entire sample, to within the resolution of the instrument (0.2 at. %). The Mg, Zn, and Cu concentration was always within 3% of the nominal value, while the concentration of the dopants was always within 10% (more typically 3%) of the nominal value.

Of the major impurities in Mg, only Mn carries a magnetic moment. We measured the Mn content directly in all alloys (and, as a check on overall purity, the Fe content in two alloys) by neutron activation.[15] In all cases (see Table I) the Mn content was less than 3 ppm and in most less than 1 ppm. The absence of magnetic impurities was further confirmed by magnetic susceptibility measurements on the three Mg-Zn samples (using a sensitive alternating force balance in a field up to 1.8 T). From the change in susceptibility between room tempera-

 

TABLE I. Physical parameters for the alloys studied. Errors: $\rho$, ±5%; $d$, ±2%; $B_{s.o.}$, ±10%; $\chi$, $\pm 2\times 10^{-8}$ emu/g; [Mn], ±1%.

| Alloy | $\rho$ ($\mu\Omega$ cm) | $d$ (g/cm$^3$) | $k_F$ ($10^8$ cm$^{-1}$) | $D$ (cm$^2$/s) | $B_{s.o.}$ (T) | $\tilde{F}_\sigma$ | $\chi_{expt}$ ($10^{-7}$ emu/g) | $\chi_{calc}$ ($10^{-7}$ emu/g) | [Mn] (ppm) |
|---|---|---|---|---|---|---|---|---|---|
| $Mg_{70}Cu_{30}$ | 44.0 | 3.11 | 1.38 | 6.9 | 0.077 | 0.50 | −0.5 | 0.1 | 2.6 |
| $Mg_{70}Cu_{27}Ag_3$ | 45.7 | 3.16 | 1.38 | 6.6 | 0.166 | 0.50 | | | 1.7 |
| $Mg_{70}Cu_{24}Ag_6$ | 46.4 | 3.30 | 1.38 | 6.5 | 0.242 | 0.50 | | | 0.3 |
| $Mg_{70}Cu_{15}Ag_{15}$ | 51.8 | 3.74 | 1.38 | 5.8 | 0.410 | 0.50 | | | 1.1 |
| $Mg_{70}Cu_{29.9}Au_{0.1}$ | 48.4 | 3.19 | 1.38 | 6.1 | 0.145 | 0.50 | | | 0.1 |
| $Mg_{70}Cu_{28.5}Au_{1.5}$ | 49.0 | 3.41 | 1.39 | 6.9 | 0.851 | 0.50 | 0.16 | −0.16 | 2.2 |
| $Mg_{70}Cu_{27}Au_3$ | 50.4 | 3.48 | 1.39 | 6.7 | 1.19 | 0.50 | 0.21 | −0.1 | 0.1 |
| $Mg_{70}Cu_{21}Au_9$ | 55.8 | 4.12 | 1.39 | 6.0 | 1.71 | 0.50 | | | 0.3 |
| $Mg_{70}Zn_{30}$ | 45.1 | 2.92 | 1.43 | 6.0 | 0.075 | 0.49 | 1.06 | 0.86 | 0.2 |
| $Mg_{70}Zn_{27}Ag_3$ | 49.8 | 3.04 | 1.43 | 5.3 | 0.151 | 0.49 | 0.71 | 0.63 | 2.3 |
| $Mg_{70}Zn_{27}Au_3$ | 50.4 | 3.23 | 1.43 | 5.3 | 1.03 | 0.49 | 0.63 | 0.53 | 0.8 |

ture and 5.5 K and also from the field dependence of the magnetization we put an upper bound of 4 ppm magnetic impurities (assuming a spin of $2.2\mu_B$).

The density of the samples was measured by Archimedes's method with toluene as the working fluid. The magnetoresistance was measured with a four-terminal ac bridge[16] to a precision of a few parts in $10^6$, in fields up to 5.6 T. The temperature was monitored with calibrated carbon-glass thermistors and was kept stable to within 2%. The resistivity temperature coefficient of the alloy is sufficiently small at the temperatures under consideration that this small temperature instability is negligible compared to the measured magnetoresistance.

Table I summarizes the characteristics of the alloys. Electrical resistivity $\rho$, magnetic susceptibility $\chi$, and density $d$, are measured quantities as mentioned above. The density of states at the Fermi level, $N(\epsilon_F)$, is calculated assuming a free-electron band structure and a valence of two for Mg and Zn and one for Cu, Ag, and Au. It should be noticed that the susceptibility of the alloys is in excellent agreement with that calculated from a free-electron model, once the core susceptibility[17] is taken into account. The diffusivity $D$ is calculated from measured $\rho$ and calculated $N(\epsilon_F)$ taking the electron-phonon enhancement into account.

## III. RESULTS AND DISCUSSION

Figure 1 shows the magnetoresistance of $Mg_{70}Cu_{30}$, $Mg_{70}Cu_{15}Ag_{15}$, and $Mg_{70}Cu_{27}Au_3$ as a function of temperature from 1.5 to 20 K. In all cases the dots represent the experimental data and the line, whether continuous or dashed, the theoretical fit, as outlined below. Similar data for the Mg-Zn system is shown in Fig. 2. Direct comparison between the two sets shows qualitatively the influence of superconducting fluctuations. In particular the dashed line (explained below) at high fields differs from the experimental data by far more in the superconducting glass than the normal glass. In Fig. 3 the effect of increasing spin-orbit scattering by adding Ag or Au is shown at one temperature and low fields.

