Upper bound on strong-coupling corrections to the second upper critical field

R. Akis, F. Marsiglio, E. Schachinger, and J. P. Carbotte
Physics Department, McMaster University, Hamilton, Ontario L8S 4M1, Canada
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An upper bound for the strong-coupling correction $\eta_{H_{c2}}$ to the second upper critical magnetic field of an Eliashberg superconductor is established as a function of Coulomb pseudopotential (μ^*) and of impurity content (t^+). Results are also given for many actual superconductors for which the electron-phonon spectral densities are known. These are found to always fall below our upper bound although in some cases they can fall fairly close to the maximum, which varies with μ^* and t^+.

I. INTRODUCTION

Superconductivity in many systems can be explained through the use of Eliashberg theory, which assumes that it is the electron-phonon interaction which is responsible for the superconductivity.[1-4] However, in the case of the recently discovered very-high-critical-temperature (T_c) oxides,[5-7] there is much evidence against the electron-phonon interaction being the mechanism exclusively responsible,[8-10] and other mechanisms have been considered.[11-16] Still, it is of interest to know the limitations placed on superconducting properties by the Eliashberg equations themselves. This is especially true since the exact mechanism involved in the oxides is still an open question and, in particular, if the mechanism still involves a boson exchange (say excitons or plasmons instead of phonons), the Eliashberg equations can still be applied, at least as a first approximation. Upper bounds on such quantities as the gap ratio $2\Delta_0/kT_c$ (Ref. 17), the normalized specific-heat jump $\Delta C(T_c)/\gamma(0)T_c$ (Ref. 18) and the zero-temperature reduced upper critical field $h_{c2}(0)$ (Ref. 19) have been explored in previous work.

The quantity we concern ourselves with in this paper is the strong-coupling correction factor to the upper critical magnetic field, $\eta_{H_{c2}}(T,t^+)$. As we have made explicit, this factor is dependent on absolute temperature T, and on an impurity parameter t^+, which is related to the impurity lifetime τ by $t^+=1/2\pi\tau$. For $T<T_c$, $\eta_{H_{c2}}(T,t^+)$ is defined through the expression[20,21]

$$H_{c2}(T,t^+)=\eta_{H_{c2}}(T,t^+)H_{c2}^{\rm BCS}(T,t^+)\ ,$$

where $H_{c2}^{\rm BCS}$ is the renormalized BCS result. Since H_{c2} vanishes at T_c, we must define $\eta_{H_{c2}}(T_c,t^+)$ using the expression

$$\left.\frac{dH_{c2}(T,t^+)}{dT}\right|_{T=T_c}=\eta_{H_{c2}}(T_c,t^+)\left.\frac{dH_{c2}^{\rm BCS}(T,t^+)}{dT}\right|_{T=T_c}\ .$$

In either case, the Fermi velocity does not appear in $\eta_{H_{c2}}(T,t^+)$ since we have not included Pauli limiting in our calculations. We shall limit ourselves to the cases $T=0$ and $T=T_c$ using values of t^+ of 0 meV (the clean limit), 50 meV (a reasonable value for real materials), and 500 meV (a value which is taken to represent the dirty limit, $t^+\to\infty$). The question we intend to answer for the above cases is, given any superconductor described by the Eliashberg equations, and with arbitrary electron-phonon spectral density $\alpha^2F(\omega)$, does the strong-coupling correction factor $\eta_{H_{c2}}$ exhibit a maximum? It should be noted that, in our attempt to answer this question, no consideration is given as to whether or not the spectral densities used are consistent with, for example, lattice stability. This is simply assumed to be the case. In Sec. II the mathematics behind $\eta_{H_{c2}}$ is discussed and this is followed by a discussion of functional derivatives in Sec. III. Section IV is concerned with proving a scaling theorem. In Sec. V results are given and in Sec. VI conclusions are drawn.

II. MATHEMATICAL THEORY OF $\eta_{H_{c2}}$

As is obvious from the way $\eta_{H_{c2}}$ is defined, we must first know what H_{c2} and $H_{c2}^{\rm BCS}$ are in order to calculate it. The strong-coupling equations for $H_{c2}(T)$, applicable for any impurity concentration, were first given by Schossmann and Schachinger.[22] These are

$$\tilde{\Delta}(i\omega_n)=\pi T\sum_m[\lambda(\omega_m-\omega_n)-\mu^*\theta(\omega_c-|\omega_m|)]\times\frac{\tilde{\Delta}(i\omega_m)}{\chi^{-1}(\tilde{\omega}(i\omega_m))-\pi t^+}\ , \tag{1}$$

