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Fractional quantum Hall effect at even-denominator filling fractions

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The recently discovered fractional quantum Hall effect at $\frac{5}{2}$ filling has been studied by the standard technique of finite electron systems in a periodic rectangular geometry. We have evaluated the low-lying excitations and the correlation functions for $\frac{1}{2}$ filling in the lowest Landau level, as well as in the next Landau level with the lowest Landau level fully filled. The results indicate that the origin of the $\frac{5}{2}$ effect is due to the same incompressible-quantum-fluid state which manifests itself at odd-denominator fractions.

The recent experimental observation[1] of the fractional quantum Hall effect (FQHE) at an *even*-denominator filling fraction of $\frac{5}{2}$ has confirmed the long-standing conjectures[2] that FQHE is not a phenomenon exclusively for odd-denominator fractions, as was found earlier.[3] The theoretical explanation of the odd-denominator effect has been put forward, quite satisfactorily, by Laughlin.[4] In this case, the electrons are expected to condense into an incompressible-quantum-fluid state with several interesting properties.[5] The observation of FQHE at even-denominator filling fractions has always been considered an interesting possibility. It was suggested by Halperin[5] that an effect at the half-filled Landau level might be possible with electron pairs. Layered electron systems are also found to be prospective systems[6] for observation of the effect at $\frac{1}{2}$ filling of the lowest Landau level. For the $\frac{5}{2}$ effect the authors of Ref. 1 have suggested the possible pairing mechanism involving spin-reversed electrons, which were earlier found to influence the FQHE at low magnetic fields.[7]

In this Rapid Communication we have studied $\frac{1}{2}$ filling in the lowest Landau level as well as in the next Landau level ($n=1$ with n being the Landau-level index). Utilizing the method of exact diagonalization for finite-size systems in a periodic rectangular geometry, we have found indications that the FQHE at $\frac{5}{2}$ is due to the incompressible-fluid state similar to the one which is expected to be present at the odd-denominator filling fractions. We found that the electron system at $\nu=\frac{1}{2}$ ($\nu=2\pi l_0^2\rho$, l_0 is the magnetic length and ρ is the areal density) is not quite a translationally invariant liquid. However, the low-lying excitations and the correlation functions in the second Landau level have a liquidlike behavior. The difference between the results at the two Landau levels is attributed to the difference in the wave functions at the two Landau levels.

In order to study the $\frac{1}{2}$ and $\frac{5}{2}$ states, we have employed the standard procedure of numerical diagonalization of the Hamiltonian for finite-size systems.[7-9] The electrons are considered to be in a fully spin-polarized state. Despite the fact that $\frac{5}{2}$ is observed at a relatively low magnetic field ($B \lesssim 5$ T), the ratio $(e^2/\epsilon l_0)/\hbar\omega_c$ is still small (~ 0.1). (Here, ω_c is the cyclotron frequency and ϵ is the background dielectric constant.) The cyclotron energy $\hbar\omega_c$ is therefore quite large compared to the interaction energy $e^2/\epsilon l_0$, which is the energy scale used throughout. Therefore, when we consider $\frac{1}{2}$ filling in the second Landau level, the influence of the lowest filled Landau level could be safely ignored.

For a finite number of electrons N_e in a rectangular cell, we impose the periodic boundary conditions on both sides[8] such that the filling fraction is $\nu=N_e/N_s$, with N_s being the number of flux quanta passing through the cell. We have chosen the Landau-gauge vector potential. As we have discussed above, when we consider the fractional filling in the second Landau level, the lowest filled Landau level could be considered as an uniform background. The Hamiltonian in the $n=0,1$ Landau levels could then be written as (ignoring the kinetic energy and single-particle terms in the potential energy which are constants),

$$\mathcal{H}=\sum_{j_1,j_2,j_3,j_4} \mathcal{A}_{nj_1nj_2nj_3nj_4} a_{nj_1}^\dagger a_{nj_2}^\dagger a_{nj_3} a_{nj_4} ,$$

$$\mathcal{A}_{nj_1nj_2nj_3nj_4}=\delta'_{j_1+j_2,j_3+j_4}\mathcal{F}_n(j_1-j_4,j_2-j_3) ,$$

$$\mathcal{F}_n(j_a,j_b)=\frac{1}{2ab}\sum_{\mathbf{q}}{}'\sum_{k_1}\sum_{k_2}\delta_{q_x,2\pi k_1/a}\delta_{q_y,2\pi k_2/b}\delta'_{j_a k_2}\frac{2\pi e^2}{\epsilon q} \times\mathcal{B}_n(q)\exp(\tfrac{1}{2}q^2 l_0^2-2\pi i k_1 j_b/N_s) ,$$

$$\mathcal{B}_n(q)=\begin{cases}1 & \text{for } n=0 ,\\ (1-\frac{1}{2}q^2l_0^2)^2 & \text{for } n=1 .\end{cases}$$

