Critical exponents of spin-glass systems

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The critical exponents at the glass temperature T_g of a spin glass are estimated by making use of predictions obtained from a phase-space percolation model, in combination with scaling arguments. With this hypothesis, the whole set of exponents for a given system, including the dynamic exponent, can be expressed simply in terms of a single nonuniversal parameter. The model is checked against data from simulations and from three sets of experiments; agreement is very satisfactory.

The critical exponents in spin-glass (SG) systems have attracted considerable attention as they are strikingly different from values observed at conventional transitions, in particular, the values obtained from experiment and simulation for the dynamic exponent $z\nu$ group around $z\nu \sim 7$, which is spectacularly high. Also, the exponents do not seem to differ drastically between simulations on Ising lattices and experiments on basically Heisenberg systems.

Continentino and Malozemoff (CM)[1] give a simple but effective discussion using scaling arguments to relate the form of the nonexponential relaxation above T_g to certain critical exponents. Their key assumption is that a cluster of s correlated spins has an effective relaxation time proportional to s^x. They find a stretched-exponential[2] long-time relaxation above T_g having exponent $(1-n)=1/(1+x)$, with the relation $x=z\nu/\beta\delta$. This in itself, however, does not lead to predictions for the critical exponents themselves.

Following a different line of approach, we have proposed that the spin-glass transition (and other glassy transitions) can be considered in terms of a percolation transition in configuration space.[3] This leads to a number of specific predictions. Those that are relevant here are (a) $(1-n)$ tends to $\frac{1}{3}$ at T_g, and (b) no specific-heat anomaly at T_g.

Prediction (a) can be checked directly by experiments on relaxation above T_g; results on a number of systems seem to be in good agreement with it.[3,4] Prediction (b) is hardly a prediction, as this is one of the best established characteristics of spin glasses.

If for the sake of argument we accept that these two predictions are strictly correct, and that the configuration-space picture and the CM scaling approach are alternative and complementary ways of representing the same physics, we conclude from (a) that $x=2$, i.e., $z\nu=2\beta\delta$. The easiest way to satisfy (b) is for α to be a negative even integer, such as -2; $C(T)$ is then regular at T_g. If we choose $\alpha=-2$, through the scaling relation $2-\alpha=\nu d$, we have $\nu=\frac{4}{3}$.

Without these two equations the set of critical exponents (including the dynamical coefficient z) can be represented by three independent parameters, the other being deduced through standard scaling relations [$\Phi=\beta+\gamma=\beta\delta$, $\nu=\gamma/(2-\eta)$]. With these two equations, we can express all critical exponents in terms of a *single* nonuniversal parameter for each system. If we choose δ to be this parameter, we have

$$\beta=4/(1+\delta),\ \gamma=4(\delta-1)/(\delta+1),$$
$$\Phi=4\delta/(\delta+1),\ z\nu=8\delta/(\delta+1)\ . \tag{1}$$

We can now test these relations. In Table I, we give experimental critical-exponent data for different systems: the Ising $\pm J$ spin glass in three dimensions,[5] *Cu*Mn,[6,7] $(Fe_{0.15}Nl_{0.85})_{75}P_{16}B_6Al_3$,[8] and $CdIn_{0.3}Cr_{1.7}S_4$.[9] In each case we give in the first column the exponents found directly from simulation or experiment together with those deduced from these through the standard scaling relations. In the second column, we give the set of exponents obtained from Eq. (1) above, using as unique input the experimental value of δ. As can be seen, the agreement is excellent. δ varies quite widely (from about 3 to 10) but because of the form of the equations, the other exponents change much less, in agreement with observation.

We can also consider the algebraic prefactor $t^{-\lambda}$ in the relaxation at T_g.[5,7-9] Ogielski[5] gives, from direct scaling arguments, $\lambda=[d-(2-\eta)]/2z$, and CM give $\lambda=\beta/z\nu$. Assuming Eq. (1), both of these lead directly to $\lambda=1/2\delta$. In Ref. 8, $\delta=10$ and $\lambda=0.05$. In Ref. 9, $\delta=4.1\pm0.5$ and $\lambda=0.105$. Thus, agreement also seems good for this prefactor.

The overall agreement appears to be very satisfactory for these four well-studied systems, suggesting that the two relations given above could have "universal" validity for spin glasses, whether Ising or Heisenberg, nearest neighbor or Ruderman, Kittel, Kasuya, and Yosida (RKKY). Data on the SG *Al*Gd (for which no value of $z\nu$ is available) also give good agreement with $\alpha=-2$.[10] However, we should note that some results reported on certain other SG systems indicate values of α that are more negative than -2.[6,11,12] In the one case among these, where $z\nu$ has also been measured, the rule $z\nu=2\beta\delta$ holds within the experimental errors.[12]

It should also be pointed out that Rosenblatt, Raboutou, Peyral, and Lebeau[13] made the remarkable observation that the critical exponents they obtained for a random granular superconductor array were similar to those for spin glasses. Their values for the granular array are also in agreement with $\alpha=-2$.

TABLE I. Values of critical coefficients obtained from simulations and experiments on different spin-glass systems, compared with values estimated using the model described in the text. In each case, the only free input parameter used in calculating the numbers in the right-hand column is the value of δ; the asterisks indicate that this has been chosen to be equal to the experimental value (see text).

Coefficient	3D Ising SG[a] Simulation	Model	Coefficient	*Cu*Mn SG[b] Experiment	Model
ν	1.3 ± 0.1	$\frac{4}{3}$	ν	1.3 ± 0.2	$\frac{4}{3}$
δ	6.7	6.7*	δ	3.3 ± 0.2	3.3*
β	0.5	0.52	β	0.9 ± 0.2	0.93
γ	2.9 ± 0.3	2.96	γ	2.1 ± 0.1	2.14
Φ	3.4 ± 0.3	3.48	Φ	3.0 ± 0.3	3.07
$z\nu$	7.9 ± 1	7.0	$z\nu$	7.0 ± 0.6	6.2
Coefficient	Amorphous Fe-Ni-P-B-Al [c] Experiment	Model	Coefficient	$CdCr_2(In)S_4$ [d] Experiment	Model
ν	1.39	$\frac{4}{3}$	ν	1.26 ± 0.2	$\frac{4}{3}$
δ	10	10*	δ	4.1 ± 0.4	4.1*
β	0.38	0.36	β	0.75 ± 0.1	0.78
γ	3.4	3.3	γ	2.3 ± 0.4	2.4
Φ	3.8	3.6	Φ	3.1 ± 0.5	3.2
$z\nu$	8.2	7.3	$z\nu$	7.0 ± 0.2	6.5

[a]Reference 5.
[b]References 6 and 7.
[c]Reference 8.
[d]Reference 9.

We should also note[14] that the stretched exponential with exponent $(1-n)=\frac{1}{3}$ appears in another context: the relaxation of one-dimensional (1D) chains with random strong pinning.[15] A segment of length N relaxes with a time constant proportional to N^2. This is the exact equivalent of $x=2$ in CM. The "cluster relaxation time" is proportional to the square of the number of spins, which is a 1D random walk in phase space with the s cluster spins having to be turned over in a definite order. This seems to be an essential ingredient which distinguishes glassy critical behavior from conventional critical behavior.

It is not clear what physical characteristic the nonuniversal parameter δ is linked to. In terms of the discussion given by CM, it is related to the effective dimensionality of the clusters.

Finally, it would be interesting to find out if similar rules linking the critical exponents to the nonexponential relaxation hold in other glassy systems (such as real glasses), as arguments we have given for the universality of the percolation approach[3] would suggest.

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