In fitting the data to QCC theories we consider three possible contributions: weak localization, enhanced electron-electron interactions (both Cooper and diffusion channel), and superconducting fluctuations. For convenience the explicit form for the various terms is given in the Appendix. We begin by considering Mg-Cu, which is not superconducting and so experiences only contributions from localization and interactions. The fitting procedure can be further simplified (and hence made more stringent) by noting that the interaction terms are functions of $B/T$ and are essentially negligible when $B/T \leq 0.4$ T/K ($\Delta\rho/\rho < 2\times 10^{-6}$). Thus by restricting the fitting to this range we need *only* consider the weak localization effect. A similar method has been used by Bergmann for two-dimensional films.[4] The fitting is carried out as follows: In Eq. (A1) the values of $\rho$, $D$ are taken from Table I, leaving only two undetermined parameters, the dephasing field $B_\phi$ and the spin-orbit scattering field $B_{s.o.}$. In fact the fit is even more restricted, since $B_{s.o.}$ must be independent of temperature. Thus each family of curves in Fig. 1 is fitted with a common $B_{s.o.}$ and one value of $B_\phi$ for each temperature. The values obtained for these parameters are insensitive to the precise value used for the $B/T$ cutoff, so long as this is chosen to be 0.4 T/K or less. It may be seen that agreement between theory and experiment is very good over the range of $B/T$ for which the fit is made (solid line), and that the quality of the fit is equally good for all temperatures (Fig. 1), and at all values of $B_{s.o.}$ (Fig. 3). We therefore conclude that *in the regime where it alone contributes,* weak localization provides an excellent description of the magnetoresistance data.

We now examine in detail the behavior of $B_\phi$ and $B_{s.o.}$. Figure 4 shows $B_\phi$ as a function of temperature for four different alloys. For the sake of clarity the values for other alloys have not been included in the graph. The solid line denotes a best fit to these points with the function $B_\phi = B_\phi^0 + AT^n$, yielding the values of $B_\phi^0 = 2.7\pm 0.4$ mT and $n = 3\pm 0.5$. The variation of $\tau_\phi^{-1}$ with temperature is similar to that seen by a number of other authors,[18,19] and is consistent with dephasing due to electron-phonon scattering at high temperatures and some temperature-independent dephasing mechanism at low temperatures. The low temperature saturation has also been seen by other authors in both three-dimensional and two-

dimensional conductors, and appears to be a universal phenomenon. It has been variously attributed to residual spin scattering due to magnetic impurities[20] or to thermal decoupling.[3] In the present case neither of these explanations is tenable. Assuming typical values of $-0.25$ eV for the conduction-electron–local-moment exchange integral[21] and a moment on the Mn ion[22] of $2.2\mu_B$ we find that $B_\phi$ due to spin scattering is 0.8 mT/ppm Mn. Thus, except perhaps in the least pure samples, residual spin scattering makes no significant contribution. As a further check, we see that $B_\phi$ saturates to the *same* value in all samples, whereas the Mn impurity content varies between 0.1 and 2.4 ppm. Thermal decoupling does not occur because the sample is immersed directly in liquid helium and the current density through the sample was kept below 25 A/cm$^2$. There are other instances in the literature[23] where it is clear that the saturation of $B_\phi$ is not caused by either of these two effects. An alternative

