I-V characteristics of coupled ultrasmall-capacitance normal tunnel junctions

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We present a theoretical study of two ultrasmall-capacitance normal tunnel junctions connected in series and driven by a dc voltage source, using the semiclassical junction approach. We show that the two junctions in series display a voltage offset similar to that of a single junction that is driven by a current source. We also show that two junctions in series can, for the right set of parameters, produce an *I-V* curve with distinct steps. A parallel array of such series units shows similar behavior. We suggest specific experimental realizations of two junctions in series and the parallel array using a scanning tunneling microscope and a granular array of small metal drops. We also compare our predictions with experiments in the literature.

I. INTRODUCTION

New techniques of device fabrication[1] have made it possible to design a host of submicrometer electronic devices whose behavior is dominated by the dynamics of single electrons, rather than those of an ensemble. At this scale the quantum-mechanical phase of the electron as well as the discrete nature of its charge can lead to nonlinear behavior not present in macroscopic systems.[2] Several novel coherent effects in superlattices,[3] mesoscopic normal metal rings,[4] and ultrasmall-capacitance tunnel junctions[5-8] have either been proposed or already demonstrated. These effects can be used as the basis of a new generation of circuit elements acting at subpicosecond speeds with vanishingly small dissipation. One such prediction is the coherent oscillation of charge in ultrasmall-capacitance normal tunnel junctions, first developed in the coherent junction[5] model and later in the semiclassical approach.[6,7] Briefly stated, both models predict that when a normal tunnel junction with an extremely small capacitance is driven by an external direct-current source I_{dc}, it will produce an alternating voltage of frequency I_{dc}/e and amplitude $e/2C$. The character of these oscillations is similar to that of the Bloch oscillations predicted in periodic superlattices placed in a constant electric field and the Bloch-type oscillations predicted in Josephson junctions.[9] In all three cases the existence of the oscillations has not been conclusively demonstrated in an experiment. The direct observation of charge oscillations has been problematic because of difficulties in fabricating submicrometer junctions and in measuring the high-frequency signal without inadvertently swamping it with external parasitic capacitances. Although these problems make it difficult to observe the oscillations in a single junction, it may be possible to do so using a pair of such devices in series, or in an array of such series units. Various combinations of these devices have been suggested[5-9] but few calculations have been done, even for such a simple arrangement. An analysis of this configuration is not only valuable for the purpose of suggesting experiments to verify the existence of coherent charge oscillations, but also to suggest electronic elements with useful *I-V* characteristics.

In this paper we use the semiclassical picture to analyze a pair of ultrasmall-capacitance normal tunnel junctions connected in series and driven by an ideal voltage source. We present numerically calculated *I-V* characteristics for different values of junction capacitance and resistance. We subsequently analyze a parallel array of such series units, again determining the *I-V* characteristics numerically for various distributions of junction capacitances and resistances. Analytic results for both cases will be presented in a separate publication.[10] We suggest simple experimental realizations for the above devices and the range of parameters required to see the effect, and compare our numerical results with experiments in the literature.

II. THE SEMICLASSICAL APPROACH

The semiclassical model has been described in detail elsewhere.[6-8] Here we give only a brief outline of the method and its physical assumptions. For a single normal tunnel junction the rate at which electrons tunnel in the direction of the induced voltage V, say from right to left, is given by

$$r(V)=\int D(E)\tau^{-1}(E)f_r(E-E_r)\times[1-f_l(E-E_l'+eV)]dE \ , \quad (2.1)$$

where $D(E)$ is the density of states, $\tau^{-1}(E)$ is the elastic tunneling rate of electrons through the barrier, and f_l and f_r denote the steady-state distribution of electrons on the left- and right-hand side of the junction, which we approximate by the equilibrium Fermi distribution function:

$$f(E)=\frac{1}{\exp(E/kT)+1} \ . \quad (2.2)$$

The energy E_r is the Fermi energy of the right hand side before the electron tunnels, and E_l' is the Fermi energy of the left-hand side *after* the electron tunnels. In metals both $D(E)$ and $\tau^{-1}(E)$ are assumed to be independent of energy, but this assumption is not crucial so long as the dependence on energy is regular. If, for example, the junction were made of a superconductor instead of metal then we would expect additional interesting effects due to the special structure of $D(E)$.[10] Since the integral of the occupation factor $f_r(E-E_r)[1-f_l(E-E_l'+eV)]$ is approximately eV at large voltages ($eV \gg kT$), we can approximate the current by

$$\langle I\rangle = er(V)\approx e^2\tau^{-1}(E_f)D(E_f)V \quad (2.3)$$

so that the Ohmic resistance of the junction is defined by

$$R\equiv\frac{1}{e^2\tau^{-1}(E_f)D(E_f)} \ , \quad (2.4)$$

where the density of states and the tunneling rate have been replaced by their value at the Fermi level. The tunneling rate in the reverse direction, $l(V)$, can be defined similarly to $r(V)$.

