Collective modes in an "ultraquantum crystal": Field-induced spin-density-wave phases. II. Coupling between longitudinal and transverse fluctuations

Didier Poilblanc* and Pascal Lederer†
Laboratoire de Physique des Solides, Université de Paris–Sud, Bâtiment 510, Centre d'Orsay, 91405 Orsay Cédex, France
(Received 21 September 1987)

It is well known that a two-dimensional anisotropic electron gas with an open Fermi surface is unstable at low temperature under the application of a uniform magnetic field perpendicular to the conducting plane. In a previous paper we have studied the collective modes of the field-induced density-wave phases within an approximation which neglects the coupling between spin-density fluctuations and charge-density fluctuations. We have found that the spin-wave mode and the phase mode were degenerate. In this paper we have taken into account the coupling between fluctuations transverse and parallel to the applied magnetic field. Our new results confirm the structure found previously, and describe the small perturbations on the rotonlike modes due to the longitudinal-transverse coupling. We find that both spin-wave and phase-fluctuation modes still exhibit, besides the trivial Goldstone bosons, a series of rotonlike modes for wave vectors of the order of the inverse magnetic length, but the degeneracy of the rotonlike energy minima is weakly lifted: the spin-wave-like modes decrease their energy slightly, while the phasonlike ones are practically unaffected, at least at their minima. The theory applies to Bechgaard salts.

I. INTRODUCTION

In a previous paper[1] (hereafter referred to as PL-I) we have derived the collective modes of field-induced spin-density-wave (FISDW) phases within an approximation which neglects the coupling between fluctuations parallel to the applied magnetic field and perpendicular to the latter; within this approximation there is no coupling between spin-density fluctuations and charge-density fluctuations. The reader is referred to PL-I for a general introduction to the problem.

We have found two degenerate types of collective modes. One is a spin-wave mode; it describes the spin fluctuations along a direction perpendicular to both the applied magnetic field and the order parameter. The other is a phase mode which describes the spin fluctuations perpendicular to the applied magnetic field and parallel to the order parameter.

We have shown that, besides the Goldstone bosons connected to the two continuous broken symmetries of the FISDW phases, the collective modes exhibit a fine structure on the scale of the inverse magnetic length x_0 of the problem. Namely, low-lying rotonlike minima appear at the ordering temperature in the single-particle energy gap and decrease relative to the latter as the temperature decreases. A numerical application to the case of Bechgaard salts, where field-induced spin-density-wave phases are observed, leads to a rotonlike energy minimum of order 30% of the single-particle energy gap.

On the other hand, Maki and Virosztek[2] have derived the FISDW collective modes by studying the spin-density correlation function parallel to the applied magnetic field and the charge-density correlation function. They have taken into account the coupling between fluctuations transverse and parallel to the magnetic field, but they have neglected the specific features of the FISDW electronic spectrum which give rise to the fine structure in the spin-correlation functions. As a result, they find an expression for collective-mode energies which is qualitatively valid in the very long wavelength limit ($q \ll x_0^{-1}$); they find nondegenerate spin-wave modes and phason modes proportional to the wave vector q, a result which corrects their previous erroneous finding of a gap in the transverse spin-fluctuation spectrum.[3]

It is thus necessary to examine the influence of the coupling between longitudinal and transverse spin fluctuations on the collective-mode structure we have found in PL-I. This is what this paper is about. Our results confirm the structure found previously and describe the small perturbations on the rotonlike modes due to the longitudinal-transverse coupling. We find that the degeneracy of the rotonlike energy minima is weakly lifted; the spin-wave-like modes decrease their energy slightly while the phasonlike ones are practically unaffected, at least at their minima.

The rest of the paper is organized as follows. Section II recalls some notations and preliminaries on the basis of the model used in PL-I. Section III sets up the equations for the spin-wave modes, when coupling to fluctuations along the external field is taken into account, and solves for the rotonlike energy minima in the presence of this coupling. Section IV deals with the phase sliding modes; they are coupled to the charge-density fluctuations. The solution for the rotonlike energy minima is given. Our results are discussed in the conclusion, Sec. V.

II. PRELIMINARIES OF FISDW PHASES

The reader is again referred to PL-I for a detailed introduction on the model.

Below the critical temperature T_c^N a staggered linearly polarized magnetization appears in the most conducting plane perpendicular to the magnetic field:

$$\langle \sigma_+(\mathbf{x}) \rangle = \langle \psi_\uparrow^\dagger(\mathbf{x}) \psi_\downarrow(\mathbf{x}) \rangle = \frac{\Delta}{\lambda} e^{i(\mathbf{Q}_N\cdot\mathbf{x}+\phi_1)} + \frac{\Delta}{\lambda} e^{-i(\mathbf{Q}_N\cdot\mathbf{x}+\phi_2)} . \quad (2.1)$$

This defines two components of the order parameters, $\Delta e^{i\phi_1}$ and $\Delta e^{i\phi_2}$, the phases of which are arbitrary. $\phi_1-\phi_2$ describes the angle of the spin direction from the x axis and $\phi_1+\phi_2$ the absolute phase with respect to the lattice. Thus, magnetic ordering breaks the spin rotational and the translational continuous symmetries.

Following Maki and Virosztek (hereafter referred to as MV),[2] it is convenient to define a four-component spinor ψ given by

$$\psi^\dagger = (\psi_{1\uparrow}^\dagger, \psi_{1\downarrow}^\dagger, \psi_{2\uparrow}^\dagger, \psi_{2\downarrow}^\dagger) , \quad (2.2)$$

where $\psi_{i\sigma}$ is the field operator of the electron on the i side of the Fermi surface and is of spin $\sigma=\uparrow,\downarrow$. The equations of motion for the Green's function of Ref. 1 are the following:[4]

$$\begin{aligned} &\left[i\omega_n + iv\frac{d}{dx} + \sigma\mu_B H \right] g_{1\sigma,1\sigma} + \tilde{\Delta}_{i_\sigma}(x) g_{2-\sigma,1\sigma} = \delta^K(x-x') , \\ &\left[i\omega_n - iv\frac{d}{dx} - v\frac{N}{x_0} - \sigma\mu_B H \right] g_{2-\sigma,1\sigma} + \tilde{\Delta}^*_{i_\sigma}(x) g_{1\sigma,1\sigma} = 0 , \end{aligned} \quad (2.3)$$

where $g_{1\sigma,1\sigma}$ and $g_{2-\sigma,1\sigma}$ are the diagonal and off-diagonal Green's functions and $\tilde{\Delta}_{i\sigma}(x)$ is the one-dimensional pseudopotential

$$\tilde{\Delta}_{i_\sigma}(x) = e^{i\phi_{i_\sigma}} \sum_n \delta_n(Q_\perp^N) e^{inx/x_0} , \quad (2.4)$$

where x_0 is the magnetic length, $x_0 = 1/eHvb$, and $Q_\perp^N$ is the actual value of the transverse component of the ordering wave vector $\mathbf{Q}_N$. $i_\sigma = 1$, 2 for, respectively, the spin indices $\uparrow$ and $\downarrow$.

The Fourier components $\delta_n(Q_\perp^N)$ open up gaps in the quasiparticle energy spectrum and separate the Landau bands.[4] These gaps are equidistant in energy at a distance $\omega_c = v/2x_0$ (cyclotron frequency).

When the coupling to the longitudinal fluctuations (along the magnetic field), is considered, the fluctuations of the two components of the order parameter are coupled, and it is essential to consider the phases ϕ_1 and ϕ_2. Nevertheless, it is always possible to choose $\phi_1=\phi_2$, which means the polarization of the spin-density wave (SDW) is chosen along the x direction. In that case, the long-wavelength transverse fluctuations of the order parameter, that is, of the y-spin component

$$\sigma_y(\bar{x}) = \frac{1}{\sqrt{2}} \Psi^\dagger(\bar{x}) \hat{\rho}_x \hat{\sigma}_y \Psi(\bar{x}) ,$$

$[\bar{x}=(\mathbf{x},\tau)]$, are coupled to the fluctuations of the spin along the magnetic field, that is, the fluctuations of the z-spin component $\sigma_z(\bar{x}) = \frac{1}{2}\psi^\dagger \hat{\sigma}_z \psi$. Here $\hat{\rho}_i$ and $\hat{\sigma}_i$ are Pauli matrices operating on spin and ordinary space as used before by MV. Similarly, the longitudinal fluctuations (to the order parameter and transverse to the field) are coupled to the charge-density fluctuations. Note that within our choice of phases we have

$$\langle \sigma_y(\mathbf{x}) \rangle = \langle \sigma_z(\mathbf{x}) \rangle = 0$$

(no staggered y- and z-spin components).

III. SPIN MODES AND TRANSVERSE (TO THE ORDER PARAMETER) ROTON MODES

When fluctuations along the field (z direction) are considered, we must extend our previous random-phase approximation (RPA) equations describing the uncoupled fluctuations of each order-parameter components. Now these components are coupled to each other and to the spin fluctuations along the field. Following MV we may write

$$\begin{aligned} \langle T_\tau \sigma_y \sigma_y \rangle &= \langle T_\tau \sigma_y \sigma_y \rangle_0 + \lambda(\langle T_\tau \sigma_y \sigma_z \rangle_0 \langle T_\tau \sigma_z \sigma_y \rangle \\ &\quad + \langle T_\tau \sigma_y \sigma_y \rangle_0 \langle T_\tau \sigma_y \sigma_y \rangle) , \\ \langle T_\tau \sigma_z \sigma_y \rangle &= \langle T_\tau \sigma_z \sigma_y \rangle_0 + \lambda(\langle T_\tau \sigma_z \sigma_z \rangle_0 \langle T_\tau \sigma_z \sigma_y \rangle \\ &\quad + \langle T_\tau \sigma_z \sigma_y \rangle_0 \langle T_\tau \sigma_y \sigma_y \rangle) , \end{aligned} \quad (3.1)$$

where λ is the dimensionless mean-field coupling constant. (We use units such that the density of state at the Fermi level is 1; T_τ means we use imaginary-time-ordered products; $\langle\ \rangle_0$ means that the thermal average is done including the interaction only in the Hartree-Fock Green's functions.)

