Acoustic plasmons in a conducting double layer

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We have investigated the acoustic plasma branch present in the longitudinal spectrum of two spatially separated parallel quasi-two-dimensional conducting layers. Our approach is based on the dielectric theory and is completely analytical within the random-phase approximation. By means of a systematic analysis we have obtained several exact results concerning the plasma dispersion relation. In particular, we have derived an exact expression for c_p the acoustic plasmon group velocity in terms of the effective masses, densities and geometrical parameters of the heterostructure. We find that when the two layers are identical the system always admits a branch of acoustic plasmons as undamped modes for any finite value of the distance between the layers.

INTRODUCTION

Momentous advances in growth techniques presently allow the fabrication of materials consisting of alternating layers of two or more semiconductors. At the various interfaces electronic or hole layers can be trapped whose low-energy dynamics is, for all practical purposes, quasi-two-dimensional.[1] The current growth techniques can actually be exploited to tailor the properties of a heterostructure to specific dynamical requirements. In particular, it is in principle possible to design heterostructures with a customized excitation spectrum.

Semiconducting electronic heterostructures have recently provided a valuable testing ground for the study of plasma excitations in various geometrical configurations.[2–4] Semiconducting superlattices are a typical example of such a class of materials. In particular, they are the only known electronic systems in which plasmons have been directly observed.[3] Recently the possibility of acoustic surface plasma modes in certain semiconducting superlattices has also been investigated.[5,6]

In this paper we focus our attention on a peculiar electronic system comprised of two spatially separated parallel quasi-two-dimensional conducting layers (Fig. 1). This problem is of current experimental interest because the system at hand is a model for double-quantum-well heterojunctions and single inversion layers with more than one populated subband.[7–9]

The collective plasma excitation spectrum in double-quantum-well electronic structures has been first investigated by numerical means.[7] A more transparent analytic treatment based on the dielectric approach has been later presented which explicitly showed the existence of two plasmon branches of which one is characterized in the electrostatic limit by an acoustic dispersion relation at long wavelength.[8] The properties of such an acoustic branch have also been studied in the context of a single electronic inversion layer with two populated subbands.[9] Recently the possibility of the plasmon mechanism for superconductivity in a double well in which the effective interaction is mediated by both plasmon branches has also been studied.[10]

The present analysis focuses on the condition for the existence as undamped modes of the acoustic plasma branch in terms of the relevant physical parameters of the heterostructure. Although our theory is based on the same formalism used in previous studies, our main results significantly differ from the ones previously reported. In what follows we will develop a systematic and analytic approach which allows us to precisely characterize and examine the plasma dispersion relation in terms of the geometry, the Fermi velocities, and the electronic effective masses. In particular, for the case of identical layers we will explicitly derive in closed form a simple and exact expression for the plasmon group velocity. Finally, we will prove that in the same situation the system always admits a branch of acoustic plasmons as undamped modes for any finite value of the layer separation.

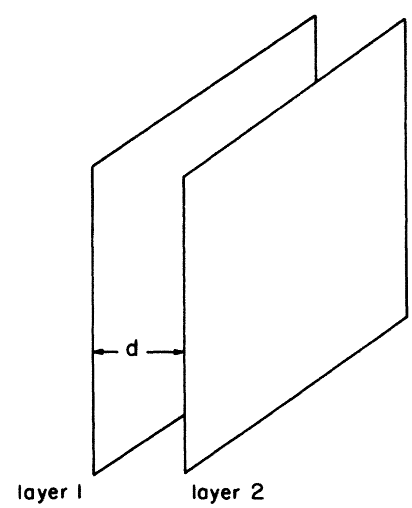

FIG. 1. Schematic of the electronic double layer system studied in the text. The layers are parallel and separated by a distance d.

DISPERSION RELATION

At long wavelength the collective plasma oscillations of a single (i.e., isolated) two-dimensional electronic layer have a dispersion relation given by[11]

$$\omega_0(q) \approx (2\pi n e^2 q/\epsilon_0 m^*)^{1/2} , \tag{1}$$

where q is the magnitude of the (two-dimensional) in-plane wave vector, n and m^* the electronic density and effective mass, and ϵ_0 is the average dielectric constant of the medium. Within the random-phase approximation (RPA), two-dimensional plasma excitations are therefore undamped for $q \approx 0$ since $\omega(q)$ lies outside the continuum of the electron-hole pair excitations. In what follows we shall refer to this type of mode as an *optical plasmon*.