Normally it is not necessary to specify at what point during the tunneling process the Fermi energy is measured, but when the capacitance is very small the change in the electrostatic charging energy due to the transfer of a single electron may become significant. If such a junction is driven by a voltage source (zero internal resistance) then the potential difference between the two sides of the junction is fixed and this distinction is irrelevant; the response of the junction is the standard Ohmic one. But when the junction is driven by a current source (infinite internal resistance) the tunneling of an electron decreases the net charge across the junction and the Fermi energy shifts by an amount

$$E_l-E_l'=\frac{e^2}{2C} \ , \quad (2.5)$$

where C is the capacitance of the junction. The electron that tunnels to the opposite side of the junction shifts the Fermi energy in doing so. If the initial voltage difference is less than $e/2C$ and an electron at the Fermi energy on the right-hand side were to tunnel to the left it would arrive at a level *below* the Fermi energy on the left. The net effect is to suppress the tunneling of electrons when the voltage is less than $e/2C$.

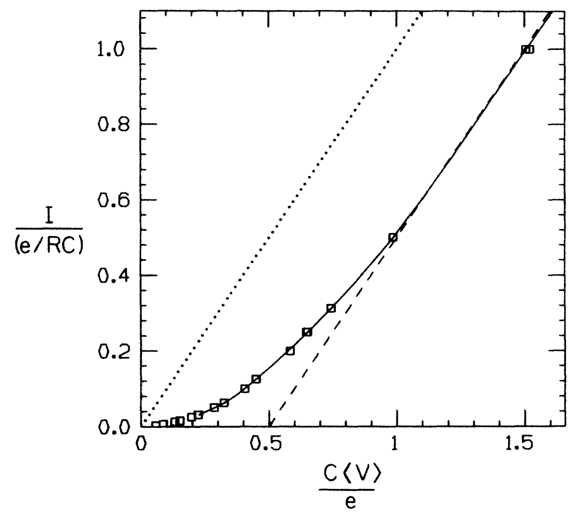


FIG. 1. Calculated I-V characteristic for a single ultrasmall-capacitance tunnel junction driven by an ideal current source. The dotted line represents the Ohmic response of the junction when it is driven by a voltage source. The dashed line is the high voltage approximation given in the text. The parameters are $R=2500\ \Omega$, $C=1$ fF, and $T=0.05$ K. The squares are the results of numerical calculations. The solid line has been added as a guide to the eye.

The dynamics of the function can be modeled by calculating these forward and backward tunneling rates. In a time Δt, the probability of an electron tunneling in the forward direction is $r(V)\Delta t$ and in the backward direction is $l(V)\Delta t$. The time evolution of the junction can be simulated by determining whether during a time segment Δt an electron tunnels in the forward direction, the reverse direction, or no tunneling occurs at all, according to these calculated probabilities. By iterating the stochastic process in time the average voltage can be calculated as a function of the applied current.

The behavior of the mesoscopic junction driven by an external current source I_{ex} differs from its macroscopic counterpart because of the above mentioned $e/2C$ shift in the Fermi levels resulting from the tunneling of an electron.[6,8] Without this shift the I-V characteristic of the junction would be the same Ohmic response as in the case of a junction driven by a voltage source. With the shift, at very low temperatures ($kT \ll e^2/2C$) and small currents ($I_{ex} \ll e/RC$) the voltage is proportional to $(I_{ex})^{1/2}$; at higher currents the I-V curve is linear but with an offset so that the zero current intercept is at a voltage of $e/2C$ (Fig. 1). The response of the junction for high currents is approximately given by

$$V\approx RI_{ex}+\frac{e}{2C} \ . \quad (2.6)$$

III. TWO JUNCTIONS IN SERIES

The methods used in the above calculations for a single junction driven by a current source can also be applied to the system of two junctions in series driven by a voltage source as shown in Fig. 2. The voltage source has zero internal resistance and so it maintains a voltage V_{ex} across the two junctions at all times independent of the size of the current. Although the *net* voltage drop is fixed, the voltage across each junction is a function of their capacitances and the charge stored between them, δQ:

$$V_1=\frac{C_2}{C_1+C_2}V_{ex}-\frac{\delta Q}{C_1+C_2}\,,$$
$$V_2=\frac{C_1}{C_1+C_2}V_{ex}+\frac{\delta Q}{C_1+C_2}\,. \tag{3.1}$$

Denoting the number of electrons that have tunneled across the first and second junction by n_1 and n_2, respectively, we can express the charge between the two junctions as $\delta Q=(n_1-n_2)e$. The forward and backward tunneling rates for each junction are functions of V, n_1, and n_2; each time an electron tunnels through one of the junctions these rates will change as the charge between the two is redistributed.