The time-ordered products $\langle T_\tau \sigma_i \sigma_j \rangle_0$ are not strictly translation invariant. They are functions of two momenta $\mathbf{q}$, $\mathbf{q}'$, the difference of which is an integer number of time of the ordering wave vector $\mathbf{Q}_N$. It is essential to point out that the magnetic momentum v/x_0 which appears in the scattering potential $\tilde{\Delta}_{i_\sigma}(x)$ does not enter in the relation $\mathbf{q}'-\mathbf{q}=m\mathbf{Q}_N$. In other words, the magnetic field pseudopotential responsible for the scattering of the electron wave function does not break the (discrete) translation invariance of the physical and macroscopic quantities as correlation functions. We keep the terms with the lowest momentum transfer but, unlike MV, (i) we do not assume $q_\perp=0$, and (ii) we do not assume $q_\parallel \ll v/x_0$ since we are interested in the roton modes[1,5] for $q_\parallel \sim mv/x_0$. In that case the collective-mode spectrum loses the $(q_\parallel, q_\perp) \to (-q_\parallel, q_\perp)$ symmetry (it always keeps the $\mathbf{q} \to -\mathbf{q}$ symmetry). Defining χ_{ij} as $\langle T_\tau \sigma_i \sigma_j \rangle$, the general expression for the poles of the diagonal and off-diagonal (in momentum) components $\chi_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)$ and $\chi_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)$, and of the off-diagonal component $\chi_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)$, can be found by solving [using (3.1)]

$$\begin{aligned}\chi_{yy}(\mathbf{Q}_N+\mathbf{q},\omega) &= \chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)+\lambda\chi^0_{yz}(\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\chi_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)+\lambda\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)\\ &\quad +\lambda\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\ ,\\ \chi_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega) &= \chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)+\lambda\chi^0_{yz}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\chi_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\\ &\quad +\lambda\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\\ &\quad +\lambda\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)\ ,\\ \chi_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega) &= \chi^0_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)+\lambda\chi^0_{zz}(\mathbf{q},\omega)\chi_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)+\lambda\chi^0_{zy}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)\\ &\quad +\lambda\chi^0_{zy}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\chi_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\ .\end{aligned} \tag{3.2}$$

The subscript 0 corresponds to the dynamical Hartree-Fock correlation functions depending on the frequency ω and the temperature T. Let us recall that the static staggered spin density is assumed to be polarized along the x direction. The previous equation, then, clearly shows that the spin-density fluctuations along y, and along the magnetic field (z), are coupled, leading to complicated algebra. The following two limits can be considered.

(i) If $q_\perp=0$ and $q_\parallel \ll 1/x_0$ (Ref. 2), then the $q_\parallel \to -q_\parallel$ symmetry is restored and the collective-mode spectrum exhibits a Goldstone mode (spin mode) which reads

$$\omega^2 \propto q_\parallel^2\ . \tag{3.3}$$

[For $q_\perp \neq 0$ see (3.20).] The constant of proportionality is 1 in our previous result. In fact, a renormalization of the Goldstone mode occurs by the spin fluctuations along z, but is not significant in the weak-coupling limit ($\lambda \ll 1$).

(ii) If the coupling terms χ^0_{yz} and χ^0_{zy} are neglected, our previous results are recovered. The spin-mode spectrum exhibits a fine structure with rotonlike minima near each multiple of the magnetic momentum $1/x_0$. Now the next step is to consider the coupling; we are going to check that it does not qualitatively change the picture of roton minima.

The general structure of the pole equation is not very simple even if we use relations like

$$\begin{aligned}\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega) &= [\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)]^*\ ,\\ \chi^0_{yz}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega) &= [\chi^0_{zy}(\mathbf{q},\pm\mathbf{Q}_N+\mathbf{q},\omega)]^*\ .\end{aligned} \tag{3.4}$$

Anticipating the results, we will show that, to a good approximation, χ^0_{yz} and χ^0_{zy} are, in fact, imaginary so that

$$\chi^0_{yz}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega) \simeq -\chi^0_{zy}(\mathbf{q},\pm\mathbf{Q}_N+\mathbf{q},\omega)\ . \tag{3.5}$$

Making use of that result the pole equation is now

$$\begin{aligned}&[1-\lambda\chi^0_{zz}(\mathbf{q},\omega)]\{[1-\lambda\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)][1-\lambda\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\omega)]-\lambda^2[\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)]^2\}\\ &\quad = -\lambda^2[\chi^0_{yz}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)]^2[1-\lambda\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)]-\lambda^2[\chi^0_{yz}(\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)]^2[1-\lambda\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\omega)]\\ &\qquad +2\lambda^3\chi^0_{yz}(\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\chi^0_{yz}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\ .\end{aligned} \tag{3.6}$$

Setting $\chi^0_{yz}=0$, we recover our previous equation,[1]

$$[1-\lambda\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},\omega)][1-\lambda\chi^0_{yy}(-\mathbf{Q}_N+\mathbf{q},\omega)]-\lambda^2[\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)]^2=0\ . \tag{3.6'}$$

General expressions for χ^0_{yy} and χ^0_{yz} are given, respectively, in PL-I and in Appendix A. For a magnetic field such that the cyclotron frequency $\omega_c=v/2x_0$ is smaller than the energy t'_b characterizing the violation of perfect nesting, a number of gaps δ_n (about $1+t'_b/\omega_c$) are comparable in magnitude. Nevertheless, some simplifications can be performed in the weak-coupling limit such that $\delta_n/\omega_c \ll 1$. In the phase labeled by N, the gap δ_N plays a special role because it sits at the Fermi level and is responsible for the stabilization of the ordered magnetic phase. It must be treated to all orders in perturbation.

The Hartree-Fock correlation functions can be computed to zero order in δ_{N+p}, $p\neq 0$; it is performed in PL-I for χ^0_{yy} and in Appendix A for χ^0_{yz} and χ^0_{zz}. Zero order is shown in a diagrammatic form in Fig. 4. It reads

$$\begin{aligned}\chi^0_{yy}(\mathbf{Q},\omega) &= \sum_n I^2_{N+n}(Q_\perp)\tilde{\tilde{\chi}}_0\left[q_\parallel-\frac{n}{x_0},\omega\right]+O((\Delta/\omega_c)^2)\ ,\\ \chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega) &= \sum_n I_{N+n}(Q^N_\perp+q_\perp)I_{N-n}(Q^N_\perp-q_\perp)\tilde{\tilde{\Gamma}}_0\left[q_\parallel-\frac{n}{x_0}\right]+O((\Delta/\omega_c)^2)\ ,\end{aligned} \tag{3.7}$$

$$\chi^0_{yz}(\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\simeq \sum_n I_{N+n}(Q_\perp^N+q_\perp)J_n(t)\tilde{\tilde{\chi}}{}_0^{yz}\left[q_\parallel-\frac{n}{x_0},\omega\right],$$

$$\chi^0_{yz}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\simeq \sum_n I_{N+n}(Q_\perp^N-q_\perp)J_n(t)\tilde{\tilde{\chi}}{}_0^{yz}\left[q_\parallel-\frac{n}{x_0},\omega\right], \tag{3.7$'$}$$

$$\chi^0_{zz}(\mathbf{q})=\sum_n J_n^2(t)\tilde{\tilde{\chi}}{}_0^{zz}\left[q_\parallel-\frac{n}{x_0},\omega\right].$$

The third direction simply disappears since we have neglected the tiny deviation from perfect nesting along the magnetic field and since we consider the collective-mode spectrum only in the plane $Q_z=Q_z^N$. $I_{N+n}(Q_\perp)$ is the coefficient of proportionality between the gap $\delta_{N+n}(Q_\perp)$ and the order-parameter Δ; t is of order $4t_b'/\omega_c$. $\tilde{\tilde{\chi}}_0$ and $\tilde{\tilde{\Gamma}}_0$ are one-dimensional correlation functions renormalized to all orders by the gap δ_N at the Fermi level, as introduced by Lee *et al.*[6] in their study of charge-density wave (CDW) collective modes. $\tilde{\tilde{\chi}}{}_0^{zz}$ and $\tilde{\tilde{\chi}}{}_0^{yz}$ are new one-dimensional elementary irreducible bubbles[2] coming from coupling between y- and z-spin fluctuations. They are computed in Appendix A. $\tilde{\tilde{\chi}}_0(\mathbf{q},\omega=0)$ and $\tilde{\tilde{\Gamma}}_0(\mathbf{q},\omega=0)$ are maximum around $\mathbf{q}=0$. Then we are expecting a series of local maxima of the diagonal and off-diagonal (in momentum) components of χ^0_{yy}. Likewise, a series of oscillations also occurs for both functions since the coefficients I_{N+n} oscillate with $Q_\perp$. But we have to go further to check that this expected fine structure in reciprocal space on a scale x_0^{-1} is not washed out by taking into account the coupling term (3.7′) into (3.6). The elementary irreducible bubbles have the following expressions:

$$\tilde{\tilde{\chi}}_0(\delta,\omega)=\int_0^{\ln 2E_0/\delta_N}dx\,\tanh\left[\frac{\delta_N}{2T}\cosh x\right]+(\omega^2-v^2\delta^2-2\delta_N^2)\tilde{F}(\omega,\delta),$$

$$\tilde{\tilde{\Gamma}}_0(\delta,\omega)=-2\delta_N^2\tilde{F}(\omega,\delta), \tag{3.8}$$

$$\tilde{\tilde{\chi}}{}_0^{yz}(\delta,\omega)=-i\sqrt{2}\omega\delta_N\tilde{F}(\omega,\delta),$$

$$\tilde{\tilde{\chi}}{}_0^{zz}(\delta,\omega)=(v^2\delta^2-\omega^2)^{-1}[v^2\delta^2-4v^2\delta^2\delta_N^2\tilde{F}(\omega,\delta)], \tag{3.8$'$}$$

with $\tilde{F}(\omega,\delta)=(\omega^2-v^2\delta^2)F(\omega,\delta)$; $F(\omega,\delta)$ has already been calculated in Refs. 1 and 2 and depends on the temperature. It simplifies for $\delta=0$ as

$$F(\omega,\delta=0)=\frac{1}{4\delta_N^2\omega^2}h(x,T)\quad\text{with } x=\frac{\omega}{2\delta_N},$$

$$h(x,T)=\int_0^\infty du\frac{\tanh\left[\frac{\delta_N}{2T}\cosh u\right]}{\cosh^2 u-x^2}. \tag{3.9}$$

In the limit $T=0$, $x<1$ it reduces to

$$h(x,T=0)=\frac{\sin^{-1}x}{x(1-x^2)^{1/2}}. \tag{3.9$'$}$$

Besides the explicit dependence on temperature, Eqs. (3.8) and (3.8′) depend on temperature through the gap $\delta_N\equiv\delta_N(T)$. The value of $\delta_N(T)$ is given by the gap equation

$$1/\lambda=\chi^0_{xx}(\mathbf{Q}_N,\omega=0)+\chi^0_{xx}(\mathbf{Q}_N,-\mathbf{Q}_N,\omega=0)$$

$$=\chi^0_{yy}(\mathbf{Q}_N,\omega=0)-\chi^0_{yy}(\mathbf{Q}_N,-\mathbf{Q}_N,\omega=0). \tag{3.9$''$}$$

This formula establishes the static properties of the low-temperature magnetic phase of quantized vector $\mathbf{Q}_N$.