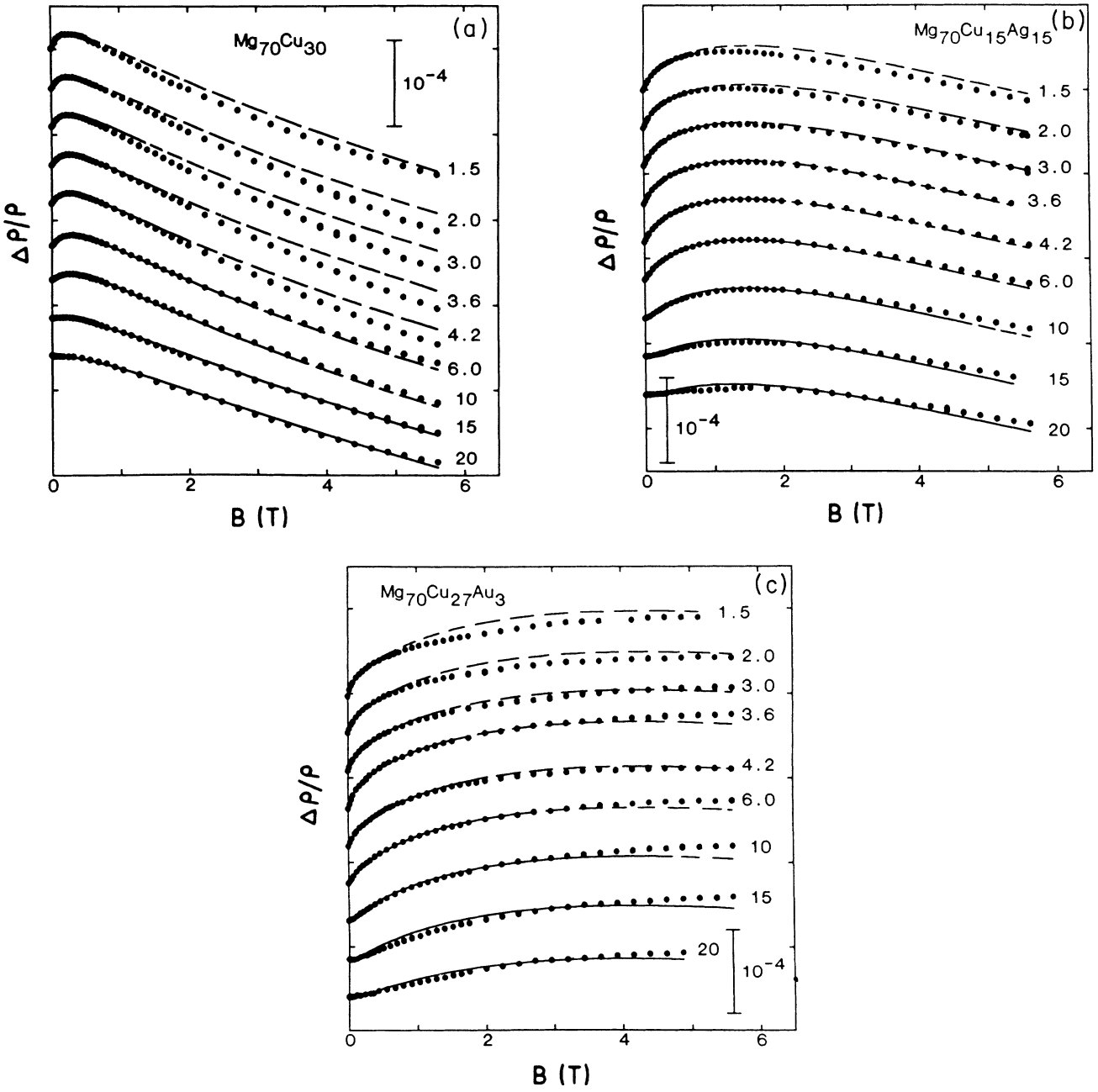


FIG. 1. Magnetoresistance of (a) $Mg_{70}Cu_{30}$, (b) $Mg_{70}Cu_{15}Ag_{15}$, and (c) $Mg_{70}Cu_{27}Au_3$. The scale and temperatures (in K) are indicated in the figures. The points are the experimental data, the solid line the fitted magnetoresistance, and the dashed line its extrapolation as described in the text.

proposal by Kumar *et al.*,[24] in which dephasing is an effect of zero-point motion, gives a low-temperature limit for $\tau_\phi$ of

$$\tau_\phi^0 \approx \frac{M}{m}\frac{\Theta_D}{T_F}\frac{D}{D_0}\frac{h}{\varepsilon_F}\left[\frac{\xi}{a}\right]^4, \quad (1)$$

where $M$ is the ion mass, $m$ the electron mass, $D_0 = h/m$ and the average interatomic spacing, and $\xi$ is the lesser of the phonon coherence length and $\sqrt{D}/\omega_D$. $\tau_\phi^0$ agrees with our saturated value of $\xi \sim 15$ Å, a reasonable value for the phonon coherence length.[25] However, doubts have been cast on the validity of the model[26] and more work is needed to resolve the question.

Figure 5 shows $1/\tau_{s.o.}$ in both Mg-Cu-Ag and Mg-Cu-Au. The most striking feature is the nonlinear behavior of $1/\tau_{s.o.}$ with concentration, $x$, for larger $x$. We have no comment on this except to say that it indicates a breakdown of the perturbation treatment of spin-orbit scattering.[27] To a certain extent the effect is consistent with previous observations. At low concentrations the data for Mg-Cu-Au agree with the data obtained by Bergmann[28]