and

$$\tilde{\omega}(i\omega_n)=\omega_n+\pi T\sum_m\lambda(\omega_m-\omega_n){\rm sgn}(\omega_m)+\pi t^+{\rm sgn}(\omega_n)\ , \tag{2}$$

where $i\omega_n=i\pi T(2n-1)$, $n=0,\pm1,\pm2,\ldots$, are the Matsubara frequencies. T and t^+ were defined in the Introduction. In Eqs. (1) and (2)

$$\lambda(\omega_m-\omega_n)=2\int_0^\infty\frac{\Omega\alpha^2F(\Omega)}{\Omega^2+(\omega_n-\omega_m)^2}d\Omega\ , \tag{3}$$

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$\tilde{\Delta}(i\omega_n)$ is the Matsubara pairing energy and ω_c is a cutoff for the Coulomb pseudopotential μ^* required for convergence. The quantity χ which is a function of the renormalized Matsubara frequencies $\tilde{\omega}(i\omega_m)$ is given by

$$\chi(\tilde{\omega}(i\omega_n))=\frac{2}{\sqrt{\alpha}}\int_0^\infty dq\, e^{-q^2}\arctan\left[\frac{q\sqrt{\alpha}}{|\tilde{\omega}(i\omega_n)|}\right], \tag{4}$$

where

$$\alpha=\left[\frac{e}{2}\right]H_{c2}(T,t^+)v_F^2 .$$

Here e is the absolute charge on an electron and v_F the Fermi velocity.

In order to find $H_{c2}^{\rm BCS}$, one must solve

$$\ln\frac{T_c}{T}=\sum_m\left[\frac{1}{|2m+1|}-\frac{[(T/T_c)/(\hat{h}^*)^{1/2}]J(\alpha_m^*)}{1-[\lambda_{\rm tr}^*/(\hat{h}^*)^{1/2}]J(\alpha_m^*)}\right], \tag{5a}$$

where

$$\hat{h}^*(T)=\tfrac{1}{2}eH_{c2}^{\rm BCS}(T)v_F^{*2}/\pi^2T_c^2 , \tag{5b}$$

$$\lambda_{\rm tr}^*=\frac{t^+}{T_c(1+\lambda)} , \tag{5c}$$

and

$$J(\alpha_m^*)=2\int_0^\infty d\omega\exp(-\omega^2)\arctan(\omega\alpha_m^*) ,$$

with

$$\alpha_m^*=\frac{(\hat{h}^*)^{1/2}}{|2m+1|(T/T_c)+\lambda_{\rm tr}^*} . \tag{5d}$$

These equations were derived from the full Eqs. (1) and (2) by Schachinger *et al.*[23] They are the same as in the work of Werthamer *et al.*,[24–26] as written also by Orlando *et al.*,[21] and Decroux *et al.*[27] except for appropriate renormalization factors of $1+\lambda$ with λ given by (3) with $n=m$.

In the clean and dirty limits the calculation is greatly simplified. Marsiglio *et al.*[28] have derived expressions for $\alpha'(T_c)$. These can be related to $H_{c2}^{\rm BCS}(T_c)$ using the relationship

$$\alpha'(T_c)=(e/2)H_{c2}'^{\rm BCS}(T_c)v_F^2 ,$$

giving us

$$H_{c2}'^{\rm BCS}(T_c)=\frac{-28.2T_c(1+\lambda)^2}{ev_F^2} , \tag{6}$$

in the clean limit and

$$H_{c2}'^{\rm BCS}(T_c)=\frac{-24t^+(1+\lambda)}{ev_F^2} , \tag{7}$$

in the dirty limit.

In the same paper, $\alpha_{c1}(0)$ was also derived so that using the relationship between α and H_{c2} gives us for the clean limit

$$H_{c2}^{\rm BCS}(0)=\frac{20.5}{ev_F^2}T_c^2(1+\lambda)^2 . \tag{8}$$

In the dirty limit, Eq. (5a) reduces to the following result given by Rainer and Bergmann:[20]

$$\ln(t)+\Psi\left[\frac{1}{2}+\frac{h_{c2}^{\rm BCS}(t)}{2\pi t}\right]-\Psi(\tfrac{1}{2})=0 \tag{9}$$

where $t=T/T_c$ and $h_{c2}^{\rm BCS}(t)$ is related to $H_{c2}^{\rm BCS}(T)$ through

$$H_{c2}^{\rm BCS}(T)=\frac{6\pi T_c(1+\lambda)t^+}{ev_F^2}h_{c2}^{\rm BCS}(t) .$$

Using Eq. (9) one can easily derive that, for the dirty limit,

$$H_{c2}^{\rm BCS}(0)=\frac{16.6}{ev_F^2}t^+T_c(1+\lambda) . \tag{10}$$

Equations (6), (7), (8), and (10) will be particularly useful in Sec. III.