The last step in the computation of collective modes at a wave vector $(q_\perp,q_\parallel=m/x_0+\delta)$ for $\delta x_0\ll 1$ and $\omega\ll\omega_c$ is to approximate (3.7′) for such a wave vector; as for $\tilde{\tilde{\chi}}_0$ and $\tilde{\tilde{\Gamma}}_0$ one can write for $n\neq m$

$$\tilde{\tilde{\chi}}{}_0^{yz}\left[q_\parallel-\frac{n}{x_0},\omega\right]\simeq O((\omega\delta_N/\omega_c^2)\ln(\Delta/\omega_c)),$$

$$\tilde{\tilde{\chi}}{}_0^{zz}\left[q_\parallel-\frac{n}{x_0},\omega\right]-1\simeq O(\omega^2/\omega_c^2). \tag{3.10}$$

In the weak-coupling limit such terms are negligible and we keep only the main terms; as a matter of fact, on a large magnetic field range the magnetic energy scale ω_c is nearly one order of magnitude higher than the condensation energy (order parameter) or than the frequencies we consider here. Then,

$$\chi^0_{yz}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\simeq I_{N+m}(Q_\perp^N-q_\perp)J_m(t)\tilde{\tilde{\chi}}{}_0^{yz}(\delta,\omega),$$

$$\chi^0_{yz}(\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\simeq I_{N+m}(Q_\perp^N+q_\perp)J_m(t)\tilde{\tilde{\chi}}{}_0^{yz}(\delta,\omega), \tag{3.11}$$

$$\chi^0_{zz}(\mathbf{q},\omega)\simeq J_m^2(t)\tilde{\tilde{\chi}}{}_0^{zz}(\delta,\omega)+[1-J_m^2(t)].$$

The same kind of approximation has already been performed for χ_{yy},[1]

$$\chi^0_{yy}(\mathbf{Q},\omega)\simeq \sum_{n(\neq m)} I^2_{N+n}(Q_\perp)\ln\left[\frac{2E_0}{|m-n|\,\omega_c}\right]+I^2_{N+m}(Q_\perp)\tilde{\tilde{\chi}}_0(\delta,\omega)\,,$$
$$\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\simeq I_{N+m}(Q^N_\perp+q_\perp)I_{N-m}(Q^N_\perp-q_\perp)\tilde{\tilde{\Gamma}}_0(\delta,\omega)\,. \tag{3.11'}$$

All of the above expressions are valid even if $\omega \gtrsim \Delta$, $\omega \ll \omega_c$. Using the formulas (3.11) and (3.11') into the pole Eq. (3.6) we immediately get

$$[1-\lambda+\bar{\lambda}-\bar{\lambda}\tilde{\tilde{\chi}}{}^{zz}_0(\delta,\omega)]\left[\left[\ln\frac{2\gamma E_0}{\pi T_{+m}(q_\perp)}-\tilde{\tilde{\chi}}_0(\delta,\omega)\right]\left[\ln\frac{2\gamma E_0}{\pi T_{-m}(q_\perp)}-\tilde{\tilde{\chi}}_0(\delta,\omega)\right]-\tilde{\tilde{\Gamma}}{}^2_0(\delta,\omega)\right]$$
$$=-2\bar{\lambda}[\tilde{\tilde{\chi}}{}^{yz}_0(\delta,\omega)]^2\left[\ln\frac{2\gamma E_0}{\pi[(T_{+m}(q_\perp)T_{-m}(q_\perp)]^{1/2}}-\tilde{\tilde{\chi}}_0(\delta,\omega)-\tilde{\tilde{\Gamma}}_0(\delta,\omega)\right]\,, \tag{3.12}$$

where $\bar{\lambda}=\lambda J^2_m(t)$ and $J_m(t)$ is Bessel's function of order m and argument t.

It has been convenient to incorporate the frequency-independent terms of (3.11') into two temperatures $T_{\pm m}(q_\perp,H)$ characterizing the virtual instability toward metastable subphases of quantum number $N\pm m$ and of transverse wave vector $Q^N_\perp\pm q_\perp$. For $m=0$ and $q_\perp=0$, $T_{\pm m}=T^N_c$, the actual ordering temperature of the Nth subphase. In general, $T_{\pm m}(q_\perp)$ are always lower than the virtual transition lines $T^{N\pm m}_c$ which can be drawn in the Nth subphase part of the phase diagram, and which represent virtual transition lines to phases with slightly larger free energy than the Nth phase and with a different wave vector $\mathbf{Q}_{N\pm m}$. $T_{\pm m}(q_\perp)$ may be equal to $T^{N\pm m}_c$ for a special value $Q^{N\pm m}_\perp$ of $Q_\perp$. In our approximation the gap equation can be explicitly written as

$$\int_0^{\ln(2E_0/\delta_N)} dx\,\tanh\left[\frac{\delta_N}{2T}\cosh x\right]=\ln\frac{2\gamma E_0}{\pi T^N_c}\,. \tag{3.12'}$$

Using (3.8), (3.8'), and (3.12'), (3.12) can be written as follows:

$$[1-\lambda+\bar{\lambda}-\bar{\lambda}(v^2\delta^2-\omega^2)^{-1}(v^2\delta^2-4\omega^2\delta^2_N\tilde{F})]\left[\left[\ln\frac{T^N_c}{T_{+m}(q_\perp)}-(\omega^2-v^2\delta^2-2\delta^2_N)\tilde{F}\right]\right.$$
$$\left.\times\left[\ln\frac{T^N_c}{T_{-m}(q_\perp)}-(\omega^2-v^2\delta^2-2\delta^2_N)\tilde{F}\right]-(2\delta^2_N\tilde{F})^2\right]$$
$$=4\bar{\lambda}\omega^2\delta^2_N\tilde{F}^2\left[\ln\frac{T^N_c}{T^{1/2}_{+m}(q_\perp)T^{1/2}_{-m}(q_\perp)}-(\omega^2-v^2\delta^2-4\delta^2_N)\tilde{F}\right]\,. \tag{3.13}$$

It is clear from the two preceding equations that the collective-mode spectrum verifies the symmetry relation $\omega(\mathbf{q})=\omega(-\mathbf{q})$. The temperature dependence has been incorporated in $\tilde{F}$. Setting $\bar{\lambda}=0$ into (3.13), we recover our previous equation (4.27) of Ref. 1. Now, when the coupling to the spin fluctuations along z is taken into account, the calculation of the collective-mode energies is still simple for special values of the magnetic field $H_m(q_\perp)$ such that

$$T_{+m}(q_\perp,H)=T_{-m}(q_\perp,H)=\theta_m(q_\perp)\,.$$

For such a field, (3.13) decouples into the following two equations:

$$\ln\frac{T^N_c}{\theta_m}=(\omega^2-v^2\delta^2-4\delta^2_N)\tilde{F}\,,$$
$$[1-\lambda+\bar{\lambda}-\bar{\lambda}(v^2\delta^2-\omega^2)^{-1}(v^2\delta^2-4\omega^2\delta^2_N\tilde{F})]\left[\ln\frac{T^N_c}{\theta_m}-(\omega^2-v^2\delta^2)\tilde{F}\right]=4\bar{\lambda}\omega^2\delta^2_N\tilde{F}^2\,. \tag{3.14}$$

The physical interpretation is clear from (3.14). The amplitude and the phase modes decouple. The amplitude mode is not renormalized by the spin fluctuations along the magnetic field. If $m=0$ and $q_\perp=0$, at all temperatures

$$\omega^2=v^2\delta^2+4\delta^2_N(T)\,. \tag{3.15}$$

On the contrary, the phason mode is coupled to χ_{zz}. Using the expression (3.9) of $\tilde{F}$ for $\delta=0$, we get

$$\ln\frac{T^N_c}{\theta_m}=(1-\lambda+\bar{\lambda})\frac{x^2h(x)}{1-\lambda+\bar{\lambda}-\bar{\lambda}h(x)}\,. \tag{3.16}$$

In the weak-coupling limit $1-\lambda+\bar{\lambda}\simeq 1$. In the denominator, $\bar{\lambda}$ is responsible for a pole at $x=x_0$ such $h(x_0)\simeq 1/\bar{\lambda}$. This equation is solved graphically in Fig. 1. At $T=0$ we always find a real solution,

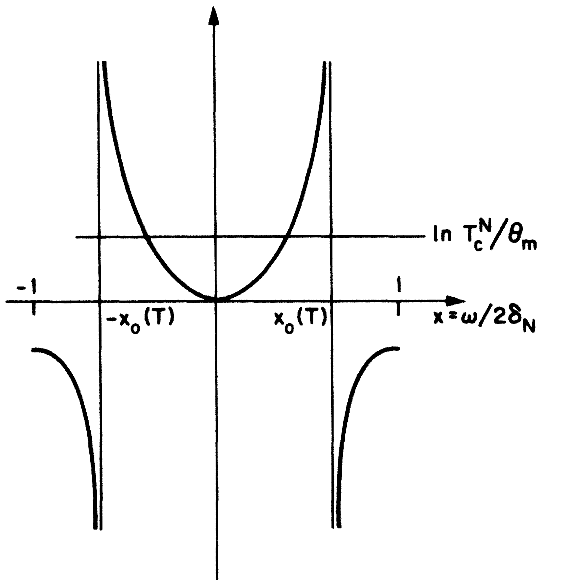


FIG. 1. Graphic solution of Eq. (3.16). The roton minimum is obtained at the crossing point between the constant $\ln(T_c^N/\theta_m)$ and the function $x \to x^2 h(x)/1-\bar{\lambda}h(x)$. If θ_m is close to T_c^N, the roton frequency is very low. When $\theta_m \to 0$, the roton frequency is bounded by $2\delta_N$.