Here a and b are the two sides of the rectangular cell and are related to the number of flux quanta as $ab=2\pi l_0^2 N_s$. The Kronecker δ with prime means that the equation is defined modN_s, and the summation over q excludes $q_x=q_y=0$. Earlier studies of the spectrum with this Hamiltonian have been very effective in obtaining the correct properties of the incompressible state at $\nu=\frac{1}{3}$.

The numerical diagonalization scheme was described in detail earlier by Haldane.[9] Here we present a brief description of the method. For every lattice vector $\mathbf{L}_{mn}$, there is a relative translation operator which commutes with the Hamiltonian. The eigenvalues of this operator are easily obtained to be $e^{2\pi i(ms+nt)/N}$, where N is the highest common divisor of N_e and N_s. The quantum numbers s and t $(s,t=0,1,\ldots,N-1)$ are related to the physical momentum by

$$kl_0=\left(\frac{2\pi}{N_s\lambda}\right)^{1/2}[s-s_{0},\lambda(t-t_0)]\,,$$

where the point (s_0,t_0), corresponding to the state $\mathbf{k}=\mathbf{0}$, is required to be the most symmetric point of the reciprocal lattice, and λ is the aspect ratio.

Let us first consider $\frac{1}{2}$ filling of the lowest Landau level. The excitation spectrum in this case is shown in Fig. 1(a) for a seven-electron system. The ground state does not appear at $\mathbf{k}=\mathbf{0}$, but for a finite value of $\mathbf{k}$, as we found earlier.[6] This value of $\mathbf{k}$ changes with the number of electrons, and we have not been able to extract any useful relation between these two quantities. The low-lying excitations have a few almost degenerate energy levels. The spectrum for larger system sizes provides quite similar results. Qualitatively similar results were also reported earlier by Fano, Ortolani, and Tosatti.[10]

In order to obtain some more information about this state, we have also calculated the pair-correlation function defined as[8]

$$g(\mathbf{r})\equiv\frac{ab}{N_e(N_e-1)}\langle\Psi|\sum_{i\neq j}\delta(\mathbf{r}+\mathbf{r}_i-\mathbf{r}_j)|\Psi\rangle\,,$$

where $|\Psi\rangle$ is one of the eigenstates. In Fig. 2(a), we present the $g(\mathbf{r})$ of the ground state for a five-electron system in the lowest Landau level. The function has very little structure and is obviously not isotropic. Qualitatively similar results were obtained for other system sizes. In Fig. 2(b), we have presented the correlation function for the same system as in Fig. 2(a), but at $\mathbf{k}=\mathbf{0}$. The function is very much isotropic (within the rectangular geometry) and has a liquidlike behavior. It should be noted, however, that this is an excited state for the half-filled lowest Landau level. The results do not change qualitatively with other system sizes. These results indicated that the half-filled lowest Landau level is presumably *not* a stable state. The translationally invariant liquid state appears as an excited state of the system. This could be presumably the explanation for the absence of the $\frac{1}{2}$ plateau in the FQHE experiments.

The situation is very different in the higher Landau levels. In the case of the fully filled lowest Landau level and the next Landau level only half-filled, the excitation spec-

