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Macroscopic dynamics of uniaxial spin glasses

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An earlier study of the long-wavelength excitations in XY spin glasses has been extended to include the LT phase (possessing both *longitudinal* and *transverse* spin-components), where the uniaxial anisotropy is not strong enough either to force the spins completely into or normal to the (x,y) plane. For an ideal case there is a Goldstone mode with linear dispersion, corresponding to rotations about the z axis. For real systems, with in-plane remanence $\mathbf{m}_0$, an external field $\mathbf{H}$ along $\mathbf{m}_0$, and in-plane anisotropy K, one finds a macroscopic mode with frequency $\omega_0=\gamma[(K+m_0H)/\chi_{zz}]^{1/2}$ in the long-wavelength limit, as for the planar spin glass.

I. INTRODUCTION

In this work we consider the theory of long-wavelength normal modes of spin glasses with uniaxial anisotropy, using the methods of macroscopic dynamics. One can realize this possibility experimentally by employing hexagonal metallic host crystals to provide the anisotropy, and various impurities to provide the magnetically active systems.[1,2] Typically, the uniaxial anisotropy D is nearly independent of the impurity concentration c, but the characteristic exchange J is roughly linear in c.

For the Sherrington-Kirkpatrick (SK) model, which has long-range interactions, such systems are predicted to possess a richer phase diagram than in the case of vanishing anisotropy D.[3–5] In the limit of large Ising-like anisotropy, so that the spins favor an axis, a low-temperature Ising-like (*longitudinal*) spin-glass phase (L) makes a transition to a high-temperature paramagnetic phase (P). In the limit of large XY-like anisotropy, so that the spins favor a plane, a low-temperature planar (*transverse*) spin-glass phase (T) makes a transition to a high-temperature paramagnetic phase (P). (Note that the correlations in these two P phases are different, due to the different anisotropies.) More interesting than these limits are the predictions, for weaker uniaxial anisotropy, that: (1) at low temperatures the system will be in a phase (LT) possessing both longitudinal (Ising-like) and transverse (XY-like) spin components (with one type dominating, according to the sign of the anisotropy D); and (2) that there will be an intermediate-temperature phase of either pure Ising-like (L) or pure XY-like (T) symmetry (according to the anisotropy D) before the high-temperature paramagnetic phase is reached.[3–5]

The dynamics of the isotropic SK spin-glass without anisotropy has been treated by Bray and Moore, using a microscopic approach.[6] (A macroscopic approach, appropriate for any isotropic spin glass in the long-wavelength limit, had been worked out earlier.[7,8]) In addition, zero-temperature dynamics of the SK model, including uniaxial anisotropy, has been studied in Ref. 9, where no energy gap was found in any of the phases L, T, or LT. This is contradicted by a more recent work, which finds that the spin waves in the L phase have a gap which goes to zero as the system goes toward the LT phase (i.e., on weakening the Ising-like anisotropy).[10]

It is the purpose of this paper to discuss the long-wavelength spin waves of the L phase and the LT phase (the spin waves for the T phase having already been discussed thoroughly). In Sec. II we briefly argue that in the L phase excitation spectrum there is a gap which should go to zero as one enters the LT phase, as found in Ref. 10. In Sec. III, we discuss the nature of the gapless excitations of the LT phase. In Sec. IV we include remanence, an external field, and random in-plane anisotropy, in order to discuss the expected ESR frequency in real materials in the LT phase. This constitutes an extension of the work of Refs. 11 and 12 from the T phase (i.e., the XY spin glass) to the LT phase. The ESR frequency, Eq. (4.7), has the same unusual field dependence as for the planar (T) spin-glass phase.

II. GENERAL ARGUMENTS

We consider the Hamiltonian

$$H=H_{\rm ex}+H_D\ , \tag{2.1}$$

where

$$H_{\rm ex}=-\tfrac{1}{2}\sum_{ij}J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j\ , \tag{2.2}$$

$$H_D=-\tfrac{1}{2}D\sum_i S_{iz}^2\ . \tag{2.3}$$

We assume that for D sufficiently large and negative, the spins are forced into the XY plane, where they are arranged by the random-exchange term $H_{\rm ex}$, yielding an XY spin glass (the T phase). Above a critical negative value of D ($0>D\geq D_T$), the spins begin to point in the z direction (the LT phase), their tips forming an oblate shape when their tails are all placed at a common origin. For $D=0$ the system is an isotropic spin glass. As D becomes positive, the spin tips form a prolate shape, and for large enough positive D ($D\geq D_L>0$) the spins point only along $\pm\hat{z}$ (the L phase).