$$\omega_{r-}^m < \left[1-\frac{\pi}{4}\bar{\lambda}\right] 2\delta_N \, . \tag{3.17}$$

There is no damping of these collective modes since they are localized in the quasiparticle energy gap at the Fermi level. We always find a lowering of the roton mode in the gap because of the longitudinal fluctuations (along the field **H**). If θ_m is well below T_c^N, the roton-mode energy is close to the gap $2\delta_N$, and the relative correction due to longitudinal spin fluctuations is of order $\bar{\lambda}$. If θ_m is close to T_c^N, the roton-mode energy is deep in the gap and we find

$$\omega_{r-}^m = 2\delta_N(1-\bar{\lambda})^{1/2}\ln^{1/2}(T_c^N/\theta_m) \, . \tag{3.18}$$

The physical meaning is clear: The roton mode is a low-energy collective-mode excitation if the metastable phases $N+m$ and $N-m$ are close enough in energy to the stable phase N. This condition could be easily fulfilled for small values of the integer m. The relative corrections are of order $\bar{\lambda}$ for every roton minima in the single-particle gap. The rotonlike mode appears in the gap as soon as $T < T_c^N$. The solution is still real and is bound for $T \simeq T_c^N$ by

$$\omega_{r-}^m \le 2\delta_N(T) \left[1-\bar{\lambda}\frac{\pi\delta_N(T)}{8T}\right] . \tag{3.19}$$

As before, we get an additional small factor proportional to $\bar{\lambda}$ and the roton-mode energy is reduced by a small factor of order $\delta_N(T)/T_c^N \, \bar{\lambda} \ll 1$. The qualitative behavior of the roton mode with temperature is not affected by longitudinal fluctuations. As the temperature is decreased, the relative distance of the roton energy from the gap increases, and this increase is maximum for $m=1$ and for

$$q_\perp^* \simeq Q_\perp^{N+1} - Q_\perp^N \simeq Q_\perp^N - Q_\perp^{N-1} \, ,$$

the values of which guarantee $\theta_{m=1}$ to be the closest to T_c^N.

It is easy to derive the Goldstone mode for $q_\perp \ll t_b'$ and $q_\parallel = \delta \ll 1/x_0$ $(m=0)$. In that case, besides the $\mathbf{q} \to -\mathbf{q}$ symmetry, Eq. (3.13) recovers the $q_\parallel \to -q_\parallel$ symmetry. Indeed, $T_0(q_\perp) \simeq T_0(-q_\perp)$, which is the critical temperature for a phase N in which the transverse component $Q_\perp$ is slightly different from its optimum value $Q_\perp^N$. It has the $q_\perp \to -q_\perp$ symmetry since $q_\perp = 0$ corresponds to the maximum of $T_0(q_\perp)$, namely $T_0(q_\perp=0) = T_c^N$. Then, (3.13) factorizes and the phase and amplitude mode decouples. We find, respectively, the phason and the amplitude modes

$$\omega^2 = (1-\lambda)\left[v^2\delta^2 + \frac{4\delta_N^2(T)}{h(x=0,T)}\ln\frac{T_c^N}{T_0(q_\perp)}\right] , \tag{3.20}$$

$$\omega^2 = 4\delta_N^2(T) + v^2\delta^2 + \frac{4\delta_N^2(T)}{h(x\to 1,T)}\ln\frac{T_c^N}{T_0(q_\perp)} \, . \tag{3.20'}$$

For $q_\perp \neq 0$ the phason and amplitudon become temperature dependent through the $h(x,T)$ thermal factor. Note that only the phason is renormalized by the transverse-longitudinal coupling. Physically, it is clear that the $(1-\lambda)$ factor is related to the Stoner factor in the static spin susceptibility. Obviously $\ln T_c^N/T_0$ is proportional to $v^2(q_\perp/t_b')^2$.

Another simple limit is $q_\perp = 0$. Then,

$$J_m(q_\perp=0) = \delta^K(m) \, .$$

If $m \neq 0$, then the coupling term χ_{yz}^0 from (3.11) vanishes, leading to decoupled order-parameter fluctuations.

From the results above we conclude that the roton minima in the collective-spin-mode spectrum are not suppressed by their coupling to the fluctuations of the longitudinal (i.e., parallel to the field **H**) component of the spin. On the contrary, their energy in general decreases and their renormalization is exactly the same as the one of the Goldstone mode predicted by MV. In the weak-coupling limit these corrections on the order-parameter spectrum are negligible. However, we agree with MV that both fluctuations should be handled at the same level to compute the longitudinal susceptibility χ_{zz} which is shown to have no change at the critical temperature. So, even if the two fluctuations should be physically treated at the same level, we find qualitatively the same results already found in Ref. 1. Now let us study the sliding mode spectrum, the degeneracy of which—with the spin mode spectrum—is lifted in our new study.

IV. SLIDING MODES AND LONGITUDINAL ROTON MODES

In our first analysis, longitudinal and transverse spin mode were degenerate. In fact, the longitudinal fluctuations of the order parameter are coupled to the charge fluctuations. Such coupling has been considered in the derivation of the charge dynamics response functions $\chi_{\rho\rho}$. Our aim, here, is somewhat different: We are going to analyze the effects of such a coupling on the order-parameter fluctuations at a wave-vector scale of the order of the magnetic wave vector $1/x_0$. Our RPA equations are then similar to MV (Ref. 2):

$$\begin{aligned}\langle T_\tau \delta\Delta_i \,\delta\Delta_1\rangle &= \langle T_\tau \delta\Delta_i \,\delta\Delta_1\rangle_0 - \lambda\langle T_\tau \delta\Delta_i \,\rho\rangle_0 \langle T_\tau \rho\, \delta\Delta_1\rangle \\ &\quad + \lambda \sum_k \langle T_\tau \delta\Delta_i \,\delta\Delta_k\rangle_0 \langle T_\tau \delta\Delta_k \,\delta\Delta_1\rangle \;, \\ \langle T_\tau \rho\, \delta\Delta_1\rangle &= \langle T_\tau \rho\, \delta\Delta_1\rangle_0 - \lambda\langle T_\tau \rho\rho\rangle_0 \langle T_\tau \rho\, \delta\Delta_1\rangle \\ &\quad + \lambda \sum_k \langle T_\tau \rho\, \delta\Delta_k\rangle_0 \langle T_\tau \delta\Delta_k \,\delta\Delta_1\rangle \;.\end{aligned} \tag{4.1}$$

The subscript i of $\delta\Delta_i$ means 1 or 2. The operators $\delta\Delta_i$ are needed to describe spin fluctuations along the direction of polarization,

$$\delta\Delta_2 \equiv \sigma'_x = \frac{1}{\sqrt{2}} \psi^\dagger \hat{\sigma}_x \psi$$

and $\delta\Delta_1$ is the x-spin component:

$$\delta\Delta_1 \equiv \sigma_x = \frac{1}{\sqrt{2}} \psi^\dagger \hat{\rho}_x \hat{\sigma}_x \psi \;.$$

ρ is the density of charge. The preceding equation simply establishes that the spin-density fluctuations along the polarization vector (namely the x direction) are coupled to the charge fluctuations of the anisotropic electron gas. The pole of χ_{xx} (or χ_{11}) is given by a 5×5 determinant as shown in Appendix C. The irreducible Hartree-Fock bubbles which couple spin and charge fluctuations can be considered as real to a very good approximation (Appendix B) so that

$$\begin{aligned}\chi^0_{x\rho}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega) &= \chi^{0*}_{\rho x}(\mathbf{q},\pm\mathbf{Q}_N+\mathbf{q},\omega) \\ &\simeq \chi^0_{\rho x}(\mathbf{q},\pm\mathbf{Q}_N+\mathbf{q},\omega)\;, \\ \chi^0_{x'\rho}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega) &= \chi^{0*}_{\rho x'}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega) \\ &\simeq -\chi^0_{\rho x'}(\pm\mathbf{Q}_N+\mathbf{q},\mathbf{q},\omega)\;,\end{aligned} \tag{4.2}$$

where x' ($\equiv 2$) corresponds to $\sigma'_x \equiv \delta\Delta_2$.

Furthermore, $\chi^0_{\rho x}$ and $\chi^0_{\rho x'}$ are related by general relations as

$$\begin{aligned}\chi^0_{\rho x'}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega) &= i\chi^0_{\rho x}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\;, \\ \chi^0_{\rho x'}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega) &= -i\chi^0_{\rho x}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\;.\end{aligned} \tag{4.2$'$}$$

Similarly $\chi^0_{x'x'}$, $\chi^0_{x'x}$, and $\chi^0_{xx'}$ are related to χ^0_{xx} (see Appendix B).

Using (4.2) and (4.2′) the pole equation becomes

$$\begin{aligned}&2\left[1+\frac{\lambda}{2}\chi^0_{\rho\rho}(\mathbf{q},\omega)\right]\{[1-\lambda\chi^0_{xx}(\mathbf{Q}_N+\mathbf{q},\omega)][1-\lambda\chi^0_{xx}(-\mathbf{Q}_N+\mathbf{q},\omega)]-\lambda^2[\chi^0_{xx}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)]^2\} \\ &\quad =\lambda^2[\chi^0_{\rho x}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)]^2[1-\lambda\chi^0_{xx}(\mathbf{Q}_N+\mathbf{q},\omega)]-\lambda^2[\chi^0_{\rho x}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)]^2[1-\lambda\chi^0_{xx}(-\mathbf{Q}_N+\mathbf{q},\omega)] \\ &\quad\quad -2\lambda^3\chi^0_{\rho x}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\chi^0_{\rho x}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\chi^0_{xx}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\;.\end{aligned} \tag{4.3}$$

The notations are similar to those of Sec. III,

$$\chi^0_{xx}(\pm\mathbf{Q}_N+\mathbf{q},\omega)=\chi^0_{yy}(\pm\mathbf{Q}_N+\mathbf{q},\omega)$$

and

$$\chi^0_{xx}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)=-\chi^0_{yy}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\;.$$

$\chi^0_{\rho\rho}$ is the usual Hartree-Fock charge-charge response function. It should be noted that the equations of Appendix C would enable us to go beyond the pole equation in computing the RPA response functions themselves. It could be shown, for example, that the static charge-charge susceptibility has the well-known Stoner form. The Hartree-Fock bubble $\chi^0_{\rho x}=\langle T_\tau \rho\sigma_x\rangle$ is responsible for the coupling between charge and spin fluctuations.