III. FUNCTIONAL DERIVATIVES

For the purpose of this work, the functional derivative of $\eta_{H_{c2}}$, with respect to the electron-phonon density $\alpha^2F(\omega)$, is required. This is defined in terms of the change in $\eta_{H_{c2}}$ and $\Delta\eta_{H_{c2}}$ when $\alpha^2F(\omega)$ is enhanced by an infinitesimal amount ϵ at a particular frequency ω_0. Taking the limit of $\Delta\eta_{H_{c2}}/\epsilon$ as $\epsilon\to0$ gives us $\delta\eta_{H_{c2}}/\delta\alpha^2F(\omega_0)$. For convenience, we use the dimensionless logarithmic form for the functional derivative following Rainer and Bergmann,[20]

$$\delta\eta_{H_{c2}}\equiv\frac{T_c}{\eta_{H_{c2}}}\frac{\delta\eta_{H_{c2}}}{\delta\alpha^2F(\omega)} . \tag{11}$$

Using the definition of $\eta_{H_{c2}}$, one finds that at $T=0$

$$\delta\eta_{H_{c2}}(0,t^+)=\delta H_{c2}(0,t^+)-\delta H_{c2}^{\rm BCS}(0,t^+) . \tag{12}$$

For $T=T_c$, we have simply that

$$\delta\eta_{H_{c2}}(T_c,t^+)=\delta H_{c2}'(T_c,t^+)-\delta H_{c2}'^{\rm BCS}(T_c,t^+) . \tag{13}$$

Expressions for δH_{c2} and $\delta H_{c2}'$ are given in the work of Marsiglio *et al.*[28] The necessary expressions are lengthy and will not be reproduced here. On the other hand, $\delta H_{c2}^{\rm BCS}(0,t^+)$ and $\delta H_{c2}'^{\rm BCS}(T_c,t^+)$ can be calculated in the clean and dirty limits using Eqs. (6), (7), (8), and (10) given in Sec. II. These yield

$$\delta H_{c2}^{\rm BCS}(0,t^+=0)=2\delta T_c+2\delta(1+\lambda) , \tag{14}$$

$$\delta H_{c2}^{\rm BCS}(0,t^+\to\infty)=\delta T_c+\delta(1+\lambda) , \tag{15}$$

$$\delta H_{c2}'^{\rm BCS}(T_c,t^+=0)=\delta T_c+2\delta(1+\lambda) , \tag{16}$$

$$\delta H_{c2}'^{\rm BCS}(T_c,t^+\to\infty)=\delta(1+\lambda) . \tag{17}$$

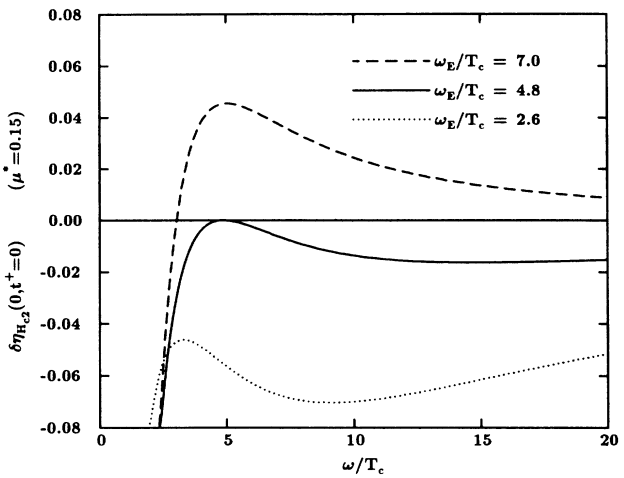

FIG. 1. The functional derivative $\delta\eta_{H_{c2}}$ vs ω/T_c for the case with $T=0$, $t^+=0.0$ meV, and $\mu^*=0.15$ is plotted for three different δ-function spectra, $\alpha^2F_E(\omega)=A\delta(\omega-\omega_E)$, where ω_E is the Einstein frequency. Note that the solid curve is negative definite and peaks at exactly zero when $\omega=\omega_E$. This indicates a local maximum. The dotted curve has the peak in $\delta\eta_{H_{c2}}$ occurring above ω_E/T_c and the dashed curve has $\delta\eta_{H_{c2}}$ peaking below ω_E/T_c.