Now consider a uniform spin rotation about the z axis, and its effect on these states and their energies. For the T and LT phases, which have spin components in the (x,y) plane, the states change (whereas for the L phase the collinear state is unaffected), but there is no change in energy. Thus, for the T and LT phases this corresponds to a broken symmetry, and, hence, one expects a Goldstone mode with vanishing frequency in the long-wavelength limit.[7,11,12] Such a mode does not exist for the L phase. However, in the L phase, as $D \to D_L$ it is not unreasonable to expect a soft mode involving a tipping of spins into the (x,y) plane (this does *not* correspond to a rotation about $\hat{z}$), thus signaling this continuous transition. Such a mode is predicted in Ref. 9: the macroscopic analysis which follows cannot confirm this, but it does indicate that the expected soft mode and the Goldstone mode of the LT phase have a *different* symmetry.

Earlier we performed a detailed analysis of the dynamics of the T phase.[11,12] Using the same methods, in the next section we analyze the LT phase, first considering the case where there is no remanence $\mathbf{m}_0$, no external field $\mathbf{H}$, and no random anisotropy.

III. MACROSCOPIC DYNAMICS

We expect the macroscopic dynamics to be described by the energy density[12]

$$\varepsilon = \frac{m_z^2}{2\chi_{zz}} + \frac{m_\perp^2}{2\chi_\perp} - \tfrac{1}{2}K_\perp(\hat{\mathbf{q}}\cdot\hat{\mathbf{z}})^2 + \tfrac{1}{2}\rho_s(\nabla\theta_z)^2 . \tag{3.1}$$

Here $\mathbf{m}$ is the magnetization, χ_{zz} is the susceptibility in the $\mathbf{z}$ direction, and includes the effects of both exchange and anisotropy in (2.1), $\chi_\perp$ is the in-plane susceptibility (due to exchange), $K_\perp$ is a measure of the anisotropy energy of the system if the normal to the spin plane $\hat{\mathbf{q}}$ is not aligned with the normal to the lattice plane $\hat{\mathbf{z}}$, ρ_s is the exchange stiffness, and θ_z is a measure of the rotation about the z axis. [No similar term in θ_x or θ_y need enter because of the much larger $K_\perp(\hat{\mathbf{q}}\cdot\hat{\mathbf{z}})^2$ term. Thus our results do not go continuously to the isotropic limit, where $K_\perp = D = 0$.]

Because of the $K_\perp(\hat{\mathbf{q}}\cdot\hat{\mathbf{z}})^2$ term, even long-wavelength rotations θ_x and θ_y of the system will be suppressed, and therefore only m_z and θ_z are the proper macroscopic variables.[11,12] Taking the usual equations of motion[7]

$$\dot{m}_z = -\gamma\frac{\delta\varepsilon}{\delta\theta_z} , \tag{3.2}$$

$$\dot{\theta}_z = \gamma\frac{\delta\varepsilon}{\delta m_z} , \tag{3.3}$$

we find that

$$\dot{m}_z = \gamma\rho_s\nabla^2\theta_z , \tag{3.4}$$

$$\dot{\theta}_z = \gamma\frac{m_z}{\chi_{zz}} \tag{3.5}$$

whose normal modes m_z, $\theta_z \sim e^{(i\mathbf{k}\cdot\mathbf{r}-\omega t)}$, have the dispersion relation

$$\omega^2 = c^2k^2, \quad c^2 \equiv \gamma^2\frac{\rho_s}{\chi_{zz}} . \tag{3.6}$$

This is precisely as in the case for the T phase, except that now χ_{zz} is less dominated by the planar anisotropy.[12] Indeed, as $D \to 0$, one has $\chi_{zz} \to \chi$, the isotropic susceptibility. [Moreover, for $D = 0$ the angles θ_x and θ_y become macroscopic variables, and one obtains two more modes satisfying (3.6).] Since $\omega \to 0$ as $k \to 0$, this is a Goldstone mode, and the terminology *hydrodynamics* is appropriate.[7]

IV. EFFECT OF REMANENCE, FIELD, AND RANDOM ANISOTROPY

The energy density in this case is expected to take the form[12]

$$\varepsilon = \frac{m_z^2}{2\chi_{zz}} + \frac{\mathbf{m}_\perp^2}{2\chi_\perp} - \tfrac{1}{4}K[(\hat{\mathbf{n}}\cdot\hat{\mathbf{N}})^2 + (\hat{\mathbf{p}}\cdot\hat{\mathbf{P}})^2] - \tfrac{1}{2}K_\perp(\hat{\mathbf{q}}\cdot\hat{\mathbf{z}})^2 - \mathbf{m}\cdot\left[\mathbf{H} + \frac{m_{0\perp}}{\chi_\perp}\hat{\mathbf{n}} + \frac{m_{0z}}{\chi_{zz}}\hat{\mathbf{q}}\right] + \frac{\rho_s}{2}(\nabla\theta_z)^2 , \tag{4.1}$$