These new terms can be treated within the same approximation as above: The gap δ_N at the Fermi level is treated to all orders in perturbation while the expansion in terms of the δ_{N+i}, $i\neq 0$ is cut at zero order (Appendix B)

$$\begin{aligned}\chi^0_{\rho\rho}(\mathbf{q},\omega) &\simeq \sum_n J_n^2(t)\tilde{\tilde{\chi}}{}^{\rho\rho}_0\left[q_\parallel-\frac{n}{x_0},\omega\right]\;, \\ \chi^0_{\rho x}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega) &\simeq \sum_n J_n(t) I_{N+n}(Q_\perp^N+q_\perp) \\ &\quad\times\tilde{\tilde{\chi}}{}^{\rho x}_0\left[q_\parallel-\frac{n}{x_0},\omega\right]\;, \\ \chi^0_{\rho x}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega) &\simeq \sum_n J_n(t) I_{N-n}(Q_\perp^N-q_\perp) \\ &\quad\times\tilde{\tilde{\chi}}{}^{\rho x}_0\left[q_\parallel-\frac{n}{x_0},\omega\right]\;,\end{aligned} \tag{4.4}$$

where the superscript $\approx$ means one-dimensional irreducible bubbles renormalized to all orders by the gap at the Fermi level δ_N. Their values are given in Ref. 2 and in Appendix B,

$$\tilde{\tilde{\chi}}{}_0^{\rho\rho}(\delta,\omega)=4(v^2\delta^2-\omega^2)^{-1}v^2\delta^2(1-4\delta_N^2\tilde{F}) ,$$
$$\tilde{\tilde{\chi}}{}_0^{\rho x}(\delta,\omega)=2^{3/2}v\delta\delta_N\tilde{F} . \quad (4.5)$$

Up to now, we limited ourselves to the computation of collective modes at a wave vector $(q_\perp, q_\| = m/x_0+\delta)$ to establish the quantum oscillations of the spectrum. The next step is to approximate (4.4) in that limit. For $n\neq m$

$$\tilde{\tilde{\chi}}{}_0^{\rho\rho}\left[q_\|-\frac{n}{x_0},\omega\right]-4\simeq O\left[\frac{\omega^2}{\omega_c^2},\frac{\delta_N^2}{\omega_c^2}\ln\frac{\Delta}{\omega_c}\right] ,$$
$$\tilde{\tilde{\chi}}{}_0^{\rho x}\left[q_\|-\frac{n}{x_0},\omega\right]\simeq O\left[\frac{\delta_N}{\omega_c}\right] . \quad (4.6)$$

Then we can approximate the Hartree-Fock bubbles in the weak-coupling limit where $\omega/\omega_c \ll 1$ and $\delta_N/\omega_c \ll 1$:

$$\chi_{\rho\rho}^0(\mathbf{q},\omega)\simeq 4[1-J_m^2(t)]+J_m^2(t)\tilde{\tilde{\chi}}{}_0^{\rho\rho}(\omega,\delta) ,$$
$$\chi_{\rho x}^0(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)\simeq J_m(t)I_{N+n}(Q_\perp^N+q_\perp)\tilde{\tilde{\chi}}{}_0^{\rho x}(\omega,\delta) , \quad (4.7)$$
$$\chi_{\rho x}^0(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)\simeq J_m(t)I_{N-m}(Q_\perp^N-q_\perp)\tilde{\tilde{\chi}}{}_0^{\rho x}(\omega,\delta) .$$

Let us recall

$$J_m(t=0)=\delta^K(m) .$$

Then, for $m\neq 0$ and $q_\perp=0$, the coupling term $\chi_{\rho x}^0$ reduces to zero and the longitudinal fluctuations of the order parameter decouple from the charge fluctuations. Then for $q_\perp=0$, $m\neq 0$ the longitudinal and transverse fluctuations (to the order parameter) are still degenerate. That is not true for $m=0$, i.e., for the spin mode and the sliding Goldstone mode in the limit $q_\|\to 0$. To check this point it is convenient to rewrite (4.3) into a new equation:

$$\left[1+\lambda-\bar{\lambda}+\frac{\bar{\lambda}}{4}\tilde{\tilde{\chi}}{}_0^{\rho\rho}(\omega,\delta)\right]\left[\left[\ln\frac{T_c^N}{T_{+m}(q_\perp)}-(\omega^2-v^2\delta^2-2\delta_N^2)\tilde{F}\right]\left[\ln\frac{T_c^N}{T_{-m}(q_\perp)}-(\omega^2-v^2\delta^2-2\delta_N^2)\tilde{F}\right]-\tilde{\tilde{\Gamma}}{}_0^2(\omega,\delta)\right]$$
$$=\frac{\bar{\lambda}}{2}(\tilde{\tilde{\chi}}{}_0^{\rho x}(\delta,\omega))^2\left[\ln\frac{T_c^N}{\pi T_{+m}(q_\perp)^{1/2}T_{-m}(q_\perp)^{1/2}}-(\omega^2-v^2\delta^2-4\delta_N^2)\tilde{F}\right] . \quad (4.8)$$

If $m=0$, $q_\perp \ll t_b'$ the amplitudon and the phason (sliding mode) decouple. Replacing $T_{\pm m}(q_\perp)$ by $T_0(q_\perp)$ and (4.5) into (4.8), one finds a renormalization of the sliding mode by charge fluctuations, although the amplitude mode is not affected:

$$\omega^2=(1+\lambda)v^2\delta^2+\frac{4\delta_N^2(T)}{h(x=0,T)}\ln T_c^N/T_0(q_\perp) , \quad (4.9)$$

$$\omega^2=4\delta_N^2(T)+v^2\delta^2+\frac{4\delta_N^2(T)}{h(x\to 1,T)}\ln T_c^N/T_0(q_\perp) . \quad (4.9')$$

In fact, for $q_\perp=0$, we recover the "poles" of $\chi_{\rho\rho}$ found by MV. Equation (4.9) is an actual pole for χ_{xx} (sliding mode) but not for $\chi_{\rho\rho}$ since the numerator of $\chi_{\rho\rho}$ also vanishes for $\omega=vq_\|=vq_\perp=0$. Actually, in this limit the charge-charge susceptibility reduces to the usual Stoner formulas. From (3.20) and (4.9) we see that the renormalization of the two Goldstone modes by fluctuations parallel to the field is different (although weak in the weak-coupling limit). Then, sliding mode and spin mode are no longer degenerate. Furthermore, only δ^2 enters in (4.8) and, on the other hand, $\tilde{\tilde{\chi}}{}_0^{\rho x}(\delta=0,\omega)=0$. This implies that the roton minimum energy for $m\neq 0$ is not affected by $\tilde{\tilde{\chi}}{}_0^{\rho x}$. The only effect of the coupling term $\tilde{\tilde{\chi}}{}_0^{\rho x}$ is to modify the curvature at the minimum. This effect does not even appear for the special value $q_\perp=0$.

V. CONCLUSION

In PL-I (Ref. 1), we had derived the peculiar rotonlike structure of the order-parameter collective modes of the FISDW phases. It was shown there that this structure is characterized by dispersion relation minima at wave vectors $q_\|=n/x_0$ (n an integer) and special values of $q_\perp$ in the plane $q_z=\pi/c$ (see Fig. 2). It was argued that the reason behind this structure is the interplay between electromagnetic gauge invariance and breaking of translation invariance by the electronic orbital motion. Spin-wave modes and sliding modes were found to be degenerate in PL-I within the approximation which neglects coupling between the spin fluctuations transverse to the applied magnetic field and spin fluctuations along this field, or charge fluctuations.

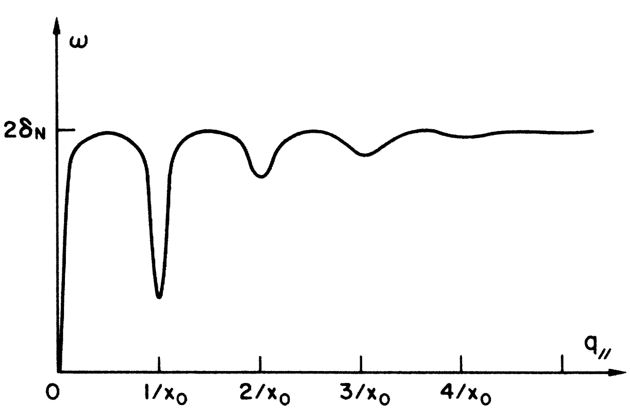


FIG. 2. Schematic view of the collective mode energy spectrum; we have shown only the minimum of $\omega(q_\|, q_\perp)$ with respect to $q_\perp$ as a function of $q_\|$. This anisotropic spectrum also gets minima in the $q_\perp$ direction. It should be noted that the whole spectrum is discontinuous at the crossing of the first-order transition lines between the quantum phase N and its neighboring phases $N+1$ and $N-1$.

In the present work we have shown that the essential results of PL-I survive in a more complete treatment which includes the previously neglected couplings, at least in the weak-coupling limit. A schematic view of both spin modes or phase-fluctuation modes is presented in Fig. 2. The degeneracy of spin-wave modes and sliding modes is lifted. Spin-wave rotonlike minima are lowered by a relative shift of order $\bar{\lambda}=\lambda J_m^2(t)$, while sliding modes have unchanged minima and their group velocity is increased by a relative shift of order $\bar{\lambda}$. At $q_\perp=0$ the collective-mode energies remain degenerate. As a result, it is a fairly sound approximation to neglect altogether the transverse-longitudinal coupling when studying the order-parameter correlation functions and some consequences of the rotonlike energy minima derived in PL-I on the physical properties of FISDW phases such as can be found in Bechgaard salts. On the other hand, it is essential to take into account the transverse-longitudinal coupling when dealing with the longitudinal correlation functions, at $\mathbf{q}\to 0$, studied by MV. It will be interesting to study the physical consequences of the "rotonlike" structure which exists in the latter in the physical properties [electron spin resonance (ESR), nonlinear conductivity] discussed by MV.