Given n_1 and n_2 at time t, their values at time $t+\Delta t$ are given by the following stochastic process:

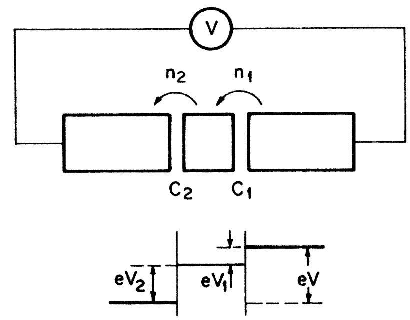


FIG. 2. Schematic representation of a pair of ultrasmall-capacitance tunnel junctions connected in series, driven by an ideal voltage source V_{ex}; below is a graph of the Fermi levels in the device. The variables n_1 and n_2 correspond to the number of electrons that have tunneled across the first and second junction, respectively.

$$n_1(t+\Delta t)=\begin{cases} n_1(t)+1 & \text{with probability } r_1(V_1)\Delta t,\\ n_1(t)-1 & \text{with probability } l_1(V_1)\Delta t,\\ n_1(t) & \text{with probability } 1-[l_1(V_1)+r_1(V_1)]\Delta t,\end{cases}$$
$$n_2(t+\Delta t)=\begin{cases} n_2(t)+1 & \text{with probability } r_2(V_2)\Delta t,\\ n_2(t)-1 & \text{with probability } l_2(V_2)\Delta t,\\ n_2(t) & \text{with probability } 1-[l_2(V_2)+r_2(V_2)]\Delta t,\end{cases} \tag{3.2}$$

where subscripts have been added to the transition rates since the two junctions may have different tunneling characteristics. The *I-V* curve is then calculated by iterating the above stochastic process for a fixed external voltage until a specified amount of charge has been transferred through the device, thereby determining the current. Alternatively it could be calculated by solving the appropriate master equation to obtain an analytic result, as we will present elsewhere.[10]

The results of the simulations are shown in Fig. 3. We initially assume that the resistances and capacitances of the two junctions are equal. The most important feature of the generated *I-V* curve is that the shift found earlier for the case of a single junction driven by a *current* source is also present when two junctions in series are driven by a *voltage* source. Although the voltage source holds the total voltage across the pair constant, the Fermi level of the central region is free to shift as δQ changes. The change in V_1 as δQ increases by one charge is $e/(C_1+C_2)$. In analogy with the single junction case we assume that the difference between the Fermi energy on the left-hand side of the first junction and the energy of the tunneling electron shifts by half this amount, that is, $E_l-E_l'=e^2/2(C_1+C_2)$. This shift in the Fermi level inhibits tunneling at voltages below $e/2(C_1+C_2)$ producing a shift in the *I-V* characteristic just as in the current driven junction described above. Since a voltage source is easier to realize in an experiment than a current source, it may be more practical to look for the predicted shift with the series configuration. However, the periodic charge oscillations present in the single current driven junction are not present in the series case. In the former the external current source serves as a "clock" because it brings charge to the junction at a fixed rate. The voltage source does not provide such a clock, so a Fourier transform of the voltage across each junction will not show a set of sharp peaks but rather a set of broad ones whose width depend upon the characteristic tunneling time.

If the resistances are equal but the capacitances are different, say $C_1<C_2$, or if the capacitances are the same but the resistances differ, say $R_1<R_2$, then there are slight steps in the *I-V* curve [Figs. 3(a) and 3(b)]. The origin of these steps can be understood qualitatively by considering the shift in the voltage across each junction as a single electron tunnels across the first. Electrons tunnel across the first junction in response to the potential difference across it. However, it is often not possible for the voltage drop across the junction to be precisely zero because it can only be altered by tunneling an integer number of electrons into or out of the region between the junctions. This amount of charge, δQ, has a

maximum value determined by the external voltage; if δQ were too large then the voltage drop across the first junction would be negative and the electron would have to tunnel to a state below the Fermi energy. The charge δQ can increase by one only when V_{ex} is raised by an amount sufficient to keep V_1 positive after the additional electron tunnels across. Since each additional electron shifts V_1 by an amount $e/(C_1+C_2)$, the above requirement can be written as

$$\frac{C_2}{C_1+C_2}\Delta V_{ex}=\frac{e}{C_1+C_2} \tag{3.3}$$

or

$$\Delta V_{ex}=\frac{e}{C_2} . \tag{3.4}$$

An external voltage increase of ΔV_{ex}, increases δQ by one electron, producing a jump $\Delta V_2=e/(C_1+C_2)$ in

