Amplification of a new surface plasma mode in the type-I semiconductor superlattice

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Mode structure and drift velocity thresholds for instability have been investigated for the type-I superlattice, and a new current-induced mode is found to exist for high drifts. Simple separation of the single-layer physics and superlattice geometry has been achieved, leading to an analytical expression for the Giuliani-Quinn mode and universality relations for the instability threshold curves of the new mode.

While the current-excited plasma waves have been studied, and are well known in gaseous plasmas,[1,2] corresponding areas in solid-state plasmas are still in the early stage of investigation. Several attempts, so far unsuccessful, have been made to observe such instabilities experimentally both in bulk and in layered semiconductors.[3–7] The major problem has been to reach the instability threshold, i.e., the carrier velocity at which the transfer of energy from the current into plasma oscillations begins. Since this threshold velocity is high, one needs a mobile plasma to achieve amplification. Recent progress in the semiconductor technology has led to layered systems in which two-dimensional (2D) plasmas of very high mobility are formed.[8] Typically a uniform electric field is applied in a direction parallel to the layered planes. The carriers moving under the influence of this field produce a current parallel to the applied field, and current-driven instabilities should be observed in such systems. Recently we has shown that such indeed is the case for a layered 2D electron-hole gas (type-II superlattice)[9] for which we obtained the threshold criteria for the current-driven instability including, in contrast to previous approaches,[10–12] the scattering and carrier heating effects. The current-induced growth balances the inherent (single-particle) absorption at the threshold drift velocity.

In this paper we investigate the surface-mode structure of a single 2D layer of electrons as well as a multilayer 2D electron system, with and without current. We show, for the first time, that amplification is possible in a single-species environment, which effectively acts as a two-species environment due to an interplay between electron-electron and electron-phonon scattering processes. In particular, (1) we show that there is a very simple relation between the dispersion equation for a single layer and that of a type-I superlattice, (2) we obtain an explicit analytical solution of the dispersion relation for the surface mode in a single layer and a type-I superlattice in the absence of an applied electric field and at zero temperature, and (3) we demonstrate the feasibility of amplification of a new, current-induced surface plasma mode in a type-I superlattice within experimentally achievable drift velocities. It becomes evident in this context that inclusion of the single-particle excitation physics is essential, i.e., one *must* go beyond the commonly used "long-wavelength" version of the susceptibility.

We start with the single 2D layer and use the full random-phase approximation (RPA) for the susceptibility[13]

$$D(\mathbf{q},\omega)=2\int\frac{d\mathbf{p}}{(2\pi)^2}\frac{f(\mathbf{p}+\mathbf{q})-f(\mathbf{p})}{(2\mathbf{p}\cdot\mathbf{q}+|\mathbf{q}|^2)\frac{\hbar^2}{2m}-\hbar\omega}\,, \tag{1}$$

where $\mathbf{q}$ is the wave vector parallel to the layer, $\mathbf{p}$ is the electron wave number, and m is the effective mass of the electron.

The dispersion relation for the longitudinal plasma modes in a single layer embedded in a dielectric of strength ϵ is given by

$$V_q D(\mathbf{q},\omega)=F(q)\,, \tag{2}$$

where $V_q=2\pi e^2/q$, and $F(q)=\epsilon$ is a "structure factor." For a layered 2D electron gas, it is possible to express the dispersion relation for the surface modes in the same form, but with a geometry-dependent structure factor (or, an effective dielectric function)

$$F(q)=\frac{\epsilon+\epsilon_0}{2}\frac{\epsilon-\epsilon_0}{\epsilon\coth(qa)-\epsilon_0}\,, \tag{3}$$

where a is the layer spacing in the medium of dielectric ϵ, ϵ_0 is the outside dielectric constant, and the first layer separates the two media. This result can be easily derived from our formalism[9] for type-II heterostructures by simply setting the density (and hence the susceptibility) of one species equal to zero in Eq. (4) of Ref. 9. It is also straightforward to show that the dispersion relation of the surface mode for type-I superlattices in the form given by Jain and Allen[14] can be reduced to Eqs. (2) and (3).[15]

By comparison with previous versions,[14,16] it should be clear that our Eqs. (2) and (3) provide a significantly simpler way of expressing the surface plasmon dispersion relation than has been realized earlier. The effects of the medium geometry and the physics of the single layer are neatly separated in this formulation. The latter is fully represented by the susceptibility $D(\mathbf{q},\omega)$ on the left side of Eq. (2), and can incorporate various physical effects such as collisions, drifts, etc., through the distribution function in Eq. (1). In fact, even the RPA can be transcended (e.g., to include exchange and correlation effects), by using a more general $D(\mathbf{q},\omega)$. All the geometrical effects merely alter the right-hand side of Eq. (2), through $F(q)$.

