Dynamics of fluxons in a system of coupled Josephson junctions

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(Received 7 July 1987)

Different dynamical processes with fluxons are analyzed in the framework of a known model describing two weakly coupled long Josephson junctions. It is demonstrated that two colliding fluxons belonging to different junctions can form a bound state due to energy dissipative losses. Collision between a fast fluxon and a large-amplitude (low-frequency) breather (bound fluxon-antifluxon state) belonging to different junctions is considered too, and a threshold condition for the possibility of breaking the breather into the fluxon-antifluxon pair is found. Some radiative effects are investigated. The equilibrium velocity of the fluxon's motion in the case when this velocity exceeds the limit velocity of the mate junction is found with regard to the "Cherenkov" energy losses. Then radiative effects accompanying collision of two fluxons are considered for the cases when the fluxons belong either to the same or to different junctions. For both cases the total emitted energy and its spectral distribution are obtained.

I. INTRODUCTION

Solitons play an important role in different branches of modern solid-state physics (see, e.g., Refs. 1–3). The concept of a soliton is especially important in dynamics of long Josephson junctions (LJJ's), where a fluxon (magnetic flux quantum) is a fundamental solitonic excitation. The standard equation to describe LJJ is[2,3]

$$\phi_{xx}-C^{-2}\phi_{tt}-\eta\phi_t=\lambda^{-2}\sin\phi+J_0/J_c\lambda^2\ , \tag{1}$$

where ϕ is the nondimensional magnetic flux, J_0 is the bias current, and C, η, and J_c are the Swihart velocity, the viscosity (coefficient of dissipative losses), and the critical current density of the junction. The junction's length is assumed to be much greater than the Josephson penetration depth λ (see, e.g., Ref. 3). The parameters $f\equiv J_0/J_c$ and $\gamma\equiv\lambda C\eta$ are often considered to be small, which enables us to apply the technique of perturbation theory for solitons. A number of problems concerning dynamics of fluxons and breathers, the latter being bound fluxon-antifluxon states, have been treated by means of this technique (see, e.g., Refs. 2–10).

A more specialized object which may demonstrate nontrivial solitonic dynamics is a weakly coupled system of two LJJ's. A model describing such a system has been elaborated in Ref. 11. According to that paper, the coupled junctions are described by the system of two equations for $\phi_1\equiv\phi$ and $\phi_2\equiv\psi$:

$$\begin{aligned}&\phi_{xx}-\phi_{tt}-\gamma_1\phi_t=\sin\phi+f_1+\alpha\psi_{xx}\ ,\\&\psi_{xx}-a^2\psi_{tt}-\gamma_2ab\psi_t=b^2\sin\psi+f_2b^2+\alpha\phi_{xx}\ ,\end{aligned} \tag{2}$$

where x and t are measured in units of, respectively, λ_1 and λ_1/C_1, $a\equiv C_1/C_2$, $b\equiv\lambda_1/\lambda_2$, α is a small coupling constant, and the sense of other notation is obvious. Note that a system of two sine-Gordon equations with a coupling of a more general form arises when one considers interaction of two linear arrays of atoms adsorbed on a metal surface.[12]

In the quoted paper[11] only one dynamical problem was considered: emission of radiation by a fluxon moving in the first junction due to its interaction with a small-amplitude periodic wave in the second one. In the present paper we shall consider several other problems, which also may be experimentally realizable.

In Sec. II we consider, in the adiabatic approximation (i.e., disregarding radiative losses), interaction of two fluxons belonging to different junctions. We demonstrate that, due to the action of dissipation, the two fluxons may fuse into a bound state which we shall call a bifluxon. Assuming $|\alpha|\gg\gamma_1^2,\gamma_2^2$, we find a threshold condition which makes the fusion possible. Then we briefly consider one more problem which is at least of methodological interest: breaking of a low-frequency (large-amplitude) breather into a fluxon-antifluxon pair due to interaction with a fast fluxon belonging to the mate junction.