Consider next two parallel conducting quasi-two-dimensional layers separated by a finite distance d (see Fig. 1). Because of the lack of translational invariance the collective modes for the system at hand are given by the zeros of the determinant of the dielectric tensor $\epsilon_{ij}(q,\omega)$.[8] The RPA expression for $\epsilon_{ij}(q,\omega)$ in this system is given by

$$\epsilon_{ij}(q,\omega) = \delta_{ij} - V_{ij}(q)\chi^0_{jj}(q,\omega), \quad i,j = 1,2 , \tag{2}$$

where $\chi^0_{jj}(q,\omega)$ is the noninteracting charge susceptibility of the jth layer and $V_{ij}(q)$ stands for the Coulomb interaction vertex between two electrons, respectively, in the ith and jth layer. In obtaining Eq. (2) we have assumed that there is no overlap between electronic wave functions on different layers. If for simplicity sake we limit our analysis strictly to literally two-dimensional electronic layers the matrix elements V_{ij}'s are simply given by $V_{11}(q) = V_{22}(q) = 2\pi e^2/\epsilon_0 q$, and $V_{12}(q) = V_{21}(q) = e^{-qd} 2\pi e^2/\epsilon_0 q$. Making use of Eq. (2) the dispersion relation is readily obtained as given by

$$D(q,\omega) = 1 - V_{11}(q)[\chi^0_{11}(q,\omega) + \chi^0_{22}(q,\omega)] + V^2_{11}(q)(1 - e^{-2qd})\chi^0_{11}(q,\omega)\chi^0_{22}(q,\omega) = 0 , \tag{3}$$

where the layer susceptibilities are accordingly evaluated for literally two-dimensional electrons.[11] Clearly this result is independent of the sign of the charge of the carriers on each layer. Equation (3) must then be then explicitly solved for ω as a function of q.

As shown in Ref. 8 the longitudinal spectrum of the total system is comprised of an optical plasmon branch [of the type of Eq. (1)], plus a new branch whose dispersion relation at long wavelength is of the type $\omega(q) \approx c_p q$, i.e., an *acoustic plasmon.* A schematic of this spectrum is shown in Fig. 2. Although it is rather simple to arrive at a rough characterization of the spectrum some care must be taken in evaluating the acoustic plasmon group velocity c_p. The physical origin of the difficulty lies in the fact that in calculating the energy as-

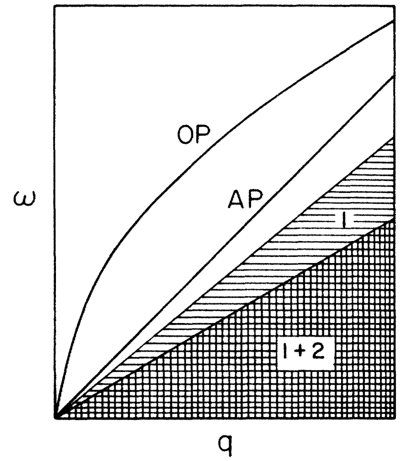

FIG. 2. Schematic of the long-wavelength region of the longitudinal spectrum ω vs q for the electronic double layer system studied in the text. The two shaded regions labeled 1 and 1 + 2 are the electron-hole pair continua for the two layers. In region 1 only pairs in layer 1 can be excited, in region 1 + 2 pairs in both layers can be excited. The two continuous thick lines represent, respectively, the optical (OP) and the acoustic branch (AP) of the plasmon spectrum. In this case the acoustic plasmon group velocity is taken to be larger than v_{F1}, the largest of the two Fermi velocities.

sociated with the plasma oscillations a cancellation due to the screening of the long-range part of the Coulomb interaction occurs in the case of the acoustic branch.

