

Calculation of spin waves in multilayered structures including interface anisotropies and exchange contributions

Burkard Hillebrands

Optical Sciences Center, University of Arizona, Tucson, Arizona 85721

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Spin-wave properties have been calculated for multilayered structures consisting of alternating ferromagnetic and nonmagnetic layers, as well as for all-ferromagnetic structures, fully taking into account magnetic interface anisotropies and exchange contributions. In the crossing regime of dipolar modes and exchange modes, the gap width is strongly influenced by the amount of interface anisotropy. In the case of all-magnetic multilayered structures, a new type of collective spin-wave excitation arising from coupled exchange modes is predicted.

Spin waves in magnetic multilayered structures have spurred increasing interest in the past few years for a number of reasons: first, they are of conceptual interest because of finite-size effects (quantization), and the possible coupling of modes in different layers, which may, for example, result in new types of collective spin-wave excitations in superlattices;¹⁻⁹ second, by measuring the mode frequencies (e.g., by means of Brillouin spectroscopy), many magnetic parameters of the system, which are often difficult to evaluate otherwise, can be determined.⁴⁻⁹

Although a large number of publications exist for bulk materials and single magnetic films, there are only few reports of spin waves in multilayered structures and in superlattices.¹⁻⁹ The first calculations were reported by Camley, Rahman, and Mills,¹ Grünberg and Mika,² and by Emtage and Daniel³ in the dipolar limit. This early work neglected exchange as well as anisotropy contributions. Although the authors were able to predict the salient features of the new collective spin-wave modes in superlattices, recent investigations by Hillebrands *et al.* show the significance of interface anisotropies for small layer thicknesses.⁹⁻¹¹ All of these calculations are valid only in a thickness regime where the dipolar modes are well separated in frequency from the exchange modes, limiting the applicability of these theories to small layer thicknesses. Experimental evidence for superlattice spin-wave modes has been reported by Grimsditch, Khan, and Schuller⁴ and Kueny, Khan, Schuller, and Grimsditch⁵ for Mo/Ni superlattices. A detailed experimental proof of the predicted magnetic properties of the collective spin-wave excitations was presented by Hillebrands *et al.*^{7,8}

In this paper, results are presented of new theoretical investigations in which spin waves in multilayered structures are calculated, properly including both magnetic interface anisotropies and exchange. This work is an extension of a recently introduced theory for the calculation of single-layer spin-wave frequencies, including interface anisotropies and exchange by Rado and Hicken.¹² Three basic questions are addressed. (1) Do exchange modes frequency split in multilayered structures, and do they eventually form a band of collective exchange-dominated spin waves in superlattices, reminiscent of the band of collective dipolar modes? (2) How strongly do interface anisotropies affect the exchange modes? (3) What are the

spin-wave properties in the regime where the exchange modes cross the dipolar modes? The calculations are carried out for two types of multilayered structures: for conventional-type structures consisting of alternating ferromagnetic and nonmagnetic layers, and for what is believed to be the first time, for all-ferromagnetic multilayered structures consisting of magnetic layers with different magnetic properties.

The calculations are based on a straightforward solution of the equations of motion (magnetostatic Maxwell equations and the Lifshitz torque equation¹³) of the magnetization \mathbf{M} in an applied magnetic field \mathbf{H} in the magnetostatic limit, as discussed in detail by Rado and Hicken for the single magnetic layer.¹² It is assumed that the static part of the magnetization lies in the plane of the layer and is parallel to the applied field, which can be achieved by a strong enough external field. The dynamic parts of the magnetization \mathbf{m} and the applied field \mathbf{h} are assumed to be small compared to \mathbf{M} and \mathbf{H} , respectively, which allows for a linearization of the equations of motion and the boundary conditions. The calculations can be summarized as follows: solving the linearized equations of motion¹³ in the bulk of each magnetic layer, including terms resulting from exchange interaction, yields six solutions for \mathbf{m} and \mathbf{h} , which are classified by the wave-vector component perpendicular to the layers. In the nonmagnetic layers (vacuum), there exist two solutions to \mathbf{h} , which exponentially increase or decrease with distance from the magnetic layer. From the equations of motion, boundary conditions are derived which couple the bulk solutions at the interfaces. The boundary conditions consist of the usual Maxwell continuity conditions for the parallel component of the magnetic field and the perpendicular component of the magnetic induction, as well as the so-called Rado-Weertman boundary condition,¹⁴ for which the summation over all contributions to the interface torque density is zero at each interface. In the case of interfaces between two magnetic materials modified Rado-Weertman boundary conditions have been reported by Hoffmann and co-workers.^{15,16} Magnetic interface anisotropies are included in a natural way by adding the corresponding surface torques to the Rado-Weertman boundary condition. It should be noted that in the special case of negligible exchange contributions (small layer