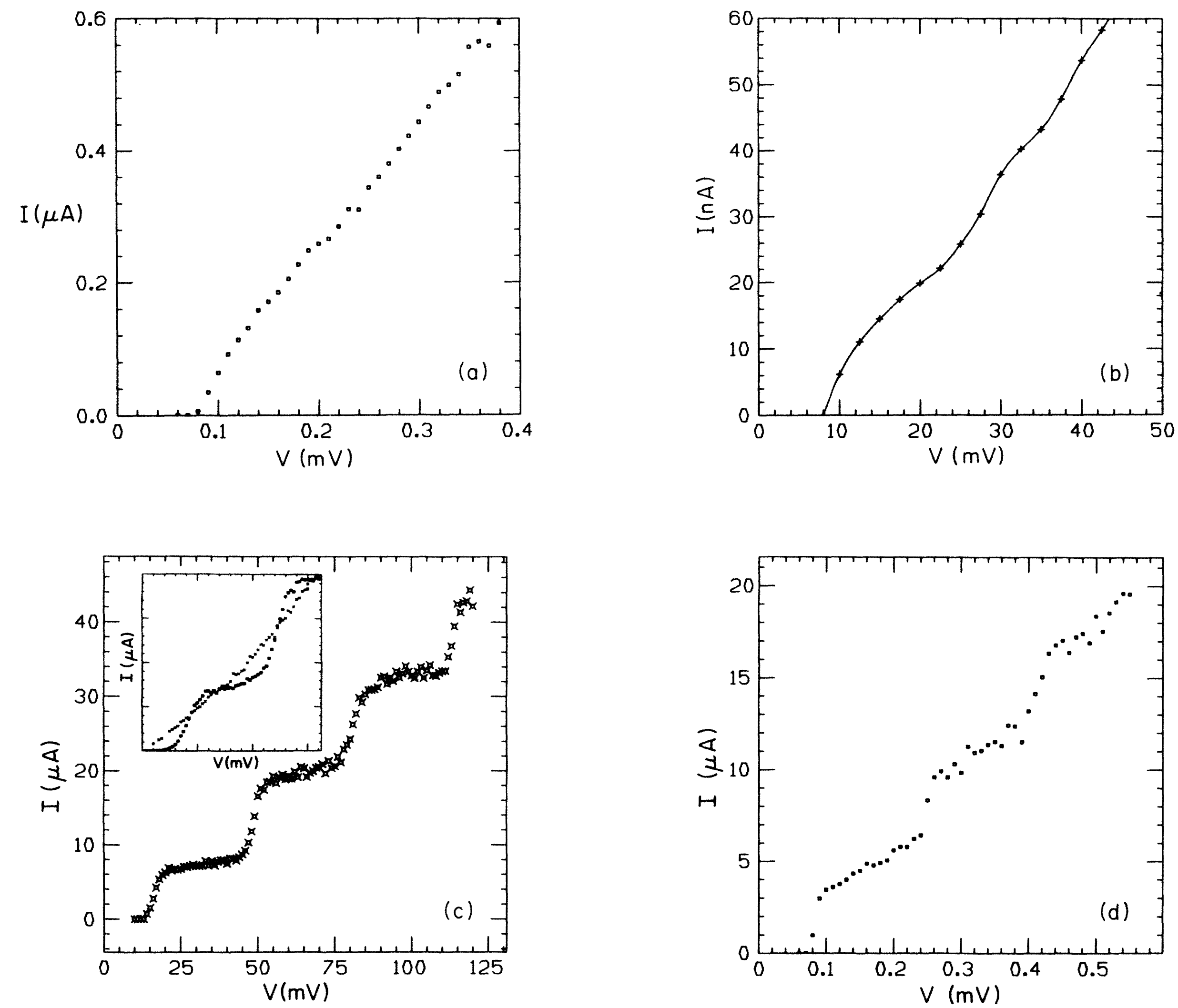


FIG. 3. (a) Calculated I-V characteristic for two ultrasmall-capacitance tunnel junctions in series driven by an external voltage source when the junctions are identical. The parameters are $R_1=R_2=250$ Ω, $C_1=C_2=0.001$ fF, and $T=10$ K. Note the voltage offset of $e/2C$ (0.08 mV) near zero current. (b) Calculated I-V characteristic for two ultrasmall-capacitance tunnel junctions in series driven by an external voltage source when $C_1<C_2$. The parameters are $R_1=R_2=250$ Ω, $C_1=0.001$ fF, $C_2=0.010$ fF, and $T=10$ K. Note that in addition to the voltage offset of $e/2C_2$ (0.08 mV), near zero current, there is a slight step in the I-V at $V=3e/2(C_1+C_2)$ (24 mV). The solid line is added only as a guide to the eye. (c) Calculated I-V characteristic for two ultrasmall-capacitance tunnel junctions in series driven by an external voltage source when $C_1<C_2$ and $R_1 \ll R_2$. The steps are $\Delta V=e/C_2=32$ mV wide and have a height of $\Delta I=e/R_2(C_1+C_2)=10.67$ μA. The parameters are $R_1=25$ Ω, $R_2=2500$ Ω, $C_1=0.001$ fF, $C_2=0.005$ fF, and $T=10$ K. Inset: The temperature dependence of the voltage steps. The steps and the voltage offset are no longer visible when kT is on the order of $e^2/2C_2$. The temperatures are $T=40$ K (squares) and 80 K (crosses). (d) I-V characteristic for two ultrasmall-capacitance tunnel junctions in series driven by an external voltage source when the $C_1=C_2$ and $R_1 \gg R_2$. The steps are still present but almost lost in the rise in current between jumps. The voltage offset is unchanged. The parameters are $R_1=25\,000$ Ω, $R_2=25$ Ω, $C_1=C_2=0.001$ fF, and $T=10$ K.

the voltage across the second junction. This jump in V_2 increases the current by

$$\Delta I = \frac{\Delta V_2}{R_2} = \frac{e}{R_2(C_1+C_2)} . \tag{3.5}$$

Similar arguments can be made when the voltage is applied in the opposite direction. Consider the case when $C_2 < C_1$, for example. The current limiting step in this case is the transfer of charge *into* the central region. Each time the voltage is increased by an amount ΔV_{ex} another charge can *leave* the central region, increasing the voltage drop across the lower capacitance junction and thereby causing a jump in the current.