The justification of the labor involved in disentangling the complicated expressions contained in Secs. III and IV of this paper, and in the Appendices, is our belief that the roton minimum described in PL-I is a real feature of the physics of Bechgaard salts under magnetic field; this belief is rooted in the general semiquantitative agreement of the theory with the experimentally observed phase diagram[4] and in numerical calculations reported in Ref. 7 which are based on the same model with the same coefficients. The main feature of the numerical work done in Ref. 7, for our purpose here, is that in any SDW subphase of index N, the virtual transition lines T_m to subphases of index $n\pm m$, are not exponentially small compared to the metal SDW actual transition temperature T_c^N, at least for $m=1$, sometimes for $m=2$. This is a necessary condition for the lowest roton-mode energy to be significantly deep in the single-particle gap. It is difficult to ascertain the degree of accuracy of our estimate, but the results displayed in Ref. 7 seem to show that the existence of at least one low-lying roton mode (at $m=1$) is a real feature in Bechgaard salts in all SDW subphases.

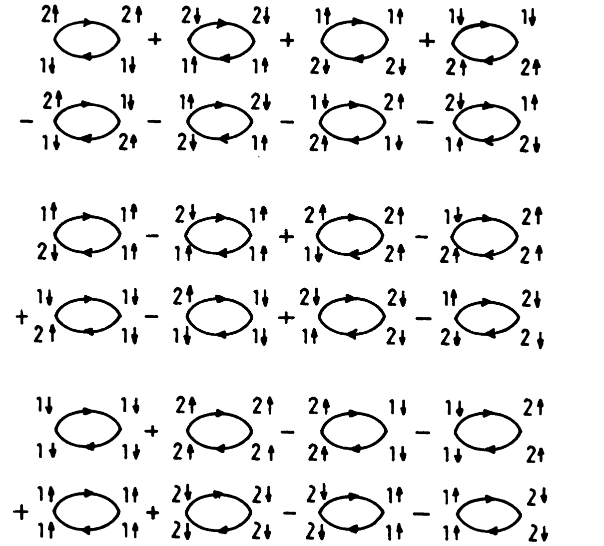


FIG. 3. Hartree-Fock bubbles $2\langle T_\tau\sigma_y\sigma_y\rangle_0$, $(2^{3/2}/i)\langle T_\tau\sigma_y\sigma_z\rangle_0$, $4\langle T_\tau\sigma_z\sigma_z\rangle_0$ obtained by use of Wick's theorem and of the expressions of the spin operators $\sigma_y=2^{-1/2}(\psi_{2\uparrow}^\dagger\psi_{1\downarrow}+\psi_{1\uparrow}^\dagger\psi_{2\downarrow}-\psi_{2\downarrow}^\dagger\psi_{1\uparrow}-\psi_{1\downarrow}^\dagger\psi_{2\uparrow})$ and $\sigma_z=\frac{1}{2}(\psi_{1\uparrow}^\dagger\psi_{1\uparrow}+\psi_{2\uparrow}^\dagger\psi_{2\uparrow}-\psi_{1\downarrow}^\dagger\psi_{1\downarrow}-\psi_{2\downarrow}^\dagger\psi_{2\downarrow})$. The only allowed contractions in the SDW phase involve separately $1\uparrow$, $2\downarrow$, and $1\downarrow$, $2\uparrow$ pairings. On the contrary, the interacting random phase approximation bubbles mix the two types of pairings. No Zeeman energy remains in the Hartree-Fock correlation functions even if it appears (as a phase factor) in each Green's function. Those correlation functions in general depend on the phases ϕ_1 and ϕ_2 (except χ_{zz}^0). With the choice $\phi_1=\phi_2$ the dependence on the phase disappears and spin indices may be dropped.

APPENDIX A: HARTREE-FOCK CORRELATION FUNCTIONS χ_{yy}^0, χ_{yz}^0, AND χ_{zz}^0

Using the expressions for the spin operators σ_y and σ_z given in the Introduction, it is easy to check that these Hartree-Fock correlation functions can be drawn as shown in Fig. 3. The last two correlation functions contain, respectively, expressions of the type

$$
\begin{aligned}
A(x,x',q_\perp)&=-T\sum_{\omega_n}\langle\langle G_{21}(x,x',p+q_\perp b,\omega_n+\omega_p)G_{11}(x',x,p,\omega_n)\rangle\rangle ,\\
B(x,x',q_\perp)&=-T\sum_{\omega_n}\langle\langle G_{11}(x,x',p+q_\perp b,\omega_n+\omega_p)G_{11}(x',x,p,\omega_n)\rangle\rangle , \qquad \text{(A1)}\\
C(x,x',q_\perp)&=-T\sum_{\omega_n}\langle\langle G_{12}(x,x',p+Q_\perp b,\omega_n+\omega_p)G_{21}(x',x,p,\omega_n)\rangle\rangle .
\end{aligned}
$$

They have been written in the mixed representation. G_{ij} are the Green's functions $\langle T_\tau \psi_i \psi_j^\dagger \rangle$ calculated in the Hartree-Fock approximation. Phase transformations connect these functions to the g_{ij} of the text. The corresponding phase factors are given in the following:

$$\Phi_{11}(x,x',p)=k_F(x-x')+\frac{x_0}{v}T_\perp\left[p-\frac{x}{x_0}\right]-\frac{x_0}{v}T_\perp\left[p-\frac{x'}{x_0}\right],$$

$$\Phi_{22}(x,x',p)=-k_F(x-x')-\frac{x_0}{v}T_\perp\left[p-\frac{x}{x_0}\right]+\frac{x_0}{v}T_\perp\left[p-\frac{x'}{x_0}\right],$$

$$\Phi_{21}(x,x',p)=k_F(x-x')-\frac{x_0}{v}T_\perp\left[p-\frac{x}{x_0}+Q_\perp^N b\right]+\frac{x_0}{v}T_\perp(p+Q_\perp^N b)$$
$$-\frac{x_0}{v}T_\perp\left[p-\frac{x'}{x_0}\right]+\frac{x_0}{v}T_\perp(p),$$

$$\Phi_{12}(x,x',p)=-k_F(x-x')+\frac{x_0}{v}T_\perp\left[p-\frac{x}{x_0}-Q_\perp^N b\right]-\frac{x_0}{v}T_\perp(p-Q_\perp^N b)$$
$$+\frac{x_0}{v}T_\perp\left[p-\frac{x'}{x_0}-Q_\perp^N b\right]-\frac{x_0}{v}T_\perp(p), \quad \text{(A2)}$$

where $T_\perp(u)=\int_0^u t_\perp(p)dp$.

Until now we use units such that $x_0=1$. The following expansions will be useful to express the functions A, B, and C above:

$$\exp\left[-\frac{i}{v}T_\perp(p-x+Q_\perp b)-\frac{i}{v}T_\perp(p-x)\right]=\sum_n I_n(Q_\perp)\exp in\left[p+\frac{Q_\perp b}{2}-x\right],$$

$$\exp\left[-\frac{i}{v}T_\perp(p+q_\perp b)+\frac{i}{v}T_\perp(p)\right]=\sum_n K_n(q_\perp)\exp in\left[p+\frac{q_\perp b}{2}\right]. \quad \text{(A3)}$$

The coefficients are easily obtained by inverting (A3),

$$I_n(Q_\perp)=\left\langle\!\!\left\langle \exp\left[\frac{i}{v}T_\perp(p+Q_\perp b)+\frac{i}{v}T_\perp(p)+in\left[p+\frac{Q_\perp b}{2}\right]\right]\right\rangle\!\!\right\rangle,$$

$$K_n(q_\perp)=\left\langle\!\!\left\langle \exp\left[-\frac{i}{v}T_\perp(p+q_\perp b)+\frac{i}{v}T_\perp(p)-in\left[p+\frac{q_\perp b}{2}\right]\right]\right\rangle\!\!\right\rangle. \quad \text{(A4)}$$

As $T_\perp$ is an odd function., it is easy to check that

$$\sum_N I_N^2=\sum_N I_N=1,$$
$$\sum_N |K_N|^2=1,$$
$$I_n(Q_\perp)=I_n(-Q_\perp)=I_n^*(Q_\perp),$$
$$K_n(q_\perp)=K_{-n}(q_\perp),\quad K_n^*(q_\perp)=K_n(-q_\perp). \quad \text{(A5)}$$

K_n is not real. It is possible to reduce it to an approximate simple expression. Using the standard expression for the transverse dispersion (see Ref. 1) we obtain

$$K_n(q_\perp)=(i)^n\sum_l(-i)^l J_{n-2l}(t)J_l(t'), \quad \text{(A6)}$$

where

$$t=\frac{8t_b}{\omega_c}\sin(q_\perp b/2),\quad t'=\frac{4t_b'}{\omega_c}\sin(q_\perp b).$$

t_b and t_b' characterize, respectively, the dispersion energy in the k_y direction and the violation of perfect nesting along k_y. We can restrict ourselves to values of $q_\perp b$ of the order of $t_b'/t_b \ll 1$ so that $t' \sim t_b'^2/\omega_c t_b \ll 1$. So we can approximate (A6) by

$$K_n(q_\perp)\simeq i^n J_n(t) \text{ with } t\simeq \frac{4t_b'}{\omega_c} . \tag{A7}$$

The Green's function g_{ij} is not space translation invariant because of the gauge periodic potential of periodicity $2\pi x_0$. So we can expand them in the following way:

$$\begin{aligned}
g_{11}(x,x',p,\omega_n)&=\sum_m \int dk \exp\left[+ik(x-x')+imx-im\left[p+\frac{Q_\perp^N b}{2}\right]\right]g_m(k,\omega_n) ,\\
g_{21}(x,x',p,\omega_n)&=\exp\left[-\frac{i}{v}T_\perp(p)-\frac{i}{v}T_\perp(p+Q_\perp b)\right]\sum_m \int dk \exp\left[+ik(x-x')+imx-im\left[p+\frac{Q_\perp^N b}{2}\right]\right]f_m(k,\omega_n) ,\\
g_{12}(x,x',p,\omega_n)&=\exp\left[\frac{i}{v}T_\perp(p)+\frac{i}{v}T_\perp(p-Q_\perp b)\right]\sum_m \int dk \exp\left[-ik(x-x')-imx+im\left[p-\frac{Q_\perp^N b}{2}\right]\right]f_m(k,\omega_n) ,\\
g_{22}(x,x',p,\omega_n)&=\sum_m \int dk \exp\left[-ik(x-x')-imx+im\left[p-\frac{Q_\perp^N b}{2}\right]\right]g_m(k,\omega_n) .
\end{aligned} \tag{A8}$$