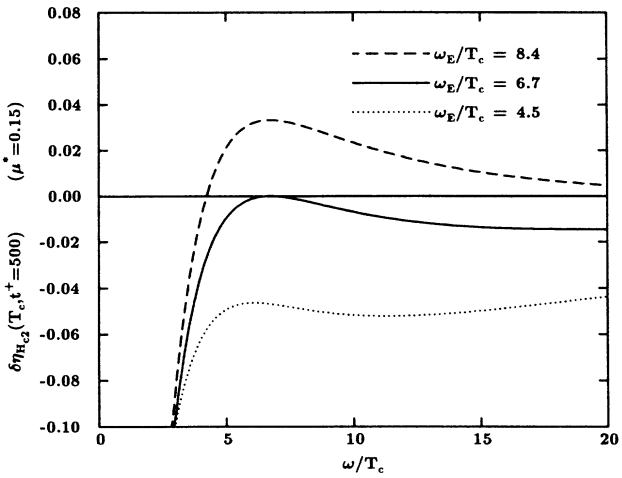

FIG. 2. The same as Fig. 1 except that now $T=T_c$ and $t^+=500.0$ meV.

As in the case for δH_{c2} and $\delta H'_{c2}$, δT_c has been evaluated in previous work[20,28] and will not be repeated here. Using Eq. (5) with $m=n$ gives the result

$$\delta(1+\lambda)=\frac{T_c}{1+\lambda}\frac{2}{\omega}\ .$$

The argument we give here for the maximization of $\eta_{H_{c2}}$ is along the same lines as that first supplied by Leavens[29] for T_c. We argue that, given that the curves for $\delta\eta_{H_{c2}}$ display some sort of maximum, then in order to increase $\eta_{H_{c2}}$ for a given $\alpha^2F(\omega)$, we should take weight from some frequency where the functional derivative is smaller than its value at maximum and transfer it to the optimum frequency, keeping the total area constant. Given that such a process is required to increase $\eta_{H_{c2}}$ leads one to believe that, for a given value of $A=\int\alpha^2F(\omega)d\omega$, a δ function should be used with all its weight at the same Einstein frequency ω_E in order to maximize $\eta_{H_{c2}}$.

Figures 1 and 2 feature plots of $\delta\eta_{H_{c2}}(0,t^+=0)$ and $\delta\eta_{H_{c2}}(T_c,t^+=500$ meV), respectively, using several different

$$\alpha^2F_E(\omega)=A\delta(\omega-\omega_E)\ ,$$

for $\mu^*=0.15$. These figures illustrate our attempts to place a δ function in the same location as the maximum in $\delta\eta_{H_{c2}}$. As both figures show, when the position of the δ function is below the peak in $\delta\eta_{H_{c2}}$, the peak value falls below zero, i.e., is negative. Similarly, when the position of the δ function occurs above the peak, the maximum in $\delta\eta_{H_{c2}}$ occurs above zero. It is only when ω_E/T_c and the position of the peaks coincide that we have that $\delta\eta_{H_{c2}}=0$ at the peak, indicating that a local extremum has been reached.

IV. A SCALING THEOREM

For all our calculations, a single value of A was used. What we intend to show in this section is that our estimates for the maxima in $\eta_{H_{c2}}$ are independent of this choice for the area under the δ function, except for small corrections due to a finite μ^*, which can be neglected. Since $\eta_{H_{c2}}$ is given by the ratio of Eliashberg and BCS results, we first return to Eqs. (1) and (2) for H_{c2}. Inserting our δ-function spectrum into them gives

$$\bar{\Delta}(i\omega_n)=\pi\bar{T}\sum_m\left[\frac{2\bar{\omega}_E}{\bar{\omega}_E^2+(\bar{\omega}_m-\bar{\omega}_n)^2}-\mu^*\theta(\omega_c-|\bar{\omega}_m|A)\right]\frac{\bar{\Delta}(\omega_m)}{\bar{\chi}^{-1}(\bar{\omega}(i\omega_m))-\pi\bar{t}^+}\ , \tag{18}$$

and

$$\bar{\omega}(i\omega_n)=\bar{\omega}_n+\pi\bar{T}\sum_m\frac{2\bar{\omega}_E}{\bar{\omega}_E^2+(\bar{\omega}_m-\bar{\omega}_n)^2}\mathrm{sgn}(\bar{\omega}_m)+\pi\bar{t}^+\mathrm{sgn}(\bar{\omega}_n)\ , \tag{19}$$

where $\bar{T}=T/A$, $\bar{\omega}_E=\omega_E/A$, $\bar{\omega}_n=\omega_n/A$, $\bar{\Delta}(i\omega_n)=\tilde{\Delta}(i\omega_n)/A$, $\bar{\omega}(i\omega_n)=\tilde{\omega}(i\omega_n)/A$, $\bar{t}^+=t^+/A$, and $\bar{\chi}(\bar{\omega}_n)=A\chi(\tilde{\omega}_n)$.