In order to see clear steps in Fig. 3(c) two conditions must hold. First we need that the tunneling rate across the first junction be much faster than that across the second so that the central region will always be "full," that is, so that δQ takes on its largest value for which V_1 is still positive. If we assume that $R_1 \ll R_2$ or $C_1 \ll C_2$ then electrons can tunnel across the first junction as fast as the second can remove them then δQ will always take on this maximum permissible value. At higher voltages the steps soon wash out because δQ will not always take on its maximum value—the second junction transfers charge out of the intermediate region as fast as the first one transfers charge into it. Second, the change in current *between* steps must be smaller than ΔI. This can also be obtained by choosing one capacitance to be smaller than the other. Choosing both $C_1 < C_2$ and $R_1 < R_2$ produces the crisp steps of Fig. 3(c). However, if we choose $R_1 < R_2$ but $C_1 > C_2$, the two effects will cancel and the shift will be present but the steps will not [Fig. 3(d)].

When the temperature is increased the voltage offset and the steps gradually disappear, as shown in the inset to Fig. 3(c). The effect of increasing the temperature is to populate states above the Fermi energy and empty states that are below it. As described above, the source of the offset was the fact that the change in the charging energy would force electrons to tunnel to states below the Fermi energy: since there are few such states at low temperature tunneling is suppressed. At high temperatures, $e^2/2C_2 \ll kT$, there are many open states within $e^2/2C_2$ of the Fermi level and so the tunneling rate is unaltered.

Note that in Fig. 3(c) the size of the first current step is half that of the subsequent steps. This result may seem odd since a plot of δQ versus V_{ex} shows a series of steps of equal size. Indeed if the response of the junction were Ohmic the current steps would all be the same height. But as pointed out in the description of the single current driven junction the response of the junction is not linear, the semiclassical model predicts a shift in the I-V curve by an amount equal to the voltage induced by half a charge. This shift means that the initial jump in current will be a height

$$\frac{\Delta V_2 - e/2(C_1+C_2)}{R_2} = \frac{\Delta I}{2} , \tag{3.6}$$

half that of the subsequent steps.