In Sec. III we study radiative effects. First we determine the equilibrium velocity of the motion of a fluxon in the case when this velocity is larger than the limit velocity C_1 of the mate junction [provided $C_1<C_2$, i.e., $a<1$, see (2)], i.e., the effective radiative dissipation due to "Cherenkov" emission in the mate junction should be taken into account. Then we consider radiative effects accompanying interaction of fluxons belonging to the same or different junctions. In the case when the two colliding (interacting) fluxons belong to different junctions we are able to calculate the energy spectral density of emitted radiation and the total emitted energy provided their relative velocity V is not too small: $V\gg|\alpha|$. In the case of two fluxons of opposite polarities belonging to the same junction we calculate the energy emitted in the mate junction. In the latter case the calculation can be explicitly performed provided $V^2 \ll 1$ or $1-V^2 \ll 1$.

II. ADIABATIC INTERACTIONS OF FLUXONS

A. Dissipative binding of two fluxons into a bifluxon

In absence of perturbations (i.e., at $f_i=\gamma_i=\alpha=0$, $i=1,2$) the system of equations (2) turns into uncoupled, exactly integrable, sine-Gordon equations. The solutions corresponding to fluxons (or antifluxons) are

$$\phi(x,t)=4\arctan\{\exp[\sigma_1(x-V_1t)/(1-V_1^2)^{1/2}]\} , \quad (3a)$$

$$\psi(x,t)=4\arctan(\exp\{\sigma_2 b(x-V_2t)/[1-(aV_2)^2]^{1/2}\}) , \quad (3b)$$

where V_i are the fluxons' velocities and $\sigma_i=\pm 1$ are their polarities. The interaction between the junctions ($\alpha\neq 0$) distorts the fluxon's form; most important is the "image" of the fluxon [Eq. (3)] in the mate junction, e.g., the image of the fluxon (3b) in the first junction is

$$\phi(Z_2)=\frac{2\alpha\sigma_2 b^2\,\mathrm{sgn}Z_2}{[1-(aV_2)^2]}[Z_2\cosh Z_2 - \sinh Z_2\ln(2\cosh Z_2)] , \quad (4)$$

where

$$Z_2=b(x-V_2t)/[1-(aV_2)^2]^{1/2} .$$

Besides, it should be taken into account that in the absence of a mate junction, but in presence of the bias current and dissipation, the velocity of the fluxon's uniform motion is uniquely determined (see, e.g., Ref. 3) as

$$V_1=\sigma_1[1+(4\gamma_1/\pi f_1)^2]^{-1/2} . \quad (5)$$

In the presence of the mate junction relative corrections to (5) are $\sim O(\alpha^2)$.

Adiabatic equations describing the interaction of two fluxons belonging to the two different junctions can be obtained in a simple way if one employs the energetic approach (see Ref. 3): Inserting the expressions (3) into the term of the Hamiltonian which accounts for the interaction between the two junctions, i.e., $H_{\mathrm{int}}=\alpha\int dx\,\phi_x\psi_x$, yields the equations of motion for the centers of the two fluxons ζ_1 and ζ_2. In the simplest case $a=b\equiv 1$ these equations are

$$\frac{d^2\zeta_1}{dt^2}=\frac{\pi f_1\sigma_1}{4}-\gamma_1\frac{d\zeta_1}{dt}-\frac{\alpha\sigma_1\sigma_2}{\sinh\zeta}(1-\zeta/\tanh\zeta) , \quad (6a)$$

$$\frac{d^2\zeta_2}{dt^2}=\frac{\pi f_2\sigma_2}{4}-\gamma_2\frac{d\zeta_2}{dt}+\frac{\alpha\sigma_1\sigma_2}{\sinh\zeta}(1-\zeta/\tanh\zeta) , \quad (6b)$$

where $\zeta=\zeta_1-\zeta_2$. Equations (6) are written in the "nonrelativistic" approximation, i.e., at $(d\zeta_j/dt)^2\ll 1$ ($j=1,2$).