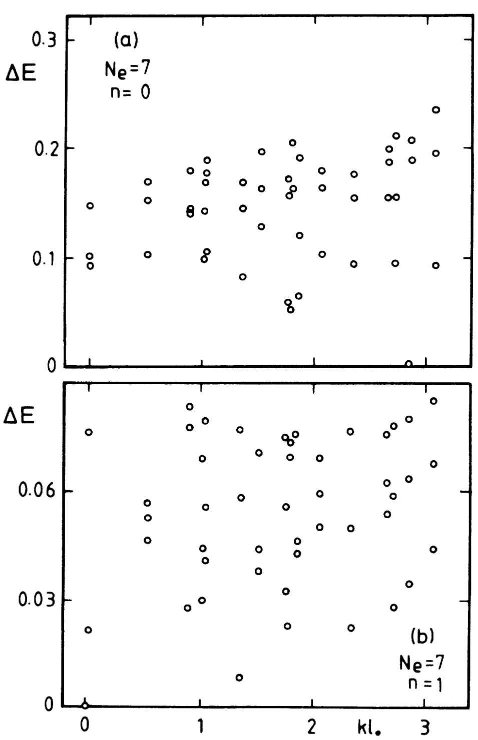

FIG. 1. Low-lying excitations (in units of $e^2/\epsilon l_0$) for the half-filled (a) lowest Landau level and (b) second Landau level, when the lowest Landau level is fully occupied, for a seven-electron system in a periodic rectangular geometry.

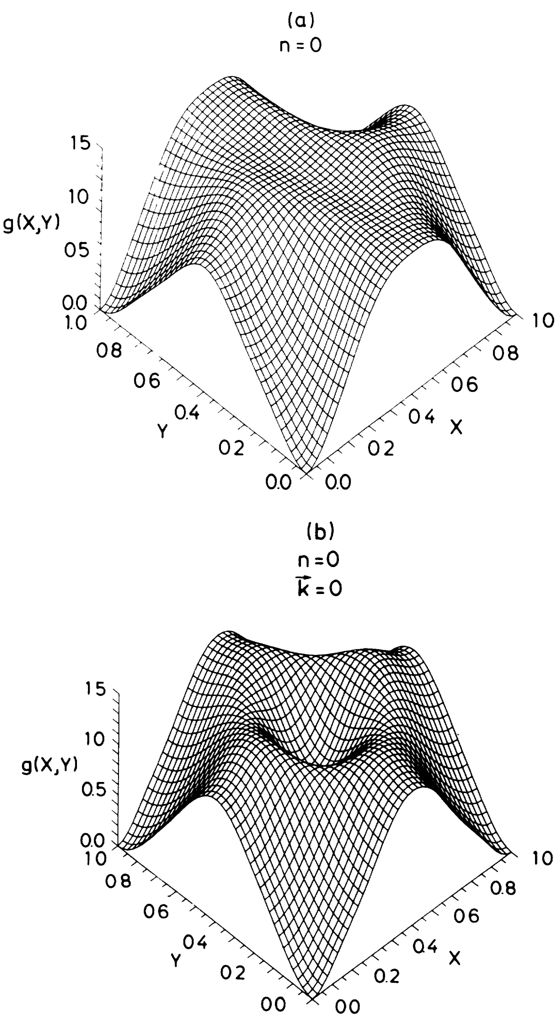

FIG. 2. Perspective view of the pair correlation function $g(\mathbf{r})$ for a five-electron system at $\nu=\frac{1}{2}$ in the lowest Landau level (a) for the ground state and (b) at $\mathbf{k}=\mathbf{0}$, which is an excited state in the system. The axes are normalized as $X=x/a$ and $Y=y/b$.

trum is markedly different. The spectrum for $N_e=5$ in the $n=1$ Landau level is shown in Fig. 1(b). The ground state appears to be at $\mathbf{k}=\mathbf{0}$, and a gap structure can be seen in the spectrum. The gap is, however, very small. This is perhaps the reason for the difficulty in obtaining a clear plateau at $\frac{5}{2}$. For all the other *odd*-number electron systems, qualitatively similar spectrum is observed. For the *even* number of electrons in the rectangular cell, the ground state does not appear at $\mathbf{k}=\mathbf{0}$, but always appears at $s=\frac{1}{2}N$, $t=0$. Therefore, we can scale the ground state to $\mathbf{k}=\mathbf{0}$ by writing

$$kl_0=\left(\frac{2\pi}{N_s\lambda}\right)^{1/2}[(s-\tfrac{1}{2}N)^2+\lambda^2t^2]^{1/2}.$$

This scaling procedure in the case of rectangular geometry preserves all the symmetries of the system. After this redefinition of the ground-state wave vector, the excitation spectrum is found to have the gap structure, qualitatively similar to those for the odd number of electrons. For the square geometry, it is not possible to redefine the ground state as above; however, the ground-state energy in this geometry is slightly higher (~ 0.007) than in the case of the rectangular geometry. Therefore, in marked contrast to the lowest Landau level, the ground state at $\frac{5}{2}$ always appears at $\mathbf{k}=\mathbf{0}$. It should be remarked that a similar procedure cannot be applied to the results in the lowest Landau level because there the ground-state wave vector changes discontinuously with system size.