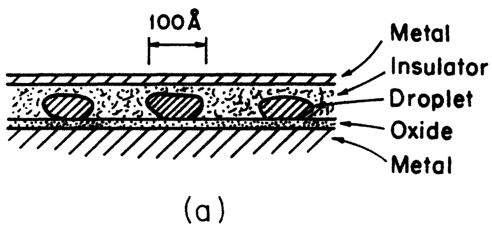
IV. POSSIBLE EXPERIMENTS

We can alter the initial configuration by attaching a lead to the central region in between the two junctions, forming a transistor. The additional lead allows us to vary δQ in a continuous fashion, bringing in an amount of charge q_{ex} from the external source. Alternatively, it may be possible to polarize the material in which the central region is embedded so as to induce the charge q_{ex}.[11] This additional charge cannot change the size of the steps, since their size is due to the fact that the tunneling charge is discrete. If q_{ex} is a multiple of the electronic charge then it will have no effect since the additional charge will simply tunnel across the second junction, adding infinitesimally to the current. However, if q_{ex} is a continuous variable then we can bring in a fraction of a charge. The interpretation of fractional charge is simply that the wave function of an electron is moved closer to the central region so that the average effect is that the charge δQ has increased by a fractional amount. This charge alters the position or "phase" of the steps, by changing the voltage across the first junction and thereby changing the voltage required before electrons can first tunnel. If the applied voltage places the junction near a step in the I-V curve then a small shift in q_{ex} can cause a large variation in current. This configuration can therefore act as an extremely sensitive transistor with a large amplification.

One possible way to create a series combination of tunnel junctions is to use a scanning tunneling microscope (STM) to probe small metal droplets deposited on an oxidized metal substrate. The STM electrode can be positioned with great accuracy above a single small metal droplet. The substrate-oxide-droplet combination forms the first tunnel junction and the droplet-gap-STM electrode combination serves as the second. Capacitances as small as 10^{-18} F can be obtained with 100-Å drops, and with appropriately chosen resistances the steps in the I-V curves are visible at temperatures up to 40 K.

If the two junctions in series are considered a single unit then it is possible to envision an array of such units, all connected in parallel. If we assume that the interdrop distance is large so that there is no coupling between the drops, then when such an array is driven by a voltage source the individual units that compose the array will act independently, and the net current through the device will be the sum of the currents through each unit. If the units are all identical then the array will display steps in its I-V characteristic as in the case of a single unit. Realistically, there will be some distribution of the junction parameters which will smooth out the step. However, if the spread is small enough then the steps are still visible. Such an array can be realized by sandwiching small metal droplets between two layers of metal separated by an insulator, as in Fig. 4(a). Distributions of 100-Å-diameter droplets can be produced with a half-width of approximately 25 Å. In order to calculate the I-V curve of the array it was assumed that the variations in resistance and capacitance stemmed from variations in droplet size, so that R_1/R_2 and

C_1/C_2 were held constant, but the capacitances were scaled with the area of the drop and the resistances were scaled inversely with the size of the area. The area of the drops as assumed to be distributed normally and so individual I-V curves were summed together, each with its own Gaussian weighting factor. As shown in Fig. 4(b), even half-widths of 25% of the average values do not smear out the steps. As stated above, the density of states was assumed to be independent of energy in all calculations. If the metal drops are treated as cubes with sides of length l then the splitting between levels is approximately $(\hbar^2/2m)(2\pi/l)^2$. For $l=100$ Å this separation is about 0.014 eV and at room temperature ($kT \approx 0.025$ eV) the density of states can be considered continuous. If smaller particles are used, quantum size effects may become important and the density of states would no longer be continuous, which would alter the I-V characteristic.[10] Additional complications may arise from the fact that the droplets are embedded in a polarizable medium, and so the droplets may have a slight fractional net charge. The effect of this charge is to shift the voltage offset.[11] In an actual experiment care must be taken to align the Fermi levels of the droplets by first holding the potential constant between the two plates. In order to observe the steps clearly the voltage must be swept at a rate faster than the droplets can alter their alignment.



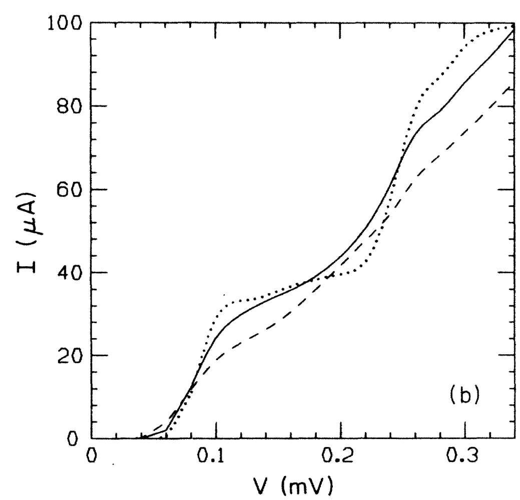


FIG. 4. (a) Experimental configuration that produces a parallel array of series units. The interdroplet distance is much larger than their radius so that there is no coupling between droplets. The array is fabricated by depositing metal on to an oxidized metal substrate, forming small metal droplets on top of the oxide layer. The droplets are then oxidized so as to give them an insulating coat, and finally a covering metal layer is evaporated on top. (b) I-V characteristic for an array of series units driven by an external voltage source when the $C_1 < C_2$ and $R_1 \ll R_2$. The area of the junctions is assumed to have a normal distribution; the capacitance is proportional to the area and the resistance is inversely proportional to it. The average values of the junction parameters are $R_1=25\ \Omega$, $R_2=2500\ \Omega$, $C_1=0.001$ fF, $C_2=0.010$ fF, and $T=10$ K. The characteristics of the array are calculated for distributions of different half-widths: (i) $\sigma=10\%$ (dotted line), (ii) $\sigma=20\%$ (solid line), and (iii) $\sigma=50\%$ (dashed line) of the average area.