The simplest adiabatic effect which can be described by these equations is the binding of the two fluxons into a bifluxon (a bound state of fluxons) due to dissipative losses. We shall study this effect in the most interesting case when the uniform motion of free fluxons is nonrelativistic, i.e., $f_j\ll\gamma_j$ [see (5)], and the coupling between the two equations is the strongest perturbation in (2), i.e., $|\alpha|\gg\gamma_j^2$. Our goal is to find a threshold condition admitting the binding of the two fluxons colliding with the velocities $V_{1,2}$ [see (5) and formula (15) below]. Using specified conditions, we may consider the problem in the nearly inertial center-of-mass reference frame (the time of braking of the center of mass will be much larger than the binding time). So, in the first approximation we may neglect the terms $\approx f_j,\gamma_j$ in Eqs. (6) to arrive at the mechanical problem for the effective particle with the reduced mass $m=4$ [see (6)] moving in the potential

$$U(\zeta)=-8\alpha\sigma_1\sigma_2\zeta/\sinh\zeta . \quad (7)$$

Evidently, the law of motion is determined by the energy equation

$$2\left[\frac{d\zeta}{dt}\right]^2+U(\zeta)=2V_0^2 , \quad (8)$$

$V_0=V_1-V_2$ being the velocity of the effective particle at infinity.

The cases of the attractive ($\alpha\sigma_1\sigma_2>0$) and repulsive ($\alpha\sigma_1\sigma_2<0$) potentials (7) are qualitatively different (as are interactions of a fluxon in a solitary junction with an attractive microinhomogeneity and with a repulsive one[13,14]). In the former case we shall assume $\gamma_j\sqrt{|\alpha|}\gg f_j$, which enables us to neglect the term $2V_0^2$ in Eq. (8). Then the law of motion of the two fluxons may be represented in the form

$$d\zeta_{1,2}/dt=\pm\sqrt{|\alpha|\zeta/\sinh\zeta} . \quad (9)$$

To obtain the threshold condition for the binding process, it is necessary to calculate the total dissipative energy loss ΔE during the collision between the fluxons: The threshold condition may be written as

$$\Delta E\geq T_1+T_2-T_b , \quad (10)$$

where $T_{1,2}\approx 4V_{1,2}^2\approx\pi^2 f_{1,2}^2/4\gamma_{1,2}^2$ are the kinetic energies of the two fluxons prior to the collision, and $T_b\approx 8V_b^2$ is the kinetic energy of an eventual bound state (bifluxon), V_b being its equilibrium velocity analogous to $V_{1,2}$. To calculate V_b, let us note that the total driving force and friction force acting upon it are, respectively, $2\pi(\sigma_1 f_1+\sigma_2 f_2)$ and $-8V(\gamma_1+\gamma_2)$, so that

$$V_b=\frac{\pi}{4}\frac{\sigma_1 f_1+\sigma_2 f_2}{\gamma_1+\gamma_2} . \quad (11)$$

The quantity ΔE can be calculated, with regard to (9), as follows:

$$\Delta E = 8\sum_{j=1}^{2}\gamma_j\int_{-\infty}^{\infty}\left[\frac{d\zeta_j}{dt}\right]^2 dt$$
$$=2(\gamma_1+\gamma_2)\int_{-\infty}^{\infty}\frac{d\zeta}{dt}d\zeta = D(\gamma_1+\gamma_2)\sqrt{|\alpha|}\ , \quad (12)$$

where

$$D \equiv 8\int_0^{\infty} dx\left[\frac{x}{\sinh x}\right]^{1/2} \approx 15.01\ .$$

Inserting (11) and (12) into (10), we obtain the binding threshold condition for the attraction case

$$(\gamma_1+\gamma_2)\sqrt{|\alpha|} \geq \frac{\pi^2}{4D}\left[\frac{f_1^2}{\gamma_1^2}+\frac{f_2^2}{\gamma_2^2}-\frac{2(f_1\sigma_1+f_2\sigma_2)^2}{(\gamma_1+\gamma_2)^2}\right]\ . \quad (13)$$

The relation (13) is valid if the potential force ($\sim\alpha$) acting upon the fluxons during their overlapping is much larger than the friction force $\sim\gamma V\sim\gamma\sqrt{|\alpha|}$, which explains the above condition $\gamma_j^2 \ll |\alpha|$.