thicknesses) interface anisotropies can also be included as effective volume anisotropies. In this way, the resulting numerical procedures are simplified significantly, allowing for spin-wave calculations in multilayered structures of up to more than a hundred layers.⁹ Writing the boundary conditions as a system of linear equations with the coefficients of the different solutions as the unknowns, the spin-wave frequencies are obtained by solving for the zeros of the determinant of the boundary condition matrix. The calculations are performed by means of appropriate numerical tools.

First, we will consider the case of a multilayer stack consisting of single-crystal Fe(110) layers separated by nonmagnetic layers of equal thickness. This structure resembles Fe/Pd superlattices where the Fe crystallites have a preferred (110) orientation,⁷⁻⁹ as well as single-crystal epitaxial Fe films on W(110) substrates.^{10,11} For the simulations, the parameters of the latter case are used for the saturation magnetization $4\pi M_s$ and the out-of-plane anisotropy constant K_s (in Gradmann's notation), i.e., $4\pi M_s = 18$ kG, $K_s = 2.8$ erg/cm². The value of the in-plane interface anisotropy constant $K_{s,p} = 0.024$ erg/cm² has been dropped because it does not affect the spin-wave frequencies in the layer-thickness regime considered here. For the other parameters, representative values have been chosen for the volume anisotropy constant $K_1 = 4.5 \times 10^5$ erg/cm³, for the spectroscopic splitting factor $g = 2.1$, for the exchange constant $A = 2 \times 10^{-6}$ erg/cm and for the wave vector $q = 1.73 \times 10^5$ cm⁻¹. The wave vector points in the [001] direction. The applied magnetic field is 1 kG.

Figure 1 shows calculated spin-wave frequencies as a function of the single-layer thickness for a multilayered

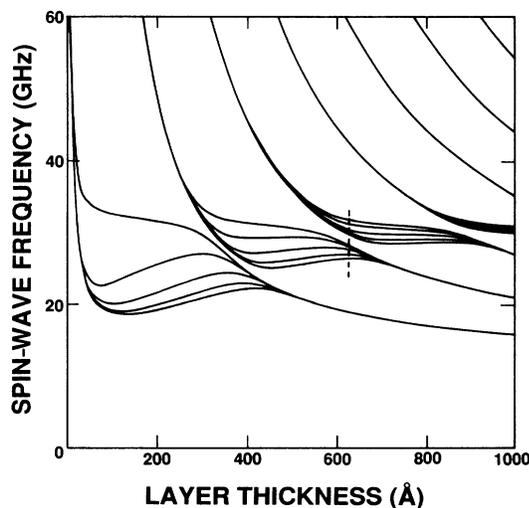


FIG. 1. Spin-wave frequencies as a function of single-layer thickness for a multilayer consisting of five layers of Fe(110) layers and interleaving nonmagnetic layers of same thickness. The parameters are $H = 1$ kG, $4\pi M_s = 18$ kG, $K_s = 2.8$ erg/cm², $K_{s,p} = 0$. The volume anisotropy constant HK_1 is 4.5×10^5 erg/cm³, the exchange constant A is 2×10^{-6} erg/cm, and the spectroscopic splitting factor (g factor) is 2.1. The dashed line indicates the thickness position referred to by Fig. 2.