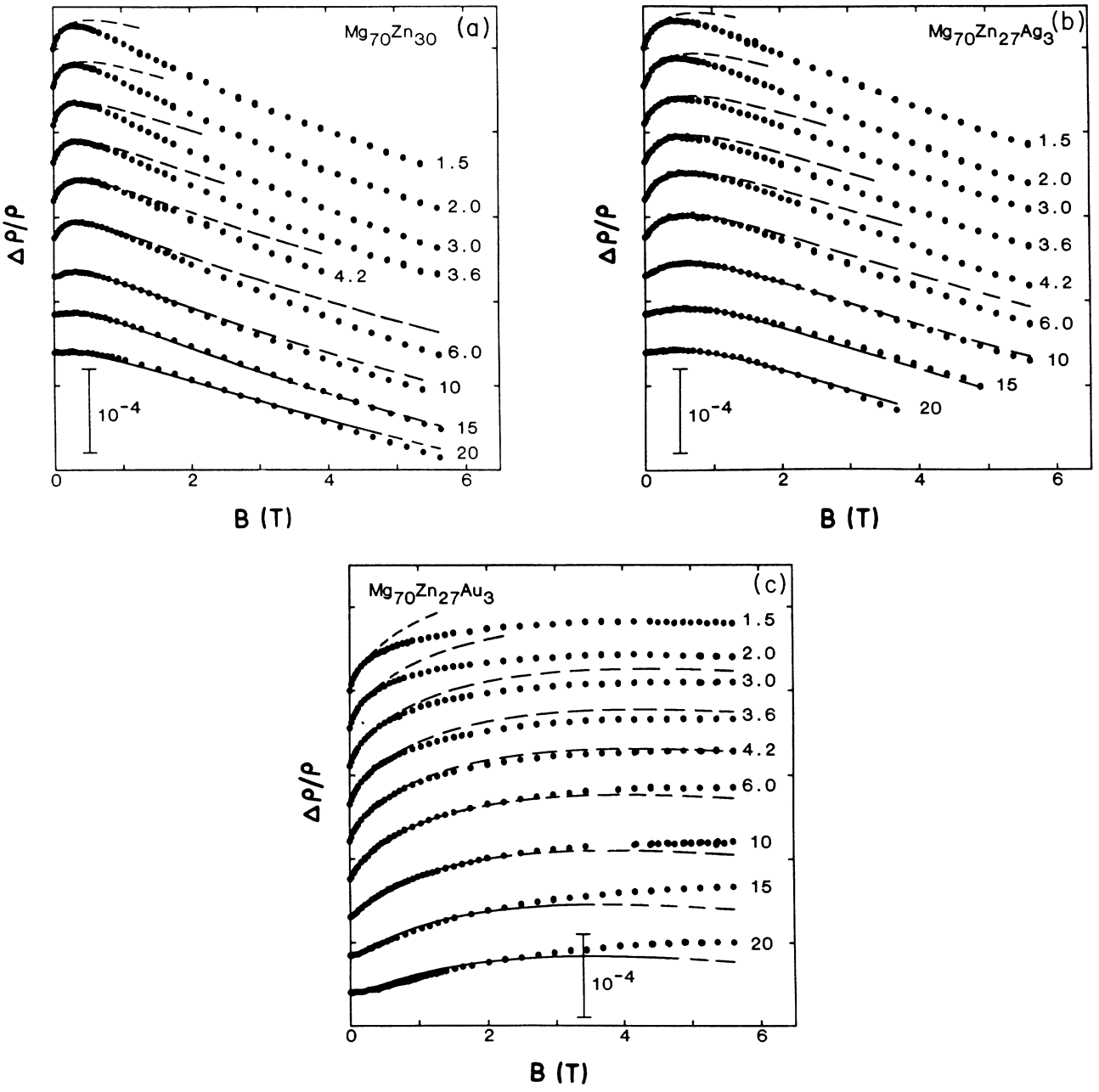


FIG. 2. Magnetoresistance of (a) $Mg_{70}Zn_{30}$, (b) $Mg_{70}Zn_{27}Ag_3$, and (c) $Mg_{70}Zn_{27}Au_3$. The scale and temperatures (in K) are indicated in the figures. The points are the experimental data, the solid line the fitted magnetoresistance, and the dashed line its extrapolation as described in the text.

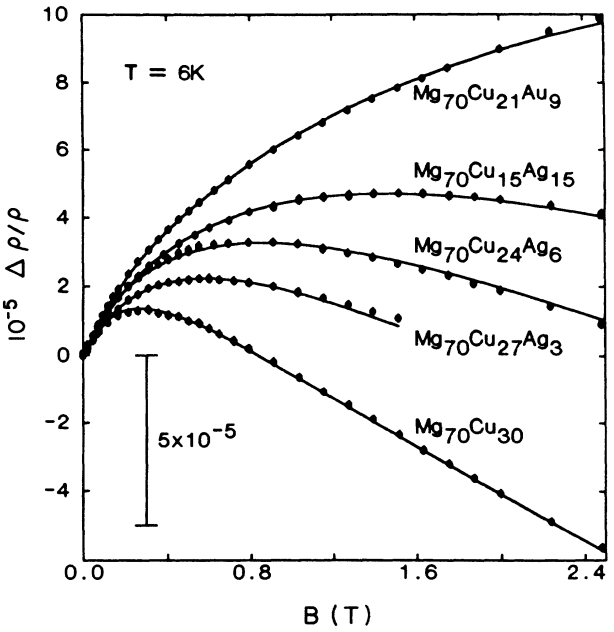


FIG. 3. Comparison of the size of the magnetoresistance of some Mg-Cu-based alloys at $T=6$ K. The scale and alloy composition is indicated in the figure. The points are the experimental data and the solid line the fitted magnetoresistance.

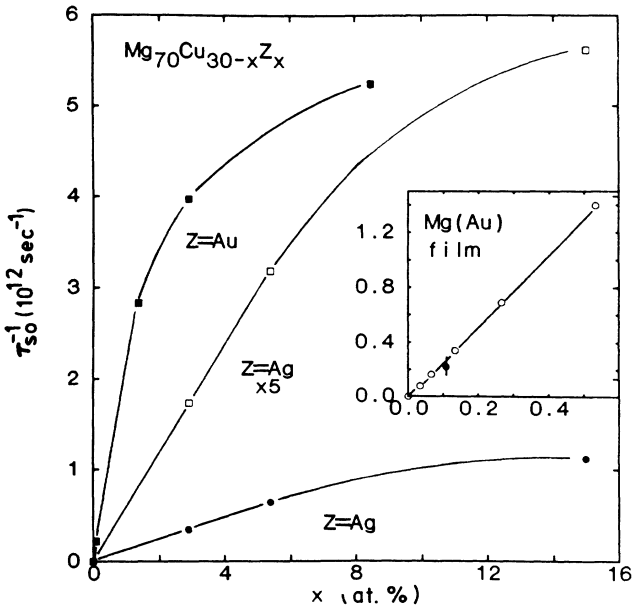


FIG. 5. The spin-orbit scattering rate $1/\tau_{s.o.}$ as a function of Au and Ag content. The insert compares our value for $Mg_{70}Cu_{29.9}Au_{0.1}$ (●) and data on Mg(Au) thin films (○) obtained by Bergmann (Ref. 28).