In the repulsion case (i.e., $\alpha\sigma_1\sigma_2<0$) obtaining the binding threshold condition is more simple: One should require that the height $8|\alpha|$ of the potential barrier (7) be larger than the same combination $T_1+T_2-T_b$ of the kinetic energies which occurs in the right-hand side of (10) (cf. an analogous condition for trapping the fluxon by a repulsive microinhomogeneity[3]),

$$|\alpha| \geq \frac{\pi^2}{32}\left[\frac{f_1^2}{\gamma_1^2}+\frac{f_2^2}{\gamma_2^2}-\frac{2(f_1\sigma_1+f_2\sigma_2)^2}{(\gamma_1+\gamma_2)^2}\right]\ , \quad (14)$$

provided the same condition $|\alpha| \gg \gamma_{1,2}^2$ holds. The latter inequality demonstrates that the condition (14) is less restrictive than (13).

The repulsive interaction of the two fluxons ($\sigma_1=\sigma_2$, $\alpha<0$) in the particular case of two identical coupled junctions [$a=b=1$, $\gamma_1=\gamma_2$ in (2)] with $f_1\neq f_2$ was considered in the recent paper by Volkov.[15] The threshold condition for dissipation-induced binding of the colliding fluxons into a bifluxon obtained in Ref. 15 is a particular case of our condition (14).

The bifluxon may perform damped internal oscillations; in particular, the frequency of small oscillations is $\omega^2 \approx |\alpha|/3$ in the attraction case, and

$$\omega^2 \approx \pi|(\sigma_1 f_1-\sigma_2 f_2)/8\alpha|$$

in the repulsion case (the latter expression is valid as long as it gives $\omega^2 \ll 1$). When the internal oscillations are absent the distance between the two fluxons bound into a bifluxon is, respectively,

$$\zeta_0 \approx \frac{3\pi}{4\alpha}|f_1\sigma_1-f_2\sigma_2|$$

and

$$\zeta_0 \approx \ln|\alpha/(f_1\sigma_1-f_2\sigma_2)|$$

for the attraction and repulsion cases [we again regard $(f_1\sigma_1-f_2\sigma_2)/\alpha$ as a small parameter].

There also exist other interesting adiabatic effects. One of them is the capture of a fluxon moving in one junction by a microinhomogeneity localized in another junction. If, for example, the microinhomogeneity is repulsive, a condition providing the possibility of the capture can be readily obtained as was done for the well-known analogous problem when both the fluxon and the inhomogeneity belonged to the same junction (see Refs. 3, 13, and 14).

B. Destructive collison between a fluxon and a low-frequency breather

Now we shall briefly consider a problem where adiabatic inelastic interaction between solitons is possible on account of interjunction coupling only, with no direct participation of the dissipation. Let us consider collision between a fluxon moving in the second junction with the velocity V_2—here we shall regard it as an independent parameter, but one should bear in mind that, as above, it is expressed in terms of f_2 and γ_2 according to the relation

$$V_2=\sigma_2[a^2+(4\gamma_2/\pi f_2)^2]^{-1/2} \quad (15)$$

similar to (5)—and a low-frequency (large-amplitude) breather resting in the first junction. In the absence of perturbations the breather is described by the solution

$$\phi_{\text{br}}(x,t) \approx 4\arctan[\omega^{-1}\sin(\omega t)\,\text{sech}x],\quad \omega \ll 1\ . \quad (16)$$