As well known within the RPA a bulk acoustic plasma branch is well defined only if it lies outside the electron-hole pair continua of the two charge components (see dashed regions in Fig. 2), i.e., the region of the q,ω plane in which the imaginary part of the susceptibilities $\chi^0_{ii}(q,\omega)$ is different from zero. If $\omega(q)$ lies inside one of the continua the plasma mode can decay into electron-hole pairs and is Landau damped. Accordingly, in RPA the condition for the existence of the acoustic branch as an undamped mode in the long-wavelength limit is then simply $c_p \geq v_{F1}$, where v_{F1} is by definition the largest of v_{F1} and v_{F2}, the Fermi velocities in the two layers. Our aim is to determine the precise condition for the validity of the above inequality.[12]

In order to obtain an exact expression for the plasmon group velocity c_p in the region of Fig. 2 in which $q \approx 0$ and $\mathrm{Im}\{\chi^0_{ii}(q,\omega)\} = 0$, we proceed as follows. We first introduce for $q \approx 0$ the power expansion

$$\omega(q) = c_p q + c_2 q^2 + c_3 q^3 + \cdots , \tag{4}$$

for the plasmon dispersion relation, and define a function $F(q)$ as

$$F(q) = D(q, c_p q + C_2 q^2 + C_3 q^3 + \cdots) , \tag{5}$$

where D is defined in Eq. (3). For $q \approx 0$, $F(q)$ can in turn be written in terms of the power expansion (in fact a Laurent-Taylor expansion)

$$F(q)=f_{-1}q^{-1}+f_0+f_1q+f_2q^2+\cdots , \tag{6}$$

where the f_i's are suitable coefficients which are derived with the use of Stern's formulas for the $\chi^0_{jj}(q,\omega)$'s (Ref. 11) in Eqs. (3) and (5). The mode condition Eq. (3) is then satisfied by requiring that all the coefficients f_i's vanish independently. As can be readily found f_{-1} depends on c_p only and by equating its expression to zero we arrive after some algebra at the following equation:

$$2k_2d-(1+2k_2d)[1-(v_{F2}/c_p)^2]^{1/2}-(m_2^*/m_1^*)(1+2k_1d)[1-(v_{F1}/c_p)^2]^{1/2} +[1+(m_2^*/m_1^*)+2k_2d][1-(v_{F1}/c_p)^2]^{1/2}[1-(v_{F2}/c_p)^2]^{1/2}\equiv 0 , \tag{7}$$

where $k_i=2m_i^*e^2/\epsilon_0\hbar^2$ is the Thomas-Fermi wave vector of the ith layer. Equation (7) is the sought condition which determines in the general case the plasmon group velocity c_p in terms of the effective masses, densities, and geometrical parameters of the heterostructure.

It should be noted that the present procedure is quite general in character and allows to obtain the dispersion relation at all orders in q. In fact, as it turns out the coefficients of the higher-order terms in the expansion for $\omega(q)$ can in principle be systematically evaluated simply by solving an equation in which only the coefficients of the lower-order terms appear. For instance, the value of c_2 can be readily obtained in terms of c_p as determined in Eq. (7) by requiring that f_0 in Eq. (6) be zero. This leads to the following equation:

$$c_2=\frac{(w_1w_2)^2[w_1w_2-2k_1k_2d^2(c_p-w_1)(c_p-w_2)]}{k_1v_{F1}^2w_2^2[2k_2d(c_p-w_2)-w_2]+k_2v_{F2}^2w_1^2[2k_1d(c_p-w_1)-w_1]} , \tag{8}$$

where we have defined $w_i=(c_p^2-v_{Fi}^2)^{1/2}$.

ACOUSTIC PLASMON: EXISTENCE CONDITION

By definition, the critical value d_c of d is the value of the layers separation for which $c_p=v_{F1}$. By using this value of c_p in Eq. (7) we obtain

$$d_c=\frac{1}{2k_2}\frac{(v_{F1}^2-v_{F2}^2)^{1/2}}{v_{F1}-(v_{F1}^2-v_{F2}^2)^{1/2}} . \tag{9}$$

For larger values of the distance, the branch lies outside the electron-hole continua. We immediately notice that our exact, and indeed very simple, expression for d_c clearly predicts a critical distance equal to zero in the particular case in which the two Fermi velocities are identical. Accordingly, in such a situation an undamped acoustic plasmon branch *always* exists for all finite of the interlayer distance d. This is at variance with the results of Refs. 8 and 9.[13] Furthermore, when $v_{F1}=v_{F2}=v_F$ and $m_1^*=m_2^*$, Eq. (7) has an exact simple closed form solution, given by

$$c_p=\frac{v_F(1+kd)}{(1+2kd)^{1/2}} , \tag{10}$$

where $k=k_1=k_2$. As illustrated in Fig. 3, c_p increases from its critical value v_{F1} at first quadratically and then (asymptotically) like a square root with kd.