From Eq. (6) for $\chi(\tilde{\omega}(i\omega_n))$, one can see that $\bar{\chi}(\bar{\omega}(i\omega_n))$ has the same form as $\chi(\bar{\omega}(i\omega_n))$ except that one must replace α with $\bar{\alpha}=\alpha/A^2$ and $\tilde{\omega}(i\omega_n)$ with $\bar{\omega}(i\omega_n)$. In writing the equations in terms of the barred quantities, we have removed the dependence on A except for a very small dependence in the cutoff associated with μ^*. Leavens[29] followed a similar approach for T_c and, following him, we likewise ignore the small correction required. Given this, $\bar{T}_c$ and $\bar{\alpha}$ become completely independent of A and are only functions of $\bar{\omega}_E$, $\bar{t}^+$, and μ^*. We can therefore write

$$\bar{T}_c=F(\bar{\omega}_E,\mu^*) \tag{20a}$$

and

$$\bar{\alpha}=H(\bar{\omega}_E,\bar{t}^+,t,\mu^*)\ , \tag{20b}$$

where $t=T/T_c$. Here, F and H represent universal functions which can be determined from Eqs. (18) and (19) and from the definition of $\bar{\chi}(\bar{\omega}(i\omega_n))$. In Eq. (20b), the $\bar{t}^+$ parameter falls out in the clean limit while the dependence is trivial in the dirty limit and does not appear at all in (20a) as it drops out in the T_c equations. Using Eqs. (20a) and (20b), we find that T_c is proportional to A and $H_{c2}(T)$ to A^2. From this, one can also conclude that $dH_{c2}(T)/dT$ is proportional to A. These results have already been obtained by Schossmann *et al.*[19] and are repeated here for the reader's convenience only.

To determine the A proportionality of $H_{c2}^{\rm BCS}$, we can turn back to Eqs. (5a)–(5d). From Eq. (5b), $H_{c2}^{\rm BCS}$ gains an A^2 dependence due to the T_c^2 being present. Using Eqs. (5a), (5c), and (5d), it follows that the functional form of $\hat{h}^*$ is given by

$$\hat{h}^*(T)=f(t,\lambda_{\rm tr}^*)\ . \tag{21a}$$

However, noting the definition of $\lambda_{\rm tr}^*$ given in Eq. (5c), it can be seen that $\lambda_{\rm tr}^*$ is itself dependent on $\bar{t}^+$ (using the A dependence of T_c) and $\bar{\omega}_E$ (for a δ-function spectrum $\lambda=2A/\omega_E$). This being the case, we can rewrite $\hat{h}^*$ as being

$$\hat{h}^*(T)=f(t,\bar{t}^+,\bar{\omega}_E)\ , \tag{21b}$$

which has no A proportionality. Given this and the functional form for T_c, a complete functional form for $H_{c2}^{\rm BCS}(T)$ can be written as

$$H_{c2}^{\rm BCS}(T)=A^2g(\bar{\omega}_E,\mu^*,t,\bar{t}^+)\ . \tag{22}$$

It in turn follows that $dH_{c2}^{\rm BCS}(T)/dT$ is proportional to A.

From the definitions of $\eta_{H_{c2}}$ for $T<T_c$ and $T=T_c$ given in Sec. I, and the proportionalities we have just determined, it becomes obvious that, for all temperatures, $\eta_{H_{c2}}$ has no explicit dependence on A (except for small corrections), which is what we wished to prove. Also, for $t^+=0.0$ or ∞, $\eta_{H_{c2}}$ depends only on $\bar{\omega}_E$ and μ^*. For intermediate impurity concentrations the constant impurity parameter is $\bar{t}^+$. As A is changed, $\bar{t}^+$ does change. It is important to keep this in mind.