The breather (16) may be treated as a bound state of the fluxon-antifluxon pair; the breather's binding energy is $E_b \approx -8\omega^2$ (see, e.g., Ref. 16). The collision may result in breaking the breather (16) into a pair of the free fluxon and antifluxon. This problem is of certain methodological interest, and it has been considered in Refs. 16 and 17 for different dissipative and conservative perturbations within the framework of one sine-Gordon equation. For the problem described by the coupled equations (2) we obtain the following result: The change of the breather's parameter ω^2 due to the interaction with a moving fast fluxon is

$$\Delta(\omega^2) = -\pi\alpha\sigma_2 b^2[1-(aV_2)^2]^{1/2}(\cos\chi)F(T_0)\ , \quad (17)$$

where

$$F(T_0)=\int_{-\infty}^{\infty}dt\frac{[\sinh(t/a)][\cosh^2(t/a)-(T_0+t)^2]}{[\cosh^2(t/a)+(T_0+t)^2]^2}\ ,$$

$T_0=(\sin\chi)/\omega_0$, ω_0 is the value of ω prior to the collision, and χ is the value of the breather's internal phase at the collision moment. The result (17) is applicable if $[1-(aV_2)^2]^{1/2} \ll 1$. The collision breaks the breather into a free fluxon–antifluxon pair if ω^2 becomes negative after the collison, i.e., if $-\Delta(\omega^2) \geq \omega_0^2$ (see also Refs. 16 and 17).

III. EMISSION OF RADIATION IN THE SYSTEM OF COUPLED JUNCTIONS

A. "Cherenkov" emission from a fast fluxon

As was emphasized in Refs. 3 and 8–11, emission of radiation by fluxons under the action of different perturbations is of significant physical interest. The simplest problem related to radiative effects in the system considered is to determine an equilibrium velocity of the fluxon's motion in the case when the velocity defined according to Eq. (15) is larger than the limit velocity C_1 of the mate junction (in our notation $C_1=1$). In this case the equilibrium velocity V_2 is determined by the energy balance where the radiative losses due to the Cherenkov emission generated by the "tachyonic" motion of the fluxon's image (4) in the mate junction should be taken into account on a level with the dissipative losses and the input term originating from a bias current. Following the general method of calculating the emission power (the energy emission rate),[18,19] it is straightforward to find the rate P of the Cherenkov radiative losses:

$$P(V_2)=\frac{2\pi^2\alpha^2 V_2}{(V_2^2-1)^2}\operatorname{sech}^2\left[\frac{\pi}{2b}\frac{[1-(aV_2)^2]^{1/2}}{(V_2^2-1)^{1/2}}\right]. \tag{18}$$

The energy balance equation has the form

$$2\pi V_2 f_2=8\gamma_2 V_2^2+P(V_2)\,.$$

Inserting the expression (18), we obtain the following equation:

$$f_2=\frac{4\gamma_2}{\pi}\frac{aV_2}{[1-(aV_2)^2]^{1/2}}+\frac{\pi\alpha^2}{b^2(V_2^2-1)^2}\operatorname{sech}^2\left[\frac{\pi}{2b}\frac{[1-(aV_2)^2]^{1/2}}{(V_2^2-1)^{1/2}}\right]. \tag{19}$$

As is well known, the fluxon's equilibrium velocity is proportional to the voltage across the junction, so that Eq. (19) is, in fact, the I-V (current-voltage) characteristic of LJJ with a trapped fluxon in the presence of a weakly coupled mate junction (Fig. 1). As we see, the differential I-V characteristic df_2/dV_2 as a function of V_2 may acquire two extrema (local maximum and minimum) in the range $1<V_2<a^{-1}$ on account of the "Cherenkov correction," provided α^2 is sufficiently large.