V. EXISTING EXPERIMENTAL EVIDENCE

Some experiments have already been performed in such systems. Giaever and Zeller[12] measured the resistance as a function of voltage for a set of droplets in which $R_1 \approx R_2$ and $C_1 \approx C_2$. As explained above, such a configuration should not show steps in its I-V response but should display an offset. The stochastic process that describes the response of the junction [Eq. (8)] can be approximated analytically in the limit of low voltage and then differentiated to obtain the resistance. At low voltages the time τ required for an electron to tunnel through the two junctions is just the sum of the time it takes to tunnel through the first when there are no electrons in the central region $[1/r_1(V_1)]$, plus the time it takes for an electron to tunnel through the second when there is one charge in the center $[1/r_2(V_2')]$, where the primed voltages are measured when $\delta Q=1$ and unprimed when $\delta Q=0$. The average current is proportional to $e/\tau = e/[1/r_1(V_1)+1/r_2(V_2')]$. However, each time an electron tunnels across the first junction it need not immediately tunnel forward through the second; it may tunnel back again through the first. The probability of forward tunneling through the second junction can be written as $r_2(V_2')/[r_2(V_2')+l_1(V_1')]$, so that the current is given by

$$I = \frac{e r_1(V_1) r_2(V_2')}{[r_1(V_1)+r_2(V_2')]} \frac{r_2(V_2')}{r_2(V_2')+l_1(V_1')} . \tag{5.1}$$

This approximation is good only for voltages sufficiently small so that δQ is never great than one. The resistance is then obtained by calculating dV/dI.

Unlike the experimentally determined result, the resistance derived from Eq. (13) diverges at low voltages. However, in the Giaever and Zeller experiment not only could electrons tunnel from the metal layer to the droplets, electrons could also tunnel directly from one metal layer to the other, through the thick oxide between droplets. This additional channel acts as a shunt resistor of some large, fixed magnitude so that the system has a finite resistance at low voltages. Calculating dV/dI from Eq. (13) and adding the resulting resistance in parallel to a large shunt resistor R_s, we obtain the result shown in Fig. 5 which exhibits the same general behavior demonstrated by experiment. Better agreement might require averaging over the unspecified droplet distribution in the Giaver and Zeller experiment, or by considering a temperature dependent shunt resistance.

Additional experiments were performed by Lambe and Jacklevic,[12] but in their system there was no tunneling between the droplets and the metal substrate ($R_2=\infty$). By driving the system with an alternating voltage and measuring the differential capacitance, they were able to show clear charge quantization effects over a range of voltage determined by the droplet distribution. In addition they found a "memory effect." The Fermi levels of the droplets need not initially be aligned, but they drift into alignment as the charged droplets polarize the medium about them. By applying a bias voltage at high temperatures to align the Fermi levels and then cooling the droplets, the polarization could be frozen in, so that the system has a record of the initial bias voltage for long times. This result gives some support to the idea of a transistor based upon the polarization of the medium in which the droplets are embedded.

Although neither of the above two experiments observed charge oscillations nor a staircase I-V curve, both observed a non-linear response due to the discrete nature of the tunneling current. These results suggest that it is possible to bias a junction by a charge less than $e/2$, and give further motivation for experiments that would look for these new effects.

In conclusion, working with two junctions in series has the advantages that the voltage source is easily implemented, the signal is not affected by parasitic capacitances, the experiments can be simply realized with a STM, and the signal can be magnified by working with an array of such devices. Finally the two-junction device shows steps in its I-V curve, not present in the single junction case. It may be possible to build transistors based upon this result.

Note added. Subsequent to the work discussed in this paper we learned that in recent independent experiments not only has the shift of the I-V curve been observed in the single junction case,[13] but that the shift *and* the steps were observed in the series case,[14] in agreement with our theoretical predictions.