which is the same as for the T phase except that we permit a remanence $\mathbf{m}_0$ out of the plane and we include the $(\nabla\theta_z)^2$ term. Here K includes the memory effects of random anisotropy in the plane,[11] $(\hat{\mathbf{n}},\hat{\mathbf{p}},\hat{\mathbf{q}})$ represents the orientation of the spin triad, where $d\hat{\mathbf{n}} = d\boldsymbol{\theta}\times\hat{\mathbf{n}}$, etc., and $(\hat{\mathbf{N}}, \hat{\mathbf{P}}, \hat{\mathbf{Q}} = \hat{\mathbf{z}})$ represents the orientation of the anisotropy triad.[13,14] (For $D = 0 = K_\perp$, $\hat{\mathbf{Q}}$ need not be fixed along $\hat{\mathbf{z}}$.) It is important to recognize that K is expected to be second order in the microscopic random anisotropy, and is unrelated to the (nonrandom) uniaxial anisotropy $K_\perp$, which is first order in the microscopic anisotropy D. (Section V discusses K and gives further references.)

The equilibrium condition on $\mathbf{m}$ is

$$0 = \frac{\delta\varepsilon}{\delta\mathbf{m}} = \overleftrightarrow{\chi}^{-1}\cdot\mathbf{m} - (\mathbf{H} + \overleftrightarrow{\chi}^{-1}\cdot\mathbf{m}_0) , \tag{4.2}$$

or

$$\mathbf{m} = \mathbf{m}_0 + \overleftrightarrow{\chi}\cdot\mathbf{H}, \overleftrightarrow{\chi} \equiv \chi_{zz}\hat{\mathbf{q}}\hat{\mathbf{q}} + \chi_\perp(\hat{\mathbf{p}}\hat{\mathbf{p}} + \hat{\mathbf{n}}\hat{\mathbf{n}}) . \tag{4.3}$$

The equilibrium condition on $\boldsymbol{\theta}$, $\delta\varepsilon/\delta\boldsymbol{\theta} = 0$, gives $\hat{\mathbf{n}} = \hat{\mathbf{N}}$, $\hat{\mathbf{p}} = \hat{\mathbf{P}}$, and $\hat{\mathbf{q}} = \hat{\mathbf{z}}$.

For simplicity, we consider $\mathbf{m}_0$ and $\mathbf{H}$ to lie in the (x,y) plane, so that $m_{0\perp} = m_0$ and $M_{0z} = 0$. The equations of motion then yield

$$\dot{m}_z = -\gamma(K\theta_z + m_0H\theta_z - \rho_s\nabla^2\theta_z) , \tag{4.4}$$

$$\dot{\theta}_z = \gamma\left[\frac{m_z}{\chi_{zz}}\right] . \tag{4.5}$$

The normal modes of (4.4) and (4.5) have the dispersion relation

$$\omega^2 = \omega_0^2 + c^2k^2 , \tag{4.6}$$

where $c^2 = \gamma^2\rho_s/\chi_{zz}$ (as in Sec. III) and

$$\omega_0^2 \equiv \gamma^2\frac{K + m_0H}{\chi_{zz}} . \tag{4.7}$$

The same type of mode was found for XY spin glasses.[12] The in-plane anisotropy K and the in-plane field both act to provide a restoring torque on the system. Because there is a gap in ω for $k \to 0$, this mode is not a Goldstone mode, and the dynamics should be called macroscopic, rather than hydrodynamic.

V. DISCUSSION

Recent work by Alloul and co-workers indicates that thermal excitations in isotropic spin glasses, which are predicted to have the form (4.6) with $m_0 \to 0$ and $\chi_{zz} \to \chi$, do not have a noticeable effective anisotropy K.[15–17] This apparently contradicts the fact that K is quite noticeable in ESR and torque experiments, for which the wave vectors $k \to 0$. The resolution may be that thermal excitations, having k finite, are sufficiently localized that the effective anisotropy is relatively less important than when $k \to 0$. Theoretical calculations have so far only focused on the effects of anisotropy for the $k \to 0$ modes.[18,19] However, uniform rotations ($k = 0$) certainly involve the anisotropy everywhere, whereas localized modes might "see" the anisotropy in a different, and less extreme fashion. Therefore, we expect that $\omega^2 \approx \omega_0^2$ will reliably describe the ESR modes, but that the anisotropy "gap" ω_0 may not be characteristic of the shorter wavelength modes which, for the reasons given above, may have a lower frequency than ω_0.

It should be noted that the analysis presented in Sec. IV only includes the case where $\mathbf{m}_0$ and $\mathbf{H}$ are collinear. Reference 12 treated the T phase in the noncollinear case, finding a complex behavior that would also be expected for the LT phase in the noncollinear case. It would be of interest to verify (4.7) experimentally, due to its unusual dependence on the field H.

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