for two-dimensional Mg films doped with Au, as can be seen in the inset in Fig. 5. At larger concentrations of Au our value for $1/\tau_{s.o.}$ is slightly less than that seen in pure Au films.[29] A nonlinear scaling of $1/\tau_{s.o.}$ has also been observed by Hickey, Greig, and Howson[19] in Cu-Ti-Au glasses. Thus we believe that the behavior of $1/\tau_{s.o.}$ reflects a real physical phenomenon rather than any systematic failure of the expression for the weak-localization magnetoresistance. We also note that the increase of spin-orbit scattering rate with Au and Ag concentration is much greater in Au-doped samples than Ag-doped samples, as expected. The ratio between the two is about a factor of 19 that is somewhat larger than the $Z^4$ dependence on atomic number found from superconductivity.[30]

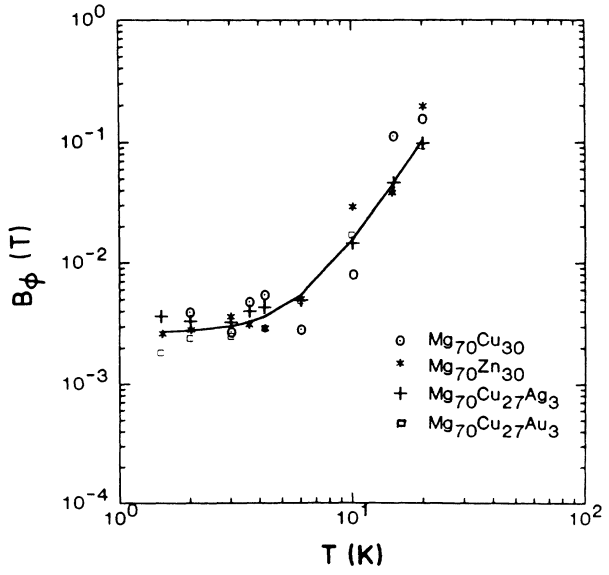


FIG. 4. The dephasing field $B_\phi$ as a function of temperature. The solid line is a fit described in the text.

Although quantum correction theories appear to work well at low $B/T$, at high $B/T$ the situation is quite different. One can see from Fig. 1 that the theoretical curve at high field for weak localization (dashed curve), in all samples and at all temperatures, lies above the experimental data. It would be tempting to assume that the difference may be accounted for by including the contributions from enhanced electron-electron interaction. That this is *not* the case, however, may be seen immediately, because both the diffusion and Cooper channel [Eqs. (A2) and (A3)] give a *positive* contribution to the magnetoresistance in Mg-Cu alloys (since the alloys are not superconductors), and so they actually increase the discrepancy between theory and experiment at lower temperatures.

Thus we conclude that the theories of quantum corrections to the conductivity cannot explain the observed magnetoresistance over the full range of field, even in so simple and well defined a system as Mg-Cu. Whether the discrepancy results from deficiencies in the high-field form of the localization and interaction theoretical expressions, or whether there is some extra contribution from a different source cannot be deduced from these results alone. Similar problems have been reported recently by Bieri, Fert, and Schuhl,[6] who point out that this may reflect a limitation of Eq. (A1), as discussed recently by Isawa.[31] However even if Isawa's result is generalized to

finite spin-orbit scattering and even if an internal consistency in his calculation is removed, a discrepancy remains which cannot be explained.

In spite of these limitations, we may still obtain an estimate of the Cooper channel and superconducting fluctuation contributions by examining the difference between comparable alloys in the Mg-Cu and Mg-Zn systems (Fig. 6). As mentioned in the Introduction, these two systems are identical except for the presence of superconductivity in the latter (and a negligible difference in the spin-orbit scattering). Therefore, if we assume that the discrepancy between theory and experiment is not influenced by the presence of superconductivity, the data in Fig. 6 represent only the changes in the contributions from superconducting fluctuations and the Cooper channel.

Recalling that the superconducting fluctuation magnetoresistance is positive, monotonic with field, and dependent on temperature primarily through $B_\phi$, it is clear that the data in Fig. 6 cannot be explained on the basis of superconducting fluctuations alone. However, these data can be described quantitatively, with no free parameters, by the combined effects of fluctuations and the Cooper channel, as we now show.