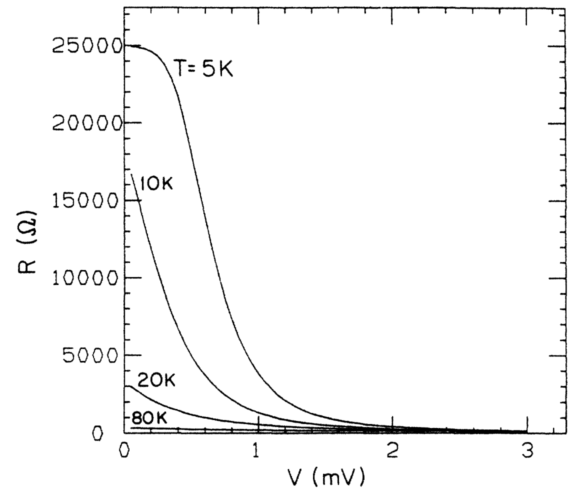


FIG. 5. Numerically calculated resistance as a function of voltage at different temperatures for an array of series units when a shunt resistor is connected in parallel. The parameters are $R_1=R_2=10\,000$ Ω, $R_s=25\,000$ Ω, and $C_1=C_2=0.08$ fF. The temperatures are 5, 10, 20, and 80 K.

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[1] *Electron-Beam Lithography in Microelectronic Fabrication,* edited by B. R. Brewer (Academic, New York, 1980); *The Physics of Submicron Structures,* edited by H. L. Gaubin, K. Hess, G. J. Iafrate, and D. K. Ferry (Plenum, New York, 1982).

[2] Y. Imry, in *The Memorial Volume in Honor of Professor Shang-Keng Ma,* edited by G. Grinstein and G. Mazenko (World-Scientific, Singapore, in press); E. Ben-Jacob, M. Buttiker, and R. A. Webb, Phys. Today (to be published).

[3] T. C. L. G. Solner, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck, Appl. Phys. Lett. **43**, 588 (1983); S. W. Kirchoefer, R. Magno, and J. Comas, *ibid.* **44**, 1054 (1984); K. Hirakawa, H. Sakaki, and J. Yoshino, Phys. Rev. Lett. **54**, 1279 (1985); S. Datta *et al.*, *ibid.* **55**, 2344 (1985).

[4] M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983); Y. Gefen, Y. Imry, and M. Ya. Azbel, Phys. Rev. Lett. **52**, 129 (1984); R. A. Webb, S. Washburn, C. Umbach, and R. A. Laibowitz, *ibid.* **54**, 2969 (1985).

[5] E. Ben-Jacob and Y. Gefen, Phys. Lett. **108A**, 289 (1985); E. Ben-Jacob, Y. Gefen, K. Mullen, and Z. Schuss, in *SQUID '85,* edited by H. D. Halhbohm and H. Lübbig (de Gruyter, New York, 1985).

[6] D. V. Averin and K. K. Likharev, in *SQUID '85,* edited by H. D. Halhbohm and H. Lübbig (de Gruyter, New York, 1985); D. V. Averin and K. K. Likharev, J. Low Temp. Phys. **62**, 345 (1986).

[7] E. Ben-Jacob, D. J. Bergman, B. J. Matkowsky, and Z. Schuss, Phys. Rev. B **34**, 1572 (1986).

[8] E. Ben-Jacob, Y. Gefen, K. Mullen, and Z. Schuss, Phys. Rev. B (to be published).

[9] A. Widon, G. Magaloudis, T. D. Clark, H. Prance, and R. J. Prance, J. Phys. Math. A **15**, 3877 (1982); R. J. Prance, J. E. Motton, H. Prance, T. D. Clark, A. Widom, and G. Megaloudis, Helv. Phys. Acta **56**, (1983); T. P. Spiller, J. E. Motton, H. Prance, R. J. Prance, and T. D. Clark, in *SQUID '85,* edited by H. D. Halhbohm and H. Lübbig (de Gruyter,

New York, 1985); A. I. Larkin, K. K. Likharev, and Y. N. Ouchinnikov, Physica **126B&C**, 414 (1984); K. K. Likharev and A. B. Zorin, J. Low Temp. Phys. **59**, 347 (1985); M. Büttiker, Phys. Rev. B **36**, 3548 (1987).

[10]E. Ben-Jacob, R. C. Jaklevic, K. Mullen, and Z. Schuss (unpublished); K. Mullen and E. Ben-Jacob, Phys. Rev. Lett. (to be published).

[11]J. Lambe and R. C. Jaklevic, Phys. Rev. Lett. **22**, 1371 (1969).

[12]I. Giaever and H. R. Zeller, Phys. Rev. Lett. **20**, 1504 (1968).

[13]T. A. Fulton and G. J. Dolan, Phys. Rev. Lett. **59**, 109 (1987).

[14]J. B. Barner and S. T. Ruggiero, Phys. Rev. Lett. **59**, 807 (1987).