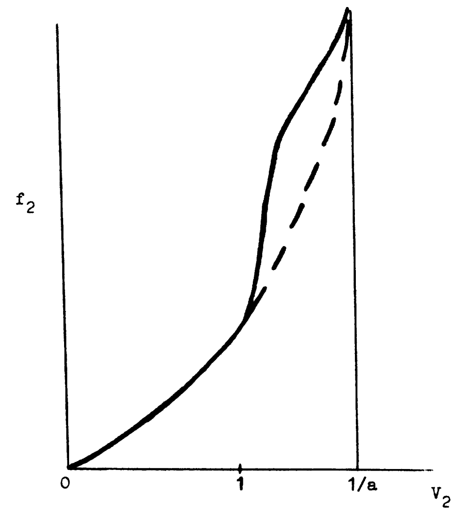

FIG. 1. The solid line depicts the I-V (current-voltage) characteristic corresponding to Eq. (19); the dashed line depicts the standard I-V characteristic of LJJ with a fluxon in absence of the mate junction.

B. Emission from colliding fluxon and antifluxon belonging to the same junction

More sophisticated problems are related to investigation of radiation emission accompanying a collision of two fluxons. If the fluxons belong to the same junction we mean a fluxon-antifluxon collision, while in the case when they belong to different junctions any relative polarity for the fluxons is possible. We shall deal with the case $f_1>4\gamma_1^{3/2}$ when the fluxon-antifluxon collision does not result in their dissipative annihilation.[16]

First let us examine the former case. A general method for calculating the emitted energy has been put forward in Ref. 17. The distinctive feature of the present problem is that colliding fluxons generate radiation in both junctions. In their own junction the emission is generated on account of the perturbating term $\sim(f_1-\gamma_1\phi_t)$ in (2). This emission has been investigated in Refs. 9 and 10. Here we shall investigate the perturbation-induced emission of radiation in the mate junction where there are no fluxons. The corresponding emitted energy can be explicitly calculated in the two limiting cases. First is the case when, as in Refs. 9 and 10, the colliding fluxon and antifluxon are "relativistic," i.e., $f_1\gg\gamma_1$ [$1-V_1^2\ll1$, see (5)]. Substitution of the exact two-soliton solution of the sine-Gordon equation into the right-hand side of the second equation from (2) and subsequent straightforward calculations yield the spectral density of the emitted energy ($a=b=1$)

$$\mathcal{E}_2(k)=\frac{\pi^3\alpha^2\sinh^2[(\pi/2\nu_1)(1+k^2)^{1/2}]}{4\nu_1^4\sinh^2\{(\pi/2\nu_1)[(1+k^2)^{1/2}+k]\}\sinh^2\{(\pi/2\nu_1)[(1+k^2)^{1/2}-k]\}}, \tag{20}$$

where $\nu_1\equiv(1-V_1^2)^{-1/2}\gg1$, V_1 is defined by (5), and k is the radiation wave number. The total emitted energy can be readily obtained from (20):

$$(E_{\rm em})_2=\int_{-\infty}^{\infty}\mathcal{E}_2(k)dk\approx\tfrac{4}{3}\alpha^2\nu_1\,. \tag{21}$$

The second tractable case is that when $f_1\ll\gamma_1$ (i.e.,

$V_1^2 \ll 1$). In this case, in the first approximation we may insert into the right-hand side of the second equation (2) the well-known exact sine-Gordon solution which describes the colliding kink (fluxon) and antikink (antifluxon) with the zero velocity at infinity:

$$\phi(x,t)=4\arctan(t\,\mathrm{sech}x) \ . \tag{22}$$

In this case calculations yield the following expression for the emitted energy spectral density:

$$\mathcal{E}_2(k)=\frac{\alpha^2}{\pi}\left|\int_{-\infty}^{\infty} dx[1-(1+k^2)^{1/2}\sinh x\tanh x]\right.$$
$$\left.\times\exp[-ikx-\cosh x(1+k^2)^{1/2}]\right|^2 , \tag{23}$$

and the total emitted energy

$$(E_{\mathrm{em}})_2\approx 38.4\alpha^2 \ . \tag{24}$$

As we see from (20) and (23), in the case $v_1^2 \gg 1$ the emitted energy is concentrated in the spectral range $k^2 \lesssim v_1^2$, while in the case $V_1^2 \ll 1$ (slow fluxons) it is concentrated in the long-wave range $k^2 \lesssim 1$.