Figure 7 shows the resistance as a function of temperature for $Mg_{70}Zn_{27}Au_3$ and $Mg_{70}Zn_{30}$ near their respective superconducting transitions. For $Mg_{70}Zn_{30}$ we find that $T_c = 0.12$ K, in good agreement with that reported by Van der Berg *et al.*[11] while for $Mg_{70}Zn_{27}Au_3$ $T_c$ is less than 90 mK. In fact we may estimate $T_c \approx 40 \pm 15$ mK for $Mg_{70}Zn_{27}Au_3$ by fitting[32] the data to the theory for fluctuation conductivity above $T_c$ (this value was also used for $Mg_{70}Zn_{27}Ag_3$). Using this value for $T_c$ and numbers from Table I, we may compute the Cooper-channel contribution for both $Mg_{70}Zn_{27}Au_3$ and $Mg_{70}Cu_{27}Au_3$ according to Eq. (A3). At the temperatures considered here ($T \gg T_c$) the Aslamasov-Larkin superconducting fluctuation magnetoresistance is negligible and only the Maki-Thompson fluctuation term contributes.[33] Its magnitude has been calculated by Larkin.[34] Computing this term using the above value for $T_c$ and values for $B_\phi$ from the fit in Fig. 4 gives a reasonable zero-free-parameter agreement between theory and experiment, as is shown as solid lines in Fig. 6. Similar results may be obtained from the difference between $Mg_{70}Zn_{30}$ and $Mg_{70}Cu_{30}$, and $Mg_{70}Zn_{27}Ag_3$ and $Mg_{70}Cu_{27}Ag_3$.

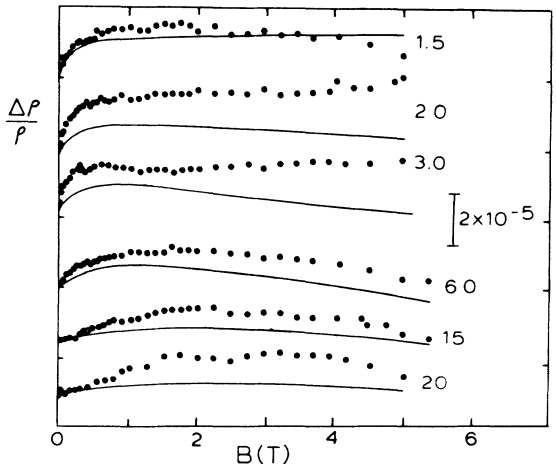


FIG. 6. Difference between the magnetoresistance of $Mg_{70}Zn_{27}Au_3$ and $Mg_{70}Cu_{27}Au_3$. The solid line is the sum of the superconducting fluctuation magnetoresistance and the difference in the respective Cooper channel magnetoresistance. Temperatures (in K) and scale are indicated in the figure.

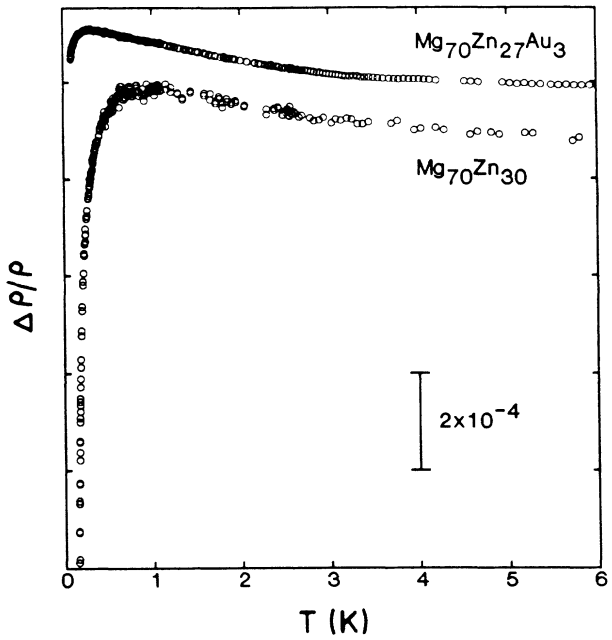


FIG. 7. Low-temperature resistance of $Mg_{70}Zn_{30}$ and $Mg_{70}Zn_{27}Au_3$.

With confidence in the validity of the Maki-Thompson term, we may now go back to fit the magnetoresistance of the Mg-Zn alloy system at low values of $B/T$ (as for Mg-Cu), including the contribution from superconducting fluctuations in addition to weak localization. Here too we fit with a common $B_{s.o.}$ for each alloy and adjust only $B_\phi$ for each temperature. For $\beta$ we use values calculated according to (A5). A representative result of the fitting is shown in Fig. 8 where the magnetoresistance is plotted against $B/T$. The values of $B_\phi$ and $B_{s.o.}$ are shown in Fig. 4 and Table I, respectively. It will be seen that these parameters are essentially the same as in Mg-Cu, providing further support for our analysis.

## IV. CONCLUSION

From our measurements in the free-electron-like amorphous alloys Mg-Cu and Mg-Zn, we conclude the following.