V. RESULTS

We have examined the μ^* dependence of $\eta_{H_{c2}}$ at $T=0$, and $T=T_c$ for $t^+=0.0$, 50.0, and 500.0 meV. The results of our calculations for the maximum $\eta_{H_{c2}}$ are given in Table I. The $t^+=0.0$ meV and $t^+=500.0$ meV results were found using the previously outlined functional-derivative technique. That is, we find the frequency ω_E for a base δ-function spectrum of arbitrary A such that its functional derivative is exactly zero at the same frequency ω_E. This means that we have reached a maximum in $\eta_{H_{c2}}$ for a δ function

$$\alpha^2F(\omega)=A\,\delta(\omega-\omega_E)\ .$$

This procedure was not possible without modifications in the case of $t^+=50.0$ meV since this falls in between the clean and dirty limits so that none of Eqs. (14)–(17) could be used. The technique in this case was to use a δ-function spectrum and vary its Einstein frequency, ω_E, until a maximum in $\eta_{H_{c2}}$ was mapped out. No attempt at calculating a functional derivative was made as this was thought unnecessary considering the amount of extra work it would have entailed. Figures 3 and 4 of $\eta_{H_{c2}}$ versus ω_E/T_c illustrate this process for two cases (at T_c with $\mu^*=0.051$ and $T=0$ with $\mu^*=0.25$). The fact that $\eta_{H_{c2}}$ displays a maximum is clearly evident in both figures. Such curves are universal, independent of A for a given $\bar{t}^+=t^+/A$. The value of A used was 7.0 meV, a value typical of a real material.

In Table II, $\eta_{H_{c2}}$ values for several real materials are

TABLE I. Calculated maximum value for the strong-coupling parameter $\eta_{H_{c2}}$ for temperature $T=0$ and $T=T_c$ and for three different impurity concentrations, namely, $t^+=0.0$ (clean limit), $t^+=50.0$ meV, and $t^+=500.0$ meV (dirty sample). Rows one to three apply to different values of the Coulomb pseudopotential, namely, $\mu^*=0.0514$, 0.15, and 0.25, which should cover the physical range.

μ^* \ T	0	T_c	0	T_c	0	T_c
t^+	0	0	50	50	500	500
0.0514	1.36	1.24	1.30	1.34	1.32	1.39
0.15	1.39	1.27	1.32	1.37	1.34	1.42
0.225	1.41	1.28	1.33	1.39	1.35	1.43

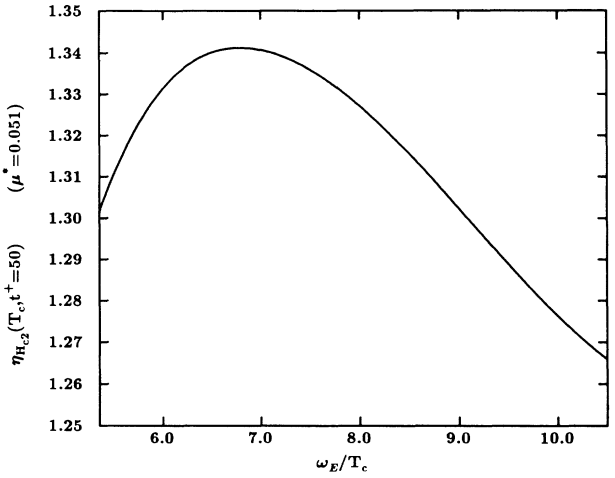

FIG. 3. The value of $\eta_{H_{c2}}$ when $T=T_c$, $t^+=50.0$ meV, and $\mu^*=0.051$ for an Einstein spectrum as a function of ω_E/T_c. The results are independent of the value used for the area (A) of the δ function for a given value of $\bar{t}^+\equiv t^+/A$. We have singled out the region in which $\eta_{H_{c2}}$ clearly reaches a maximum.

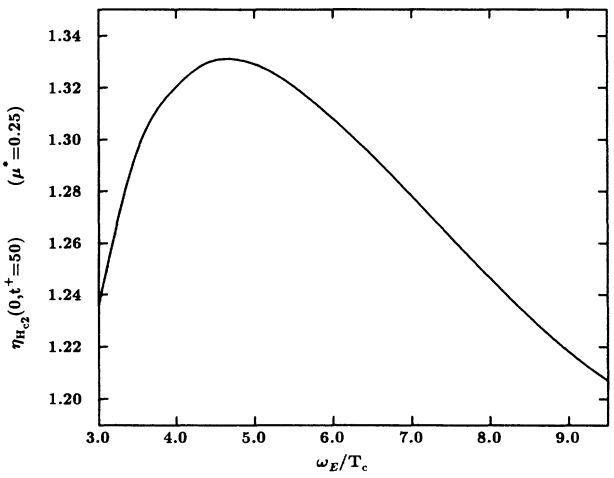

FIG. 4. The same as Fig. 3 except that $T=0$ and $\mu^*=0.25$.