One of the by-products of this formulation is that it allows us to find an explicit solution of the dispersion relation for a multilayered plasma described by a cold Fermi distribution. The susceptibility of the cold 2D electron gas is well known[13,14] ($\mathrm{Im}\omega > 0$),

$$D(q,\omega) = -\frac{nk_F}{\epsilon_F q}\left\{\frac{q}{k_F} - \left[\frac{1}{4}\left[\frac{2\omega}{qv_F}+\frac{q}{k_F}\right]^2 - 1\right]^{1/2} + \left[\frac{1}{4}\left[\frac{2\omega}{qv_F}-\frac{q}{k_F}\right]^2 - 1\right]^{1/2}\right\}, \quad (4)$$

where n is the surface density of electrons and k_F, v_F, and ϵ_F the Fermi wave vector, velocity, and energy. Combined with Eq. (2), it provides the explicit result

$$\omega^2 = \frac{2\pi ne^2 q}{mF(q)}[1+\rho QF(q)]^2\{[1+\tfrac{1}{2}\rho QF(q)]^{-1} + \tfrac{1}{2}\rho Q^3 F(q)\}, \quad (5)$$

where $Q = q/k_F$, $\rho = \hbar v_F/2e^2 = v_F/2\alpha c$, and α is the fine-structure constant. For the type-I superlattice [with $F(q)$ given by Eq. (3)], Eq. (5) provides the full analytical expression (not known before) for the Giuliani-Quinn (GQ) surface mode.[16] The previously reported,[17] approximate, long-wavelength version of Eq. (5) is obtained from our result by simply suppressing the Q terms in the brackets, leading to $\omega^2 = 2\pi ne^2 q/mF(q)$. With $F(q) = \epsilon$, Eq. (5) provides a similar generalization of the well-known dispersion relation[13] for a 2D layer in a uniform dielectric. For the superlattice, the range of validity of the mode described by Eq. (5) is the domain between the bulk-plasmon band and the single-particle continuum. This mode touches the single-particle continuum curve tangentially at $q_{\max}$ given by $\rho Q^2 F(q)[1+\frac{1}{2}\rho F(q)Q] = 1$, and there is no physical solution beyond this.

We next consider the effect of applying a uniform electric field. The electron distribution function $f(\mathbf{v})$ satisfies a Boltzmann equation which in general contains the one-body (electron-phonon or electron-impurity) and two-body (electron-electron) collision operators.[18] These operators can be modeled using the constant collision time approximations. The resulting form of the Boltzmann equation which includes the collisional and heating effects is[18,19]

$$\frac{e\mathbf{E}}{m}\cdot\frac{\partial f(\mathbf{v})}{\partial \mathbf{v}} = -\nu_{e\text{-ph}}[f(\mathbf{v}) - f_{\mathrm{eq}}(\mathbf{v},T_1)] - \nu_{ee}[f(\mathbf{v}) - f_{\mathrm{eq}}(\mathbf{v}-\mathbf{v}_{dr},T_2)], \quad (6)$$

where the drift velocity $\mathbf{v}_{dr} = e\mathbf{E}/m\nu_{e\text{-ph}}$, ν_{ee} and $\nu_{e\text{-ph}}$ are, respectively, the electron-electron and electron-phonon collision frequencies, T_1 is the lattice temperature, and T_2 the electron temperature. The equilibrium distribution function is the Fermi distribution of a 2D electron gas. The electron temperature T_2 is determined by the drift from kinetic energy conservation condition.[9] The first term on the right-hand side of Eq. (6) drives the distribution towards the lattice temperature equilibrium, while the second term provides the drifting, heated component. Thus, we have essentially two "species" (groups of electrons) moving with different average velocities.

The exact solution to Eq. (6) can be obtained in the form[20]

$$f(\mathbf{v}) = \int_0^\infty dx\, e^{-x}[\alpha_1 f_{\mathrm{eq}}(\mathbf{v}-\alpha_1 x\mathbf{v}_{\mathrm{dr}},T_1) + \alpha_2 f_{\mathrm{eq}}(\mathbf{v}-(1+\alpha_1 x)\mathbf{v}_{\mathrm{dr}},T_2)], \quad (7)$$

with $\alpha_1 = \nu_{e\text{-ph}}/(\nu_{ee}+\nu_{e\text{-ph}})$, $\alpha_2 = \nu_{ee}/(\nu_{ee}+\nu_{e\text{-ph}})$, $\alpha_1+\alpha_2 = 1$, leading to