(1) At low fields, where enhanced electron-electron interaction effects are negligible, weak localization (including contributions from superconducting fluctuations for Mg-Zn glasses) provides an excellent description of the magnetoresistance, both in the low and high spin-orbit scattering regime. In particular the size of the effect is

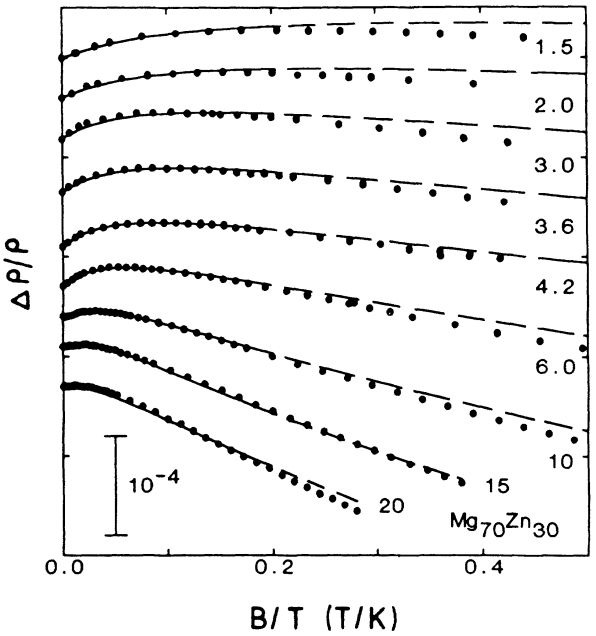


FIG. 8. The magnetoresistance of $Mg_{70}Zn_{30}$ plotted against $B/T$. The scale and temperatures (in K) are indicated in the figure. The points are the experimental data, the solid line is the fitted magnetoresistance, and the dashed line its extrapolation as described in the text.

given accurately by the theoretical expression of Fukuyama and Hoshino[27] (A1), without the need for an arbitrary scaling factor $\alpha$, as has been used by some authors.[6] Recently Baxter, Richter, and Strom-Olsen[35] have shown how this may be used to determine the resistivity of a sample in a way which is independent of the sample geometry.

(2) At high fields there are significant deviations between quantum correction theories and measured magnetoresistance, and this cannot be attributed to the contributions from enhanced electron-electron interactions. The weak-localization calculation of Fukuyama and Hoshino is inaccurate at these fields.

(3) The dephasing time above about 4 K is controlled by inelastic electron-phonon scattering, but below 4 K saturates. The saturation is not consistent with residual spin scattering, but is consistent with a simple model calculation for dephasing due to zero-point motion.

(4) The spin-orbit scattering rate obtained by adding Ag and Au increases with concentration, but not linearly. The values at low concentrations of Au compare well with those in Au-covered two-dimensional Mg films, and those at high concentrations are consistent with values seen in pure Au films.

(5) The *difference* between the magnetoresistance in Mg-Cu and Mg-Zn based alloys can be quantitatively explained (with no free parameters) by a combination of Cooper-channel interactions and the Maki-Thompson term for superconducting fluctuations.

## ACKNOWLEDGMENTS

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## APPENDIX

### Weak localization

In the presence of spin-orbit scattering and inelastic scattering, and including the splitting of the spin-up and spin-down bands in the applied field, quantum interference in the weakly localized regime contributes a term[27,36]

$$\left[\frac{\delta\rho}{\rho}\right]_{\rm WL}=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\left\{\frac{1}{2\sqrt{1-\gamma}}\left[f_3\left[\frac{B}{B_-}\right]-f_3\left[\frac{B}{B_+}\right]\right]-f_3\left[\frac{B}{B_2}\right]-\left[\frac{4B_{\rm s.o.}}{3B}\right]^{1/2}\left[\frac{1}{\sqrt{1-\gamma}}(\sqrt{t_+}-\sqrt{t_-})+\sqrt{t}-\sqrt{t+1}\right]\right\} \tag{A1}$$

where

$$B_\pm=B_i+2B_s+\frac{2(B_{\rm s.o.}-B_s)}{3}(1\pm\sqrt{1-\gamma})\ ,$$

$$B_2=B_i+\tfrac{4}{3}B_{\rm s.o.}+\tfrac{2}{3}B_s\ .$$

The scattering fields $B_x$ are related to the various scattering times $\tau_x$ (inelastic scattering $\tau_i$, spin-orbit scattering $\tau_{\rm s.o.}$, or scattering of local magnetic moments, $\tau_s$) by the relation $B_x=\hbar/4eD\tau_x$, where $D$ is the diffusivity. For the purpose of the discussion here we have defined the dephasing field $B_\phi$, as $B_\phi=B_i+2B_s$. The other parameters are defined as

$$t=\frac{3(B_i+2B_s)}{4(B_{\rm s.o.}-B_s)}\ ,$$

$$t_\pm=t+\tfrac{1}{2}(1\pm\sqrt{1-\gamma})\ ,$$

$$\gamma=\left[\frac{3g^*\mu_BB}{8eD(B_{\rm s.o.}-B_s)}\right]^2\ ,$$

where $g^*$ is the effective $g$ factor, $g^*=2$ in metals and

$$f_3(x)=\sum_{n=0}^{\infty}a_n(x)$$

with

$$a_n(x)=2\left[n+1+\frac{1}{x}\right]^{1/2}-2\left[n+\frac{1}{x}\right]^{1/2}-\left[n+\frac{1}{2}+\frac{1}{x}\right]^{-1/2}\ .$$

This term dominates the magnetoresistance in the temperature and field regime under consideration. It should be noted that for systems with a large electron diffusivity (say $D > 3$ cm$^2$/sec) it reduces to the expression given by Altshuler and Aronov.[2]