structure consisting of five bilayers. The thickness d of the magnetic layers equals that of the nonmagnetic layers. Two kinds of modes are observed: between about 18 and 27 GHz there are five dipolar modes (Damon-Eshbach modes) separated in frequency because of their dipolar interaction across the nonmagnetic spacer layers. The frequency splitting decreases with increasing d because of a corresponding decrease in the interlayer coupling. For very small layer thicknesses ($d < 30$ Å), the dipolar modes exhibit a characteristic increase in frequency, because interface anisotropies become dominant in this regime. It should be noted that in this regime, the results are very close to those obtained by an approach in which the interface anisotropies are treated as effective volume anisotropies and the exchange contributions are neglected.⁹ The highest-frequency mode is the Damon-Esbach surface spin-wave mode of the total multilayer stack. For $d \rightarrow 0$, all modes with the exception of the highest-frequency mode become degenerate. An analysis shows that, as a result of the dominating interface anisotropies, the dipolar mode in each layer becomes bulk-mode-like, with minor stray fields in the spacer layers, thus exhibiting reduced coupling.⁹ For $d > 250$ Å, the regime of dipolar modes is crossed by the exchange modes. The latter are characterized by their typical $1/d^2$ behavior. Their stray fields are very weak into the spacer layers, resulting in virtually no mode splitting apart from the crossing regime. For small layer thicknesses, a weak but still significant dependence of the exchange mode frequencies on the interface anisotropy constant was established. In the crossing regime, the dipolar modes and the exchange modes exchange their mode type, leading to a pronounced frequency gap. Although in this thickness regime the energetic contributions of the interface anisotropies are very small, the gap width is determined primarily by K_s . This is demonstrated in Fig. 2, where the spin-wave frequencies are plotted for $d = 630$ Å (indicated by the dashed line in Fig. 1) as a function of the interface anisotropy constant

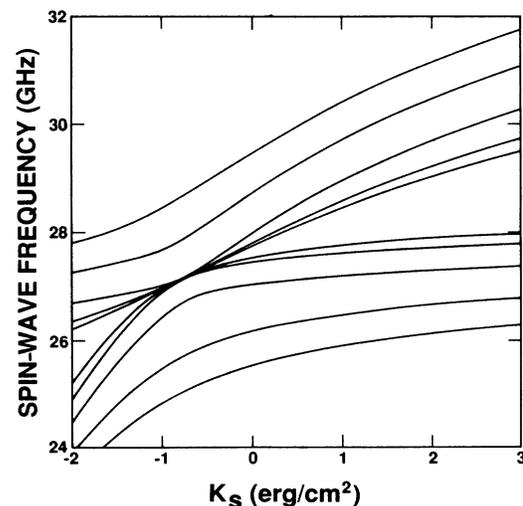


FIG. 2. Spin-wave frequencies as a function of the out-of-plane interface anisotropy constant K_s for $d = 630$ Å (indicated in Fig. 1 by the dashed line).

K_s . For negative values of K_s , the gap width shrinks to virtually zero and then increases for even smaller values of K_s . A study of the form of the dispersion curves as a function of the number of bilayers, N , revealed no strong dependence of the gap width on N . However, because of numerical instabilities, N could not be chosen larger than 7. An extrapolation of the results implies that the properties of the crossing regime are much the same for larger N (superlattices), in particular the gap width and the importance of the interface anisotropy.

Although the method for calculating spin waves in multilayered structures of alternating magnetic and nonmagnetic layers is straightforward, special attention must be given to boundary conditions at the interfaces between two magnetic layers. It is necessary to introduce an interlayer exchange mechanism, described by an interlayer exchange constant A_{12} , which keeps the spins on each side of the interface aligned either parallel or antiparallel, depending on the sign of A_{12} . The appropriate modified Rado-Weertman boundary conditions have been reported by Hoffmann and co-workers.^{15,16}

Next, a multilayer stack consisting of three Fe layers of equal thickness interleaved by two Ni layers of the same thickness is considered. The system Fe-Ni has been chosen since the single-film dispersion curves of the exchange modes of both materials are nearly degenerate for the lowest-lying mode. The left side of Fig. 3 shows the spin-wave frequencies as a function of the single-layer thickness. The parameters used are the bulk literature values for the exchange constants, the saturation magnetizations, and the g factors. Interface anisotropies have been set to zero; calculations using nonzero values exhibit-