$$D(\mathbf{q},\omega) = \alpha_1\tilde{D}_{\mathrm{eq}}(\mathbf{q},\omega,T_1,\alpha_1) + \alpha_2\tilde{D}_{\mathrm{eq}}(\mathbf{q},\omega-\mathbf{q}\cdot\mathbf{v}_{\mathrm{dr}},T_2,\alpha_1), \quad (8)$$

$$\tilde{D}_{\mathrm{eq}}(\mathbf{q},\omega,T,\alpha_1) = \int_0^\infty dx\, e^{-x} D_{\mathrm{eq}}(\mathbf{q},\omega-\mathbf{q}\cdot\mathbf{v}_{\mathrm{dr}}\alpha_1 x,T),$$

which expresses the complete susceptibility as a sum of two collision-weighted averages of the more elementary equilibrium susceptibility functions $D_{\mathrm{eq}}(\mathbf{q},\omega,T)$. These are for $T=0$ the analytically known Eq. (4), and are otherwise evaluated by numerical integration of Eq. (1) or by its expansion in a series of plasma dispersion functions.[21] The two terms in Eq. (8) describe the unshifted cold species, maintained by the electron-phonon scattering, and the Doppler-shifted and heated species provided by the electron-electron scattering.

At the threshold of instability, $\mathrm{Im}\omega = 0$. We also take $\mathbf{q}$ along $\mathbf{v}_{\mathrm{dr}}$. The dispersion relation then provides the threshold boundary curve, connecting q and v_{dr}. The interior domain is unstable. The boundary curves so obtained for a single layer as well as for a type-I superlattice are shown in Fig. 1. The parameters, typical for a GaAs-$\mathrm{Ga}_{1-x}\mathrm{Al}_x$As type-I modulation-doped superlattice, are $m = 0.0665m_e$, $\epsilon = 13.1$, $\epsilon_0 = 1.0$, $n = 10^{11}\ \mathrm{cm}^{-2}$. The dashed lines describe the instability threshold for a single surface layer (separating dielectrics ϵ_0 and ϵ) for two values of the ratio of the collision frequencies $R = \alpha_2/\alpha_1 = \nu_{ee}/\nu_{e\text{-ph}}$. These values ($R = 10$ and 5) span the estimated range for R in Ref. 18 for GaAs.[22] There is no instability for any q until $v_{\mathrm{dr}} \approx 2.5v_F$. The lowest threshold v_{dr} occurs for $q = 0$. As q increases, larger v_{dr} are required to achieve the threshold for amplification and eventually, above a characteristic q (for a given R), instability becomes impossible. The mode being amplified is an acoustic mode. On the boundary curve (e.g., for $R = 10$) the phase velocity $\approx 1.25v_F$. The solid lines describe the threshold curves for the superlattice with $a = 300$ Å. These computed curves, displayed in Fig. 1, can be easily understood as follows. As shown in Eq. (3), the only difference between a single layer and the superlattice arises through a geometrical factor [the last factor in Eq. (3)]. For small q, D is primarily a function of the ratio ω/q. Then for a given v_{dr} and R, the imaginary part of Eq. (2), $\mathrm{Im}D(\omega/q) = 0$, implies that the single layer and the superlattice modes have the same phase velocity ω/q, and the real part leads to the scaling relation for $\epsilon > \epsilon_0$,

$$q_1 = q_2\frac{\epsilon-\epsilon_0}{\epsilon\coth(q_2 a)-\epsilon_0}, \quad (9)$$

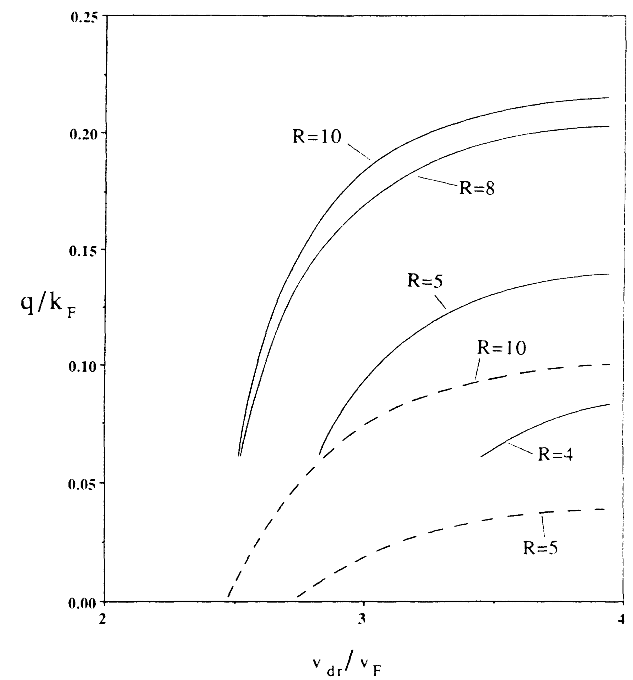

FIG. 1. Instability boundary curves for the type-I superlattice (solid lines) and single layer (dashed lines) for different values of the scattering ratio $R=\nu_{ee}/\nu_{e\text{-ph}}$. The dotted lines indicate continuation of the threshold boundary curves below the cutoff $q^*=0.064k_F$. Parameters are $m=0.0665m_e$, $\epsilon=13.1$, $\epsilon_0=1.0$, $n=10^{11}$ cm^{-2}, and superlattice constant $a=300$ Å.