calculated using $t^+=0.0$ and 50.0 meV only. These are combined with the results in Table I to produce Figs. 5–8. These figures feature plots of $\eta_{H_{c2}}$ versus μ^* for the cases $T=0$ and $t^+=0.0$ meV, $T=T_c$ and $t^+=0.0$ meV, $T=0$ and $t^+=50.0$ meV, and $T=T_c$ and $t^+=50$ meV, respectively. In all these figures, the solid dots represent the $\eta_{H_{c2}}$ values for the real materials while the solid lines represent maximum value of $\eta_{H_{c2}}$ calculated for a given μ^*. In all cases, the real materials fall below the solid lines. While all we can claim rigorously is that we have found local maxima, this fact gives us confidence that we have indeed determined the absolute maximum for $\eta_{H_{c2}}$ in each case for realistic $\alpha^2F(\omega)$ spectra. It should be noted that in Figs. 7 and 8, we have shown the maximum curve for a fixed $t^+=50$ meV and an A of 7 meV so that $\bar{t}^+=7.1$. At the same time, the calculations for real materials are for fixed $t^+=50.0$ meV while they should be for the same $\bar{t}^+=7.1$. As the area under $\alpha^2F(\omega)$ will vary from material to material, so will $\bar{t}^+$. The differences implied in $\eta_{H_{c2}}$ are, however, not large, and since no real material falls very close to the maximum curve we need not be concerned about small differences.

VI. CONCLUSIONS

In conclusion, we have found local maxima for $\eta_{H_{c2}}(T,t^+)$ for $T=0$ and $T=T_c$ with $t^+=0.0$, 50.0, and

TABLE II. Values of the strong-coupling correction $\eta_{H_{c2}}$ for various superconductors. The third to sixth column apply, respectively, to temperature $T=0$ and impurity content $t^+=0.0$ (clean limit), $T=T_c$ with $t^+=0.0$, $T=0$ with $t^+=50.0$ meV, and $T=T_c$ with $t^+=50.0$ meV. The values of Coulomb pseudopotential are entered in column two. All entries are for systems in which reliable tunneling data on the electron-phonon spectral density $\alpha^2F(\omega)$ exists. Details are given in the paper of Mitrovic *et al.* (Ref. 30).

	μ^*	T / t^+	0 / 0	T_c / 0	0 / 50	T_c / 50
Nb	0.1158		1.105	1.093	1.117	1.166
Nb_3Sn	0.1513		1.196	1.133	1.189	1.213
V_3Si	0.1357		1.117	1.119	1.12	1.188
Pb	0.1392		1.246	1.198	1.223	1.307
a-Ga	0.1711		0.942	0.899	1.052	0.970
Va	0.2193		1.04	1.045	1.06	1.097
$Tl_{0.9}Bi_{0.1}$	0.1116		1.068	1.071	1.091	1.144
Ta	0.1187		1.042	1.047	1.064	1.102
$Pb_{0.65}Bi_{0.35}$	0.0866		1.246	1.097	1.238	1.195
Nb_3Al	0.1245		1.189	1.115	1.187	1.195
V_3Ga	0.088		1.15	1.122	1.1148	1.196
a-Mo	0.0695		1.108	1.095	1.119	1.168

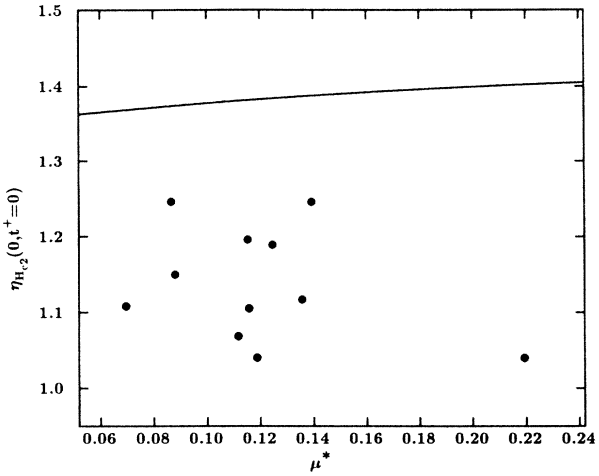

FIG. 5. The maximum possible value of $\eta_{H_{c2}}$ with $T=0$ and $t^+=0.0$ meV is plotted as a function of μ^*. The solid dots represent theoretical values for real materials (see Table II).