The coefficients g_m and f_m, as defined above, do not depend on $p=k_\perp b$ and are given by solving a Dyson equation (see Ref. 1). Putting (A2) and (A8) into (A1), making use of (A3), and finally averaging over p leads to functions of $x-x'$ as expected from general gauge invariance arguments. Then it is possible to perform Fourier transform relatively to $x-x'$; we find

$$\begin{aligned}
A^F&=-T\sum_{\omega_n}\sum_{n,n',m}\int dk\, I_{N+n}(Q_\perp)K_{n'}(q_\perp)e^{i\phi_A}f_{N+m}(-k-q_\parallel,\omega_n+\omega_p)g_{n+n'-m}(-k+m-n,\omega_n) ,\\
B^F&=-T\sum_{\omega_n}\sum_{n,n',m}\int dk\, K_n^*(q_\perp)K_{n'}(q_\perp)e^{i\phi_B}g_m(-k-q_\parallel,\omega_n+\omega_p)g_{n'-m-n}(-k+m+n,\omega_n) ,\\
C^F&=-T\sum_{\omega_n}\sum_{n,n',m}\int dk\, K_n^*(q_\perp)K_{n'}^*(q_\perp)e^{i\phi_C}f_{N+m}(k+q_\parallel,\omega_n+\omega_p)f_{N+m-n-n'}(-k+n-m,\omega_n) ,
\end{aligned} \tag{A9}$$

with

$$\begin{aligned}
\phi_A&=(n+n'-2m-N)\frac{Q_\perp b}{2}-n'\frac{Q_\perp^N b}{2} ,\\
\phi_B&=(n'-n-2m)\frac{q_\perp b}{2}+(n-n')\frac{Q_\perp^N b}{2} ,\\
\phi_C&=n\frac{Q_\perp^N b}{2}+(2m+2N-n-n')\frac{q_\perp b}{2} .
\end{aligned}$$

These formulas, although general, are too complicated to be used in the text. At sufficiently low temperatures, as explained in the text, we perform an expansion to zero order in powers of δ_{N+p}, $p\neq 0$ (for details see Ref. 1). So we keep only

$$\begin{aligned}
f_{N+m}(k,\omega_n)&\simeq f_{N+m}^{(0)}(k,\omega_n)\equiv -\delta^K(m)\delta_N/\mathcal{D}_0 ,\\
g_m(k,\omega_n)&\simeq g_m^{(0)}(k,\omega_n)\equiv \delta^K(m)(i\omega_n+k)/\mathcal{D}_0 ,
\end{aligned} \tag{A10}$$

with $\mathcal{D}_p=(i\omega_n+k+p)(i\omega_n-k-p)-\delta_N^2$.

On the other hand, $K_n(q_\perp)$ can be approximated by using (A7). Assuming $q_\perp b \ll 1$, $Q_\perp^N b\simeq\pi$ it is easy to check that all extra phase factors could be taken to 1 to a good approximation.

Then, (A9) can be rewritten as

$$\begin{aligned}
A^F&=-T\sum_{\omega_n}\sum_n J_n(q_\perp)I_{N+n}(Q_\perp)\int dk\, f_N^{(0)}(q_\parallel-n+k,\omega_n+\omega_p)g_0^{(0)}(-k,\omega_n) ,\\
B^F&=-T\sum_{\omega_n}\sum_n J_n^2(q_\perp)\int dk\, g_0^{(0)}(-k,\omega_n)g_0^{(0)}(-(q_\parallel-n-k),\omega_n+\omega_p) ,\\
C^F&=-T\sum_{\omega_n}\sum_n J_n^2(q_\perp)\int dk\, f_N^{(0)}(-k,\omega_n)f_N^{(0)}(q_\parallel-n+k,\omega_n+\omega_p) .
\end{aligned} \tag{A11}$$

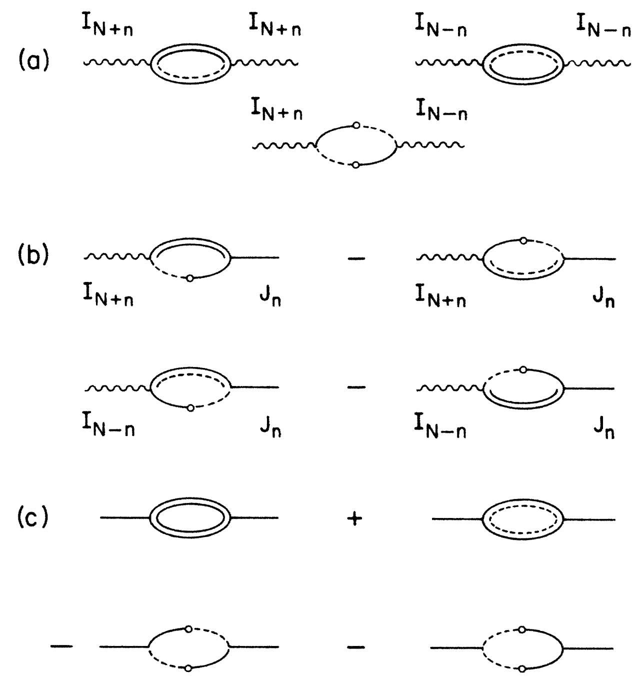
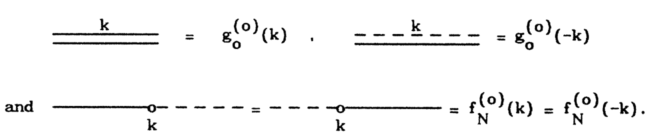


FIG. 4. Hatree-Fock (HF) correlation functions of Fig. 3 can be approximated as explained in Appendix A. The general rules consist in attributing a factor $I_{N\pm n}(Q_{\perp}^{N}\pm q_{\perp})$, $[J_{-n}(t)]$, to each vertex with diffusion to the other (same) side of the Fermi surface. Figure 4(a) represents, respectively, $\chi_{yy}^{0}(\mathbf{Q}_N+\mathbf{q})$ or $\chi_{yy}^{0}(-\mathbf{Q}_N+\mathbf{q})$ and $\chi_{yy}^{0}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q})$. Figure 4(b) represents $\chi_{yz}^{0}(\mathbf{Q}_N+\mathbf{q},\mathbf{q})$ and $\chi_{yz}^{0}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q})$, χ_{zz}^{0} is diagonal in wave vector and is drawn Fig. 4(c) for the argument $\mathbf{q}$. The $q_{\perp}$-dependent factors of all the terms involved in each HF correlation function factorize and the remaining factors are then one dimensional depending only on $q_{\parallel}$. We have used the same notation as in PL-I



The frequencies $\omega_n+\omega_p$ (ω_n), and the pseudomomentum $k+q_{\parallel}$ $(k+n)$, are associated with the upper (lower) lines.

Equation (A11) shows that each bubble which enters the expression of χ_{yz}^{0} and χ_{zz}^{0} (see Fig. 3) can be expanded in the same way with, respectively, the same factors J_nI_{N+n} and J_n^2 leading to formula (3.7′) of the text. Such identities are expressed diagrammatically in Fig. 4 where it is then possible for the reader to extract the general rules for constructing higher-order terms and other correlation functions. χ_{yy}^{0} is a combination of terms called χ_{+-}^{0} and Γ_{++}^{0} in Ref. 1 and calculated in the same reference. After factorizing the $q_{\perp}$-dependent terms, the $q_{\parallel}$ dependence is left in so-called irreducible one-dimensional bubbles $\tilde{\tilde{\chi}}_0^{yz}$ and $\tilde{\tilde{\chi}}_0^{zz}$ (and also $\tilde{\tilde{\chi}}_0$ and $\tilde{\tilde{\Gamma}}_0$ for χ_{yy}^{0}).

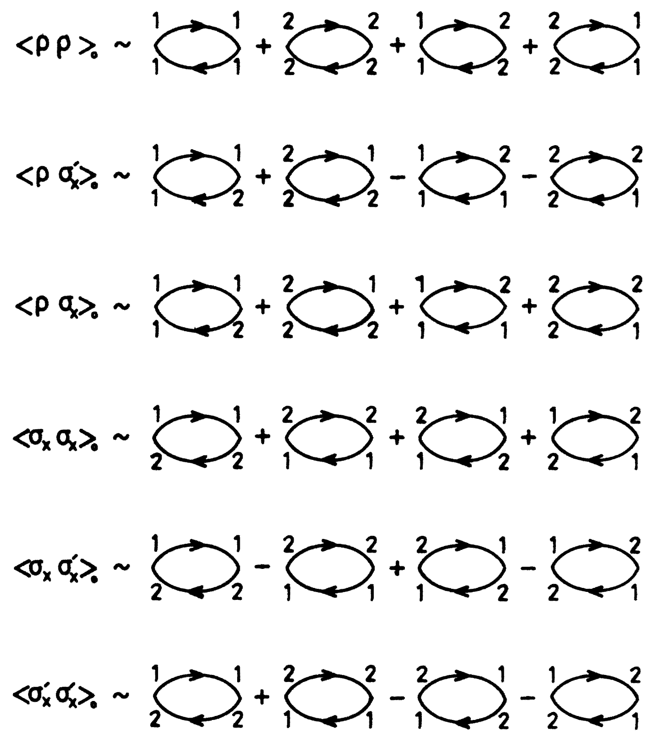


FIG. 5. Wick's theorem has been applied to get Hartree-Fock T_τ products using the following definitions of the density operators: $\rho=\sum_{i=1,2;\ \sigma=\uparrow,\downarrow}\psi_{i\sigma}^{\dagger}\psi_{i\sigma}$, $\sigma_x\equiv\delta\Delta_1=2^{-1/2}\sum_{i=1,2;\ \sigma=\uparrow,\downarrow}\psi_{i\sigma}^{\dagger}\psi_{-i-\sigma}$, and $\sigma_x'\equiv\delta\Delta_2=i/\sqrt{2}(\psi_{1\uparrow}^{\dagger}\psi_{2\downarrow}+\psi_{1\downarrow}^{\dagger}\psi_{2\uparrow}-\psi_{2\uparrow}^{\dagger}\psi_{1\downarrow}-\psi_{2\downarrow}^{\dagger}\psi_{1\uparrow})$. The straight lines labeled i and j stand for the fermion propagators $\langle T_\tau\psi_{i\sigma}\psi_{j\sigma'}\rangle$. We have chosen the polarization along x so that $\phi_1=\phi_2$ $(=0)$. It enables us to drop the spin indices since there are no more extra phase factors $e^{i\phi_\alpha}$ in the Green's functions depending on the spin. The input or output momentum at a "longitudinal" vertex 1,1 or 2,2 is $\mathbf{q}$ and $\pm\mathbf{Q}_N+\mathbf{q}$ at a "transverse" one 1,2 or 2,1.