We also want to stress the fact that for any finite value of d greater then d_c there *always* exists an acoustic branch. The plasma modes in each layer are decoupled strictly only for $d=\infty$.[14]

With a procedure similar to that used to obtain Eq. (7) we have also studied the solution for the acoustic mode in the region which is outside the electron-hole continuum of the layer with smaller Fermi velocity, but inside the continuum of the electron-hole excitations of the layer with greater Fermi velocity (the shaded region labeled 1 in Fig. 2). In this region $\mathrm{Im}\{\chi^0_{11}(q,\omega)\}$ is different from zero. The solution for $\omega(q)$ is then necessarily complex, and the acoustic branch is damped. One can verify that when c_p approaches (from below) the limiting

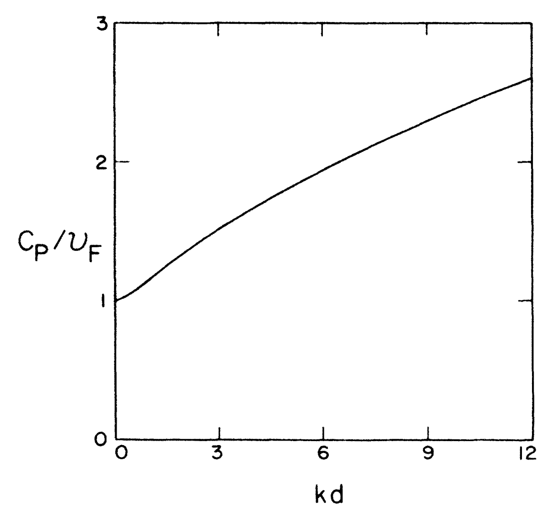

FIG. 3. Plot of the ratio of c_p, the plasmon group velocity, to v_F the Fermi velocity vs kd, the product of the Thomas-Fermi wave vector and the layers distance, for the case in which the two layers are identical. Notice that for small kd, the value of c_p increases quadratically from its critical value v_f. For large kd, c_p increases like $(kd)^{1/2}$

value v_{F1} the imaginary part of $\omega(q)$ is much smaller than the real part and is exactly zero for $c_p = v_{F1}$. The latter situation occurs when $d = d_c$. The two solutions for $c_p(d)$ obtained in this way perfectly match. This is in contrast with the conclusions reached in Ref. 8. There the RPA is held responsible for a series of odd results that are instead a consequence of an incorrect analysis.

It must be mentioned here that Landau damping of the acoustic branch is still possible via intersubband excitations. This will, of course, have an energy threshold close in value to the subband separation and will therefore be negligible in the long-wavelength limit.[15]

In conclusion, we have characterized the acoustic part of the longitudinal spectrum of an electronic double layer system within the RPA. By means of a systematic analytical approach we have obtained several exact results concerning the plasma dispersion relation. In particular, we have derived an exact expression for c_p, the acoustic plasmon group velocity, and d_c, the critical distance for their existence, in terms of the effective masses, densities, and geometrical parameters of the structure. Finally, an interesting and exact result is that when the two layers are identical, for any finite value of their distance, the system always admits a branch of acoustic plasmons as undamped modes.

A similar analysis can be carried out also for the optical plasmon. In such a case, however, the energy is mainly determined by the long-range part of the Coulomb interaction which makes the problem rather trivial.

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[12] Multipair excitations beyond the RPA will in general always lead to a small damping of the plasmon but will be neglected here for simplicity.

[13] For the case of the double quantum well GaAs/$Al_xGa_{1-x}As$ with equal Fermi velocities, the authors of Ref. 8 arrived at the (incorrect) result $d_c \approx 150$ Å. It is straightforward to see that the assorted results for d_c obtained by the authors of Refs. 8 and 9 can be simply traced to the use of invalid approximations.

[14] Here, care must be taken in correctly handling the limiting procedures involved. The solution with two decoupled optical plasmons given in Ref. 8 never exists for finite albeit large d.

[15] This phenomenon is similar to the absence of Landau damping for surface plasmons in semiconducting superlattices first discussed in G. F. Giuliani and J. J. Quinn, Phys. Rev. Lett. **51**, 919 (1981).