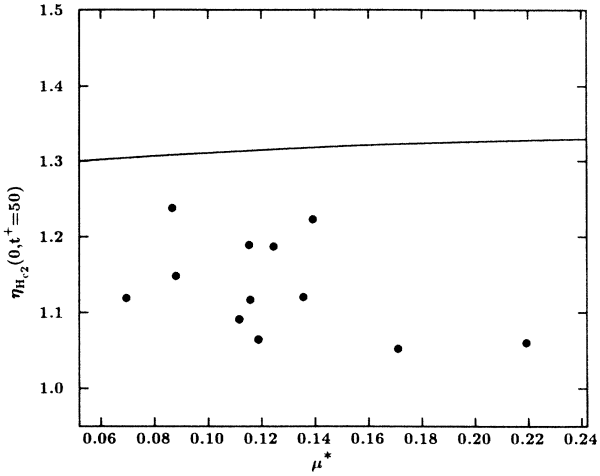

FIG. 7. The same as Fig. 5 except that $t^+=50.0$ meV.

500.0 meV, respectively. We believe that these maxima are absolute at least as far as realistic electron-phonon spectral densities are concerned.

For all cases considered, the maximum in $\eta_{H_{c2}}$ is a slowly varying function of μ^*. The maxima in $\eta_{H_{c2}}$ do not appear to be radically higher than the $\eta_{H_{c2}}$ values found in real materials.

It is worth noting again that in as much as other boson exchange mechanisms such as plasmon or exciton exchange can be cast within the framework of Eliashberg theory, to a first approximation the maxima we have found here could be applied (with discretion) to these other possible mechanisms as well.

Another possible application of our results has to do with the Sommerfeld constant $\gamma(0)$. For example, there exists a formula for the dirty limit[26]

$$\left.\frac{dH_{c2}(T)}{dT}\right|_{T_c} = 4.48\times 10^4 \eta_{H_{c2}}(T_c)\gamma(0)\rho \ \text{Oe K}^{-1} , \tag{23}$$

where ρ is the residual resistivity in units of Ω cm. From our results, we know that for the dirty limit $\eta_{H_{c2}}(T_c)\lesssim 1.4$. Since $[dH_{c2}(T)/dt]\,|_{T_c}$ and ρ are measurable quantities, a lower limit on $\gamma(0)$ could possibly be placed.

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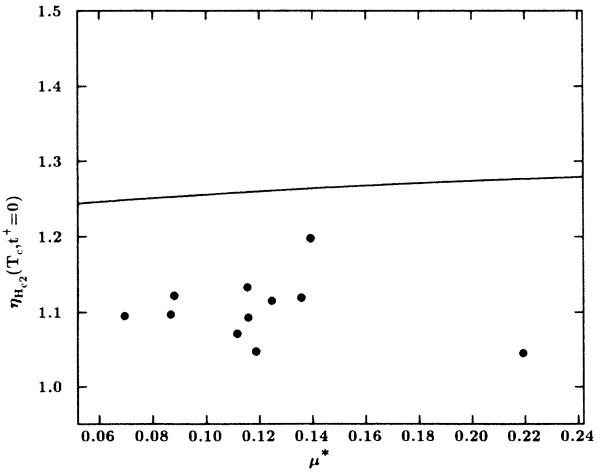

FIG. 6. The same as Fig. 5 except that $T=T_c$.

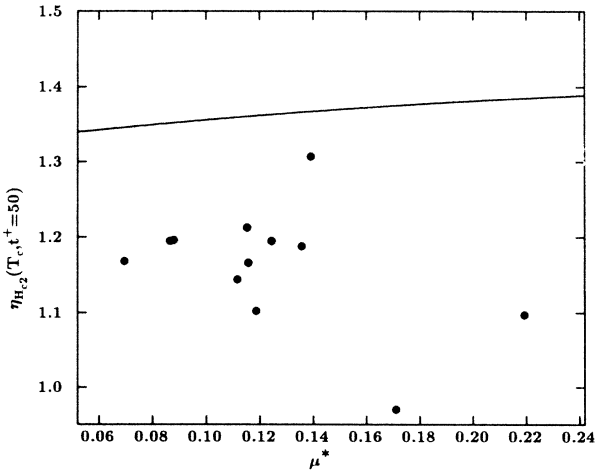

FIG. 8. The same as Fig. 5 except that $T=T_c$ and $t^+=50.0$ meV.

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