It is pertinent to note that, provided that $(E_{\mathrm{em}})_2$ defined in (24) is larger than the total kinetic energy $8V_1^2$ of the colliding fluxon and antifluxon, the considered radiative losses will result in a fluxon-antifluxon annihilation into a bound state (breather).

C. Emission from colliding fluxons belonging to different junctions

In the case when the colliding fluxons belong to different junctions the emitted energy can be explicitly found for all the values of the fluxons' velocities except for the case when their relative velocity V is very small, $V^2 \lesssim \alpha^2$, as in this case the collision may result in radiative binding of the fluxons into a bifluxon (dissipative binding was considered in Sec. II).

Straightforward calculations yield the following expressions for the spectral densities of the energy emitted in both junctions (as above, $a=b=1$):

$$\mathcal{E}_1(k)=\frac{\pi^3\alpha^2(1-V_1^2)^2}{(V_2-V_1)^4\sinh^2(\pi\kappa_1/2)\cosh^2(\pi\kappa_2/2)} , \tag{25}$$

$$\mathcal{E}_2(k)=\mathcal{E}_1(k;\ V_1 \rightleftarrows V_2) , \tag{26}$$

where

$$\kappa_1\equiv(1-V_1^2)^{1/2}[(1+k^2)^{1/2}-kV_2]/\,|\,V_2-V_1\,| \ ,$$
$$\kappa_2\equiv(1-V_2^2)^{1/2}[(1+k^2)^{1/2}-kV_1]/\,|\,V_2-V_1\,| \ ,$$

V_1 and V_2 are determined according to (5) and (15).

The corresponding total emitted energies can be calculated in the same two limiting cases that have been distinguished above: $1-V_j^2 \ll 1$ and $\alpha^2 \ll (V_2-V_1)^2 \ll 1$. In the former case

$$(E_{\mathrm{em}})_1=8\alpha^2 v_2^3/3v_1^2 ,$$
$$(E_{\mathrm{em}})_2=8\alpha^2 v_1^3/3v_2^2 , \tag{27}$$

where $v_j\equiv(1-V_j^2)^{-1/2}$, $j=1,2$. In the latter case

$$(E_{\mathrm{em}})_1\approx(E_{\mathrm{em}})_2=16\pi^3\alpha^2(V_2-V_1)^{-7/2}$$
$$\times\exp(-2\pi/\,|\,V_2-V_1\,|\,) \ . \tag{28}$$

The expressions (25)–(28) demonstrate the same qualitative features as Eqs. (20), (21), and (23).

In conclusion let us note that the effects which have been analyzed theoretically in the present paper should manifest themselves in an experiment as peculiarities of I-V characteristics of the systems of coupled junctions. Details will be presented elsewhere.

Note added in proof. The collision of a bifluxon with a free fluxon may give rise to interesting inelastic effects. The simplest one is breakup of a bifluxon into a pair of free fluxons. Indeed, the total potential $U_{\mathrm{tot}}(\zeta)$ of the interaction between two fluxons bound into a bifluxon is given by expression (7) plus a contribution from the bias currents:

$$U_{\mathrm{tot}}(\zeta)=-8\alpha\sigma\zeta/\sinh\zeta-\pi(\hat f_1-\hat f_2)\zeta , \tag{29}$$

where $\sigma\equiv\sigma_1\sigma_2$, $\hat f_j\equiv\sigma_j f_j$ $(j=1,2)$. It is straightforward to see that, provided $|\,\alpha\,| \gg |\,\hat f_1-\hat f_2\,|$, the potential (29) has a local minimum and a local maximum separated by the distance ζ_0 determined by the expression

$$\exp(\zeta_0)\approx\{16\,|\,\alpha\,|\,/[\pi(\hat f_1-\hat f_2)]\}$$
$$\times\ln\{16\,|\,\alpha\,|\,/[\pi(\hat f_1-\hat f_2)]\} \tag{30}$$