### Electron-electron interactions

The two contributions here come from the particle-particle diagrams of small total momentum (the Cooper channel), and particle-hole diagrams of small momentum transfer (the diffusive channel). The diffusion channel contribution to the magnetoresistance (MR) has been calculated by Lee and Ramakrishnan[37,38] who give the expression

$$\left[\frac{\delta\rho}{\rho}\right]_{\rm DC}=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\frac{\tilde{F}_\sigma}{2\sqrt{\pi}}\left[\frac{\pi k_B T}{2DeB}\right]^{1/2}\times g_3\left[\frac{g\mu_B B}{k_B T}\right] \tag{A2}$$

where

$$\tilde{F}_\sigma=-\frac{32}{3F}\left[1+\frac{3F}{4}-(1+F/2)^{3/2}\right],$$

$$F=\frac{\int d\Omega\, V(q=2k_F\sin(\Theta/2))}{\int d\Omega\, V(q=0)},$$

$$g_3(x)=\int_0^\infty d\omega\left[\frac{d^2}{d\omega^2}[\omega N(\omega)]\right]\times(\sqrt{\omega+x}+\sqrt{|\omega-x|}-2\sqrt{\omega}).$$

$V(q)$ is the Fourier transform of the static screened Coulomb potential, $N(\omega)=1/(e^\omega-1)$, and $F$ is the average of the interaction on the Fermi surface over the solid angle $\Omega$. In contrast to transition metals, where $F$ is difficult to evaluate, one can calculate it quite easily in free electron metals by using Thomas-Fermi screening theory. By applying this to the alloys studied here it gives values of $\tilde{F}_\sigma=0.5$.

The Cooper-channel (CC) term has been calculated by Isawa and Fukuyama[39] for the case of electron repulsion (nonsuperconductors). It takes the form

$$\left[\frac{\delta\rho}{\rho}\right]_{\rm CC}=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\frac{3\pi^2}{2}\left[\frac{k_B T}{4eDB}\right]^2\times g(T,B)\Phi_F(B,T), \tag{A3}$$

where

$$\Phi_F(B,T)=-\sum_{k=0}^{\infty}k\left[\zeta(\tfrac{5}{2},\tfrac{1}{2}+(\gamma+k)/\gamma h)-\frac{2}{3}\left[\frac{\gamma h}{\gamma+k}\right]^{3/2}\right],$$

with

$$\gamma=\frac{2eDB_i}{\pi k_B T},\quad h=B/B_i,$$

$\zeta$ is the generalized Riemann zeta function,

$$g(T,B)=\frac{1}{\frac{2}{F}+\ln\left[\frac{1.13T_F}{T^*}\right]},$$

$$T^*=\max\left\{T,\frac{4DeB}{k_B}\right\}.$$

It should be noted that this expression is valid for superconductors as well, if the electron coupling $g$ is replaced by $g^{-1}(T,B)=-\ln(T^*/T_c)$. Indeed for long phase coherence times it is equivalent to the expression derived by Altshuler and Aronov[2,40] for superconductors provided the prefactor in Ref. 40 is corrected for a missing factor of $1/\pi$. Note that Eq. (A3) does not depend on the spin-orbit scattering strength, because the spin-orbit-scattering-dependent triplet Hartree term cancels with the exchange term.[2,30]

### Superconducting fluctuations

Fluctuation effects can influence the conductivity of a superconductor at temperatures above $T_c$ either directly, by carrying a current, or indirectly through their influence on the (normal) quasiparticles. The direct contribution (known as the Aslamazov-Larkin term[41,42]) is only important quite close to the superconducting transition,[43] and we can ignore it in the present measurements as the lowest temperatures considered are larger than $12T_c$. The field dependence of the indirect (or Maki-Thompson) term has been derived by Larkin,[34] and may be expressed as

$$\left[\frac{\delta\rho}{\rho}\right]_{\rm MT}=\rho\frac{e^2}{2\pi^2\hbar}\left[\frac{eB}{\hbar}\right]^{1/2}\beta(T)f_3\left[\frac{B}{B_i}\right], \tag{A4}$$

where $f_3$ was defined above. In the limit of small applied fields and small inelastic fields,

$$B,B_i\ll\frac{k_B T}{4De}\ln(T/T_c)$$

the function $\beta$ can be written as

$$\beta(T)=\frac{\pi^2}{4}\sum_{m=-\infty}^{\infty}(-1)^m\Gamma(|m|)-\sum_{n=0}^{\infty}\Gamma''(2n+1), \tag{A5}$$

where

$$\Gamma(|m|)=[-g(T)^{-1}+\Psi\left[\frac{1}{2}+\left|\frac{m}{2}\right|\right]-\Psi(\tfrac{1}{2})]^{-1},$$

$\Psi$ is the digamma function, and $g(T)$ was given earlier in (A3). Because of the logarithmic temperature dependence of $\beta$ the Maki-Thompson superconducting fluctua-

tion magnetoresistance must be considered even well above $T_c$. The reader should note that the expression Larkin has derived for $\beta(T)$ contains a typographical error causing a singular behavior of $\beta$ at some temperatures above $T_c$. Correcting for it (see Lopes Dos Santos and Abrahams[44]) and recalculating the table Larkin presented for $\beta$ yields values that are essentially the same as in his paper.

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