where q_1 and q_2 are, respectively, the corresponding wave numbers of the single layer and the superlattice. This scaling relation is universal, valid for each v_{dr} and R. For $q_2a \ll 1$, this reduces to $q_2a=(q_1a)^{1/2}(1-\epsilon_0/\epsilon)^{-1/2}$. Consequently the minimum threshold drift velocity for the superlattice ($q_2=0$) is the same as that of the corresponding single layer ($q_1=0$). The superlattice curves are shown by a dotted line for $q<q^*=0.064k_F$ or $qa<q^*a=\ln|(\epsilon+\epsilon_0)/(\epsilon-\epsilon_0)|=0.153$ which corresponds to the still valid limit[16] at which the penetration depth of the surface mode in the direction of the superlattice axis becomes infinite. Thus only the solid-line portions describe the surface-confined modes for the superlattice. This cutoff in q decreases inversely with a and thus there is no forbidden wavelength for the single layer.

The unstable mode with the threshold boundaries described in Fig. 1 is a *new* mode, which arises from an interplay between the cold and the hot components of the electron distribution, and is quite distinct from the GQ mode. In order to understand the physical mechanism for the generation of this mode, we can consider the cold and the hot electrons as two species[23] as mentioned above Eq. (7). For a single species it is impossible to obtain a mode unless it is far from the single-particle absorption continuum. Otherwise the mode is damped due to single-particle excitations, or screened out due to the high polarizability. On the other hand, a combination of two species with a relative drift, under appropriate conditions, allows a cancellation of the high individual polarizabilities (to reach net polarizability $=-1$) and a compensation of the absorption by inverse Landau damping, thus leading to a "normal" mode. This interplay is quantitatively represented by Eq. (8). An immediate inference from the structure of D functions is that this mode is governed by the more stringent polarizability condition ($v_{\mathrm{dr}}>2v_F$), rather than the usual inverse Landau damping condition [$v_{\mathrm{dr}}>(\omega/k)\approx v_F$]. For small v_{dr} (and large R), the drifting hot component of the susceptibility dominates and the dispersion equation provides only one mode, the Doppler-shifted, slightly damped GQ mode. For large enough v_{dr}, however, the new, current-induced mode arises as described above. The GQ mode, on the other hand, remains slightly damped due to collisional effects and, in contrast to the new mode, has a very high phase velocity ($>10v_F$) for the domain of parameters of Fig. 1.

To observe the new mode, the drift velocity must exceed 3.5×10^7 cm/sec for the parameters of Fig. 1. Since this value is close to the maximum of experimentally observed drift velocities in GaAs superlattices, high-quality samples would be necessary. The threshold drifts, which scale as v_F, can be lowered by decreasing the electron density or increasing the effective mass.

In conclusion, we have investigated the surface-mode structure and instability conditions for the type-I superlattice, as well as a single layer. We found a simple form of the dispersion relation, which separates the 2D layer physics from the geometrical structure effects of the superlattice. This simple form yields an analytical formula for the full range of the GQ mode at $T=0$ and $v_{\mathrm{dr}}=0$, as well as a universal scaling property for the boundary curves for the instability of the new, current-induced, surface mode. This mode arises through a novel mechanism, an interplay between the two components of the distribution function. The existence of the two components is related to the presence of the two physical processes: the electron-phonon collisions and the electron-electron scattering. While we have modeled both of these as constant collision time processes in the present work, more refined collision models can certainly be devised, and we expect that the main results of this work would survive.

From the theoretical point of view, it is evident that one must include the single-particle effects as done here, for they are essential for the study of plasma instabilities. Any formulation which ignores these effects (e.g., long-wavelength limit) is generally inadequate. From the experimental point of view, the current-driven *unstable* modes would be easier to detect, and should be looked for above the drift thresholds indicated here. The study of this area is of importance for device applications as well, since with an appropriate coupling mechanism, the plasma energy of the amplified mode can be converted to electromagnetic radiation, leading to a potential new class of solid-state radiation sources.

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