The correlation function for the five-electron system in the $n=1$ Landau level is shown in Fig. 3. There is more structure in this function, as compared to the $g(\mathbf{r})$ in the lowest Landau level. At $\nu=\frac{1}{3}$, MacDonald[11] generalized the Laughlin wave function for the $n=1$ level. We note that the structures in our correlation function are qualitatively similar to his result. No long-range order is noticeable in this function. The qualitative features of the correlation functions described in the present work are the same for larger systems. Five-electron system results are the simplest ones where the features are most clearly seen.

Earlier studies of the FQHE states for odd-denominator

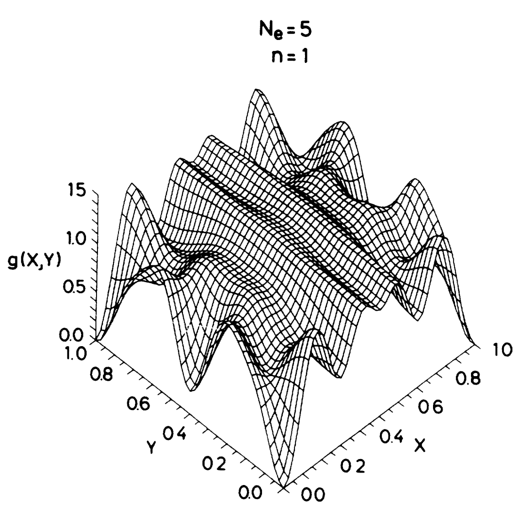

FIG. 3. Same as in Fig. 2(a), but in the second Landau level.

TABLE I. Ground-state energies (in units of $e^2/\epsilon l_0$) for four- and six-electron systems at $\nu=\frac{5}{2}$ for various values of the total spin S. The Zeeman energy is not included in the energy values.

N_e	$S=0$	$S=1$	$S=2$	$S=3$
4	−0.3644	−0.3655	−0.3849	. . .
6	−0.3782	−0.3783	−0.3785	−0.3797

fillings in the higher Landau levels[12] also found a very different behavior as compared to the case of fractional filling of the lowest Landau level. For example, in the periodic rectangular geometry at $\nu=\frac{1}{3}$ in the second Landau level, the spectrum showed a *gapless* compressible mode,[12] in contrast to the lowest Landau level, where there is a large gap. This difference in the results for the two Landau levels can be attributed to the presence of nodes in the $n=1$ wave function at the center of the cyclotron orbit. The situation is similar to the present half-filled systems.

Considering the low magnetic fields where $\frac{5}{2}$ is experimentally observed, it is quite tempting to include the spin reversal of some of the electrons in this state. The Zeeman energy, however small it is, would then of course add a nonvanishing contribution to the exchange energy.[7] The Zeeman energy (per particle) for different spin polarization is given as $E_Z=(1-2p)g\mu_B Bs$, where p is the ratio of the number of spins parallel to the field to the total number of spins, g is the Landé g factor, $\mu_B=e\hbar/2mc$ is the Bohr magneton, and $s=\frac{1}{2}$. For GaAs with all spins parallel to the field, $E_Z=-0.008e^2/\epsilon l_0$ for $B=5$ T and $g\simeq 0.52$, and $\epsilon\simeq 13$. We have studied the ground state for four- and six-electron systems at $\nu=\frac{1}{2}$ in the second Landau level for different spin polarizations. The results are presented in Table I. The system so far has a fully spin-*polarized* ground state, even in the absence of the Zeeman energy. The energy difference between the various spin states is very small. However, for a spin-reversed system to be energetically favored, the energy of this state must be larger than that of the spin-polarized state by at least the Zeeman-energy contribution. Such a situation might occur for larger systems than the ones considered here, as the even-denominator system results are known to be very much system-size dependent. It should be noted, however, that the few characteristics of the Laughlin-type incompressible fluid state, which we have found for the fully spin-polarized state, have to be retained by the spin-reversed state, in order to explain the FQHE at $\frac{5}{2}$. Therefore, we do not expect any major change in the present results, even if the spin reversal wins over the Zeeman energy for large systems. From these observations, we wish to conclude that the state $\frac{5}{2}$ is the same incompressible quantum fluid state that Laughlin originally proposed for the odd-denominator filling fractions.

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