(for definiteness, we assume $\hat f_1-\hat f_2>0$). On the other hand, it is well known that, in the absence of perturbations, a collision between two sine-Gordon kinks (fluxons) moving with nonrelativistic velocities V_j $(V_j^2 \ll 1)$ gives rise to the shifts $\Delta\zeta_{1,2}$ of the centers of the kinks,

$$\Delta\zeta_{1,2}\approx\mathrm{sgn}(V_{1,2}-V_{2,1})\ln[4/(V_1-V_2)^2] \ . \tag{31}$$

Straightforward analysis based upon Eqs. (30) and (31) brings us to the following conclusion (for the sake of simplicity, hereafter we set $\gamma_1=\gamma_2\equiv\gamma$): Collision between a bifluxon with $\alpha\zeta>0$ and a free fluxon which has the same polarity σ_j as a corresponding (jth) component of the bifluxon results in the breakup of the bifluxon under the condition

$$(\hat f_1-\hat f_2)\lesssim 32\gamma^2/\pi\,|\,\alpha\,| \ . \tag{32}$$

Expression (32) is relevant provided $|\,\alpha\,|^{3/2}\lesssim\gamma \ll |\,\alpha\,|$. Bifluxons with $\alpha\zeta<0$ are more stable against the collision.

In the opposite case, when the polarity σ_3 of the free fluxon is $-\sigma_j$, the situation is more complicated. If, for instance, the free fluxon belongs to the first junction, and $\hat f_1 > |\,\hat f_2\,|$, the breakup takes place under the condition

$$(3\hat f_1+\hat f_2)^2/(\hat f_1-\hat f_2)$$
$$\lesssim 16\gamma^2/\{\pi\,|\,\alpha\,|\,\ln[16\,|\,\alpha\,|\,/\pi(\hat f_1-\hat f_2)]\} , \tag{33}$$

irrespectively of the sign of $\alpha\sigma$ Expression (33) is relevant provided $\alpha^2 \ll \gamma \ll |\alpha|$. However, in the present case ($\sigma_3 = -\sigma_1$), two other channels of inelastic interaction between the bifluxon and the free fluxon are possible. If $\alpha\sigma > 0$, the fluxons with the polarities σ_1 and $\sigma_3 = -\sigma_1$ colliding in the first junction annihilate (due to dissipative losses) into a breather under the condition

$$|3\hat{f}_1 + \hat{f}_2| \le 8\sqrt{3}\gamma^{3/2}, \tag{34}$$

so that eventually there will remain a free fluxon in the second junction and nothing in the first one. If $\alpha\sigma < 0$ and $|\alpha| \gg \gamma$, the collision leads, under the same condition (34), to another result ("recharge"): The two fluxons in the first junction switch roles, so that eventually one will see a free fluxon with the polarity $\sigma_3 = +\sigma_1$ in the first junction, and a bifluxon with the reversed sign of $\alpha\sigma$ ($\alpha\sigma > 0$).

Analogous consideration reveals that a collision between a bifluxon with $\alpha\sigma > 0$ and a corresponding antibifluxon results in their dissipative annihilation provided

$$|\hat{f}_1 + \hat{f}_2| \le 8\gamma^{3/2}. \tag{35}$$

A colliding bifluxon and antibifluxon with $\alpha\sigma < 0$ turn, due to the dissipative losses and under the same condition (35), into a bifluxon-antibifluxon pair with $\alpha\sigma > 0$. Finally, collision between two bifluxons with the polarities (σ_1, σ_2) and $(\sigma_1, -\sigma_2)$ gives rise to two free unipolar fluxons in the first junction and dissipative annihilation in the second one under the condition $|\hat{f}_2| \le 4\sqrt{2}\gamma^{3/2}$.

ACKNOWLEDGMENTS

The authors are indebted to L. I. Glazman, A. A. Golubov, A. M. Kosevich, K. K. Likharev, and M. R. Samuelsen for useful discussions.

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