ed changes close to those discussed above. In order to determine the case of maximum exchange coupling across the interfaces the calculations for Fig. 3 are carried out in the limit of A_{12} large compared to the product of either A_1 or A_2 with the spin-wave wave vector. The most striking feature is the lifting of the degeneracy for the exchange modes. With the exception of the lowest two modes (labeled 1 and 2 in Fig. 3), the observed exchange modes can be always subdivided into groups of five modes, and each group can be further subdivided in a two-mode and a three-mode subgroup. Each subgroup corresponds to a single-layer exchange mode for either Fe or Ni. For the lowest-lying group (labeled 3 to 7 in Fig. 3), the single-layer Fe-exchange mode and the corresponding Ni exchange mode are nearly degenerate, as reflected by the nearly equal spacing of all five modes. In the limit of an infinite number of layers (superlattice structure), the modes will eventually form a band of collective exchange modes, similar to the band of collective dipolar modes in magnetic-nonmagnetic-type superlattices. The frequency splitting of the exchange modes depends strongly on the interface exchange constant A_{12} and becomes zero for $A_{12} \rightarrow 0$. Two dipolar modes (labeled D_1 and D_2 in Fig. 3) are identified. They are in frequency regimes of single-layer Fe and Ni Damon-Eshbach modes, respectively. The right side of Fig. 3 displays the perpendicular component of the dynamic part of the magnetization as a function of the location in the multilayer stack, for the modes indicated in the dispersion curves. Besides the two dipolar modes, the number of nodes in the field amplitude increases with increasing mode order. The dipolar mode D_2 clearly shows properties similar to those of a single-layer Damon-Eshbach mode: the amplitude has a maximum on one side of the stack and decreases towards the other side. The main contribution comes from the Fe layers. This is in contrast with the mode D_1 , which has its largest amplitudes in the Ni layers.

In the crossing regions, the modes exhibit a frequency gap similar to conventional-type multilayered structures described above. Simulations show, that the gap width is strongly influenced by the interface anisotropy constant K_s .

In conclusion, results are presented for spin-wave properties in conventional-type alternating magnetic-nonmagnetic-layer multilayers, as well as for all-magnetic multilayered structures. Both exchange contributions and interface anisotropies are included. In the regime where the exchange modes cross with dipolar modes, a gap occurs which is determined primarily by the amount of interface anisotropy. In the case of all-magnetic multilayered structures, a new type of collective spin waves, i.e., coupled exchange modes, are observed, reminiscent to dipolar collective spin-wave excitations in superlattices of the conventional type. Since their mode splitting depends strongly on the interface exchange constant A_{12} , a measurement of the spin-wave frequencies in the region of interest might allow for the determination of A_{12} . It should be feasible to test these predictions by means of Brillouin scattering experiments, with the potential of gaining new ways for evaluating multilayer specific properties.

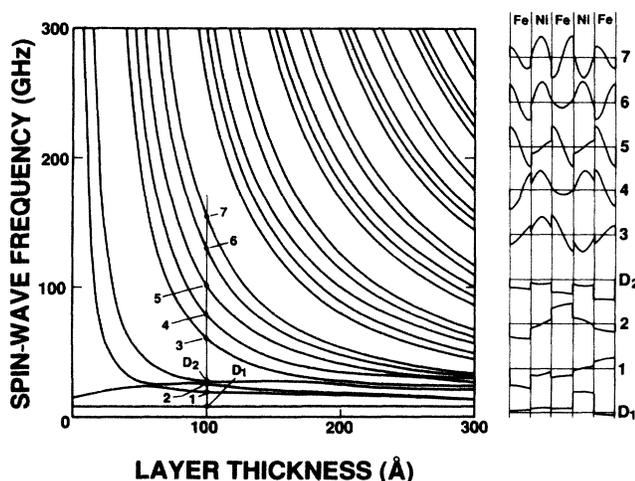


FIG. 3. Spin-wave frequencies as a function of single-layer thickness for a Fe-Ni-Fe-Ni-Fe multilayer structure (left side) and profiles of the perpendicular component of the dynamic part of the magnetization for some modes (right side), indicated in the dispersion curves. D_1 and D_2 indicate the two dipolar modes, the numbers denote the exchange modes. The parameters are Fe: $4\pi M_s = 21$ kG, $g = 2.1$, $A = 2 \times 10^{-6}$ erg/cm; Ni: $4\pi M_s = 6$ kG, $g = 2.2$, $A = 0.7 \times 10^{-6}$ erg/cm. The applied magnetic field is 1 kG.

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