Using Fig. 4 and Eq. (A10) it is straightforward to get

$$\tilde{\tilde{\chi}}_0^{yz}(q,\omega)=\frac{i}{\sqrt{2}}T\sum_{\omega_n}\int dk\frac{\delta_N i\omega_p}{\mathcal{D}_0\mathcal{D}_q'}\ ,$$

$$\tilde{\tilde{\chi}}^{zz}(q,\omega)=T\sum\int dk\frac{\omega_n(\omega_n+\omega_p)-k(k+q)+\delta_N^2}{\mathcal{D}_0\mathcal{D}_q'}\ . \tag{A12}$$

Analytical continuation $\omega_p\to\omega+i0^+$ is performed at the end of the calculation and $\mathcal{D}_q'$ means $(i\omega_n+i\omega_p-k-q)(i\omega_n+i\omega_p+k+q)-\delta_N^2$. Summation over Matsubara frequencies and integration over k leads to the formula (3.8′).

Note that for the special value $q_{\perp}=0$, χ_{yz}^{0}, and χ_{zz}^{0} lose their fine structure. Accordingly, using $J_n(q_{\perp}=0)=\delta^K(n)$, formula (3.7′) reduces to

$$\chi^0_{yz}(\pm\mathbf{Q}+\mathbf{q},\ \mathbf{q}=(q_\parallel,0),\ \omega)=I_N\tilde{\tilde{\chi}}{}_0^{yz}(q_\parallel,\omega)\ ,$$

$$\chi^0_{zz}(q_\parallel,\ q_\perp=0,\ \omega)=\tilde{\tilde{\chi}}{}_0^{zz}(q_\parallel,\omega)\ . \tag{A13}$$

APPENDIX B: HARTREE-FOCK BUBBLES OF CHARGE AND SPIN-DENSITY COUPLED FLUCTUATIONS

The HF bubbles which enter RPA Eq. (4.3) can be calculated with the method described in Appendix A. Their general expression is represented in Fig. 5. The first term of the expansion in powers of δ_{N+p}, $p\neq 0$, are shown diagrammatically in Fig. 6. The $q_\perp$-dependent term factorizes and the remaining terms are one-dimensional correlation functions depending on $q_\parallel$ the expressions of which are deduced from Fig. 6

$$\tilde{\tilde{\chi}}{}_0^{\rho\rho}(q,\omega)=4T\sum_{\omega_n}\int dk\frac{\omega_n(\omega_n+\omega_p)-k(k+q)-\delta_N^2}{\mathcal{D}'_q\mathcal{D}_0}$$

$$=4[\tilde{\tilde{\chi}}{}_0^{zz}(q,\omega)+2\tilde{\tilde{\Gamma}}_0(q,\omega)]\ , \tag{B1}$$

$$\tilde{\tilde{\chi}}{}_0^{\rho x}(q,\omega)=\sqrt{2}T\sum_{\omega_n}\int dk\frac{q}{\mathcal{D}_0\mathcal{D}'_q}=2\frac{q}{i\omega}\tilde{\tilde{\chi}}{}_0^{yz}(q,\omega)\ .$$

Simple analytical manipulations lead to formula (4.5).

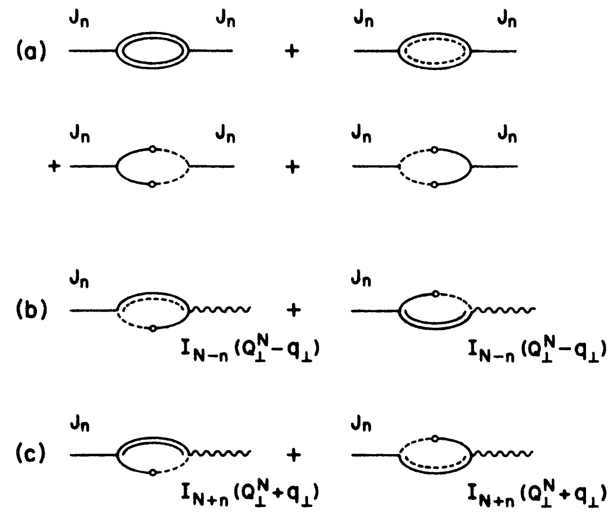


FIG. 6. The HF bubbles are approximated by use of the technique described in Appendix A in terms of one-dimensional propagators. The upper (lower) propagators are taken at a Matsubara frequency $\omega_n+\omega_p$ (ω_n) and at a wave vector $k+q_\parallel$ ($k+n$). (a) represents the diagonal term (in momentum) component $\chi^0_{\rho\rho}(\mathbf{q},\omega)$. As $\chi^0_{zz}(\mathbf{q},\omega)$, it does not depend on the polarization of the SDW in the XY plane. The sum over the integer n has to be performed (although not explicitly written). (b) and (c) correspond, respectively, to $\chi^0_{\rho x}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q},\omega)$ and $\chi^0_{\rho x}(\mathbf{q},\mathbf{Q}_N+\mathbf{q},\omega)$. On the other hand, the HF correlation functions $\langle T_\tau\sigma_x\sigma_x\rangle_0$, $\langle T_\tau\sigma_x\sigma'_x\rangle_0$, and $\langle T_\tau\sigma'_x\sigma'_x\rangle$ (not drawn here) involve exactly the same one-dimensional bubbles as in Fig. 4(a) (with different signs) and are not shown here.

APPENDIX C: RPA APPROXIMATION FOR CHARGE AND SPIN-DENSITY CORRELATION FUNCTIONS

Equation (4.1) can be explicitly written as (the frequency ω is omitted)

$$\begin{aligned}\chi_{11}(\mathbf{Q}_N+\mathbf{q})=\chi^0_{11}(\mathbf{Q}_N+\mathbf{q})&-\frac{\lambda}{2}\chi^0_{1\rho}(\mathbf{Q}_N+\mathbf{q},\mathbf{q})\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{11}(\mathbf{Q}_N+\mathbf{q})\chi_{11}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{11}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{12}(\mathbf{Q}_N+\mathbf{q})\chi_{21}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{12}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\ ,\end{aligned}$$

$$\begin{aligned}\chi_{21}(\mathbf{Q}_N+\mathbf{q})=\chi^0_{21}(\mathbf{Q}_N+\mathbf{q})&-\frac{\lambda}{2}\chi^0_{2\rho}(\mathbf{Q}_N+\mathbf{q},\mathbf{q})\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{21}(\mathbf{Q}_N+\mathbf{q})\chi_{11}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{21}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{22}(\mathbf{Q}_N+\mathbf{q})\chi_{21}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{22}(\mathbf{Q}_N+\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\ ,\end{aligned}$$

$$\begin{aligned}\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})=\chi^0_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})&-\frac{\lambda}{2}\chi^0_{\rho\rho}(\mathbf{q})\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{11}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{\rho 1}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{\rho 2}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{21}(\mathbf{Q}_N+\mathbf{q})\\&+\frac{\lambda}{2}\chi^0_{\rho 2}(\mathbf{q},-\mathbf{Q}_N+\mathbf{q})\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\ ,\end{aligned} \tag{C1}$$

$$\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})=\chi^0_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})-\frac{\lambda}{2}\chi^0_{1\rho}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q})\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})$$
$$+\frac{\lambda}{2}\chi^0_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{11}(\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{11}(-\mathbf{Q}_N+\mathbf{q})\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})$$
$$+\frac{\lambda}{2}\chi^0_{12}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{21}(\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{12}(-\mathbf{Q}_N+\mathbf{q})\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q}) ,$$

$$\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})=\chi^0_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})-\frac{\lambda}{2}\chi^0_{2\rho}(-\mathbf{Q}_N+\mathbf{q},\mathbf{q})\chi_{\rho 1}(\mathbf{q},\mathbf{Q}_N+\mathbf{q})$$
$$+\frac{\lambda}{2}\chi^0_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{11}(\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{21}(-\mathbf{Q}_N+\mathbf{q})\chi_{11}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})$$
$$+\frac{\lambda}{2}\chi^0_{22}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q})\chi_{21}(\mathbf{Q}_N+\mathbf{q})+\frac{\lambda}{2}\chi^0_{22}(-\mathbf{Q}_N+\mathbf{q})\chi_{21}(-\mathbf{Q}_N+\mathbf{q},\mathbf{Q}_N+\mathbf{q}) .$$

This leads to a 5×5 determinant which is expanded in (4.3). Furthermore, by solving this linear system it is even possible to obtain a general expression of the RPA spin-spin and charge-charge correlation functions.

---

[*]Present address: Institut für Theoretische Physik, Eidgenössische Technische Hochschule Zürich–Hönggerberg, CH-8093 Zürich, Switzerland.

[†]Present address: Institute for Solid State Physics, University of Tokyo, 7-22-1 Roppongi, Minato-ku, Tokyo 106, Japan.

[1]D. Poilblanc and P. Lederer, preceding paper, Phys. Rev. B **37**, 9650 (1988).

[2]K. Maki and A. Virosztek (unpublished).

[3]A. Virosztek and K. Maki, Phys. Rev. B **35**, 1954 (1987).

[4]D. Poilblanc, M. Héritier, G. Montambaux, and P. Lederer, J. Phys. C **19**, L321 (1986); G. Montambaux and D. Poilblanc, Phys. Rev. B (to be published); K. Maki, *ibid.* **33**, 4826 (1986); A. Virosztek, L. Chen, and K. Maki, *ibid.* **34**, 3371 (1986).

[5]P. Lederer and D. Poilblanc, C. R. Acad. Sci. (Paris) **304**, 251 (1987); P. Lederer, D. Poilblanc, and G. Montambaux, Jpn. J. Appl. Phys. **26.3**, 573 (1987).

[6]P. A. Lee, T. M. Rice, and P. W. Anderson, Solid State Commun. **14**, 703 (1974).

[7]P. Lederer, D. Poilblanc, and G. Montambaux, Europhys. Lett. (to be published).