Rate theory of dislocation motion: Thermal activation and inertial effects

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A rate theory of dislocation motion is developed which incorporates both thermal activation and inertial effects and that accounts for the enhanced plasticity in superconductors. The equations of Brownian motion are applied to dislocations and are shown to describe viscous effects in thermal activation and thermal fluctuations during inertial effects. The viscosity reduces the activation rate only for high values of the damping whereas inertial effects occur only at low values. For dislocations the damping is in the range where inertial effects, and not a change in the frequency factor, occur for pure samples in the superconducting state and for dilute alloys in both the normal and superconducting state. The enhanced plasticity is therefore explained by an increase in the average distance traveled by dislocations after thermal activation accounting for the dependence on both the viscosity and the temperature.

I. INTRODUCTION

The discovery of an enhanced plasticity in superconductors has revealed an inadequacy in standard rate-theory treatments of plastic flow. A purely mechanical inertial model gives a fair account of the stress changes which occur in normal to superconducting transitions, but cannot account for the temperature dependence of the flow stress observed in the superconducting state. Evidently, a rate theory is needed which takes account of inertial effects. It is the purpose of this paper to formulate such a theory. This is done by taking explicit notice of the effect of thermal fluctuations and of a viscous drag on both the thermally activated and the inertial overcoming of obstacles by dislocations. The rate-limited velocity is given by $v = d\Gamma$, where d is the average distance traveled after each activation and Γ is the activation rate. The enhanced plasticity is found to be an inertial effect that depends on the viscosity through d and not Γ while the temperature dependence is due to the familiar Arrhenius factor Γ. This approach is generally applicable to rate processes and is not restricted to dislocation motion. However, dislocation motion provides a favorable and possibly unique system because the required parameters in the theory, e.g., viscosity and normal frequencies, can be measured with internal friction, thereby providing a means for testing the theory. Furthermore, viscosity becomes a controllable parameter by using superconducting materials.

The rate of plastic flow and the internal friction of solid solutions is normally determined by the mobility of dislocations in the crystal. The average velocity of the dislocations depends upon the applied stress and the temperature and is limited by the largest force arising from four mechanisms: (1) the interaction with obstacles such as point defects and intersecting dislocations, (2) the interaction with parallel dislocations, (3) the interaction with a viscous medium, or (4) relativistic effects. The interaction with parallel dislocations through their mutual long-range stress fields is supposed to give rise to a background stress level which will be ignored here. The limits imposed by each of the other three mechanisms are sketched schematically in Fig. 1 for a crystal with obstacles of equal strength.

For low temperatures and for stresses just below the mechanical breakaway stress, the velocity depends strongly on the temperature and on the stress because the activation energy for overcoming the point obstacles is stress dependent. For low enough stresses, thermal backjumping over the obstacles occurs, and the net velocity becomes linear in the stress, given by an Einstein mobility.

For sufficiently high stresses or high enough tempera-

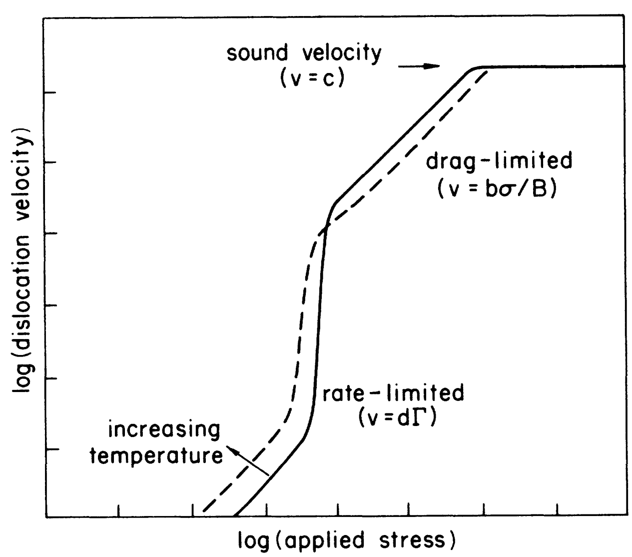

FIG. 1. Schematic relationship between the average dislocation velocity and the applied stress showing the mechanism that limits the velocity in each stress range.

tures so that the dislocation-point defect interaction is negligible, the velocity is limited by a viscous drag. The scattering of electrons and phonons as well as the radiation of elastic waves during acceleration provides an effective viscous force on the dislocation. The velocity is then linear with stress and the proportionality constant is inversely proportional to the damping. Since the phonon damping increases with temperature, the velocity in this region decreases with increasing temperature.

At very high stresses relativistic effects become important. The dislocation velocity then reaches a limiting value equal to the speed of sound in that medium. In this paper we consider only those stress regions for which relativistic effects are negligible.

To account for the enhanced plasticity in superconductors, Kojima and Suzuki[1] first suggested that the effect occurred in the drag-limited regime since the component of the viscosity B due to electron scattering would be absent in the superconducting state. However, that model could not explain the strain-rate dependence of the enhanced plasticity.[2] In the rate-limited regime, other workers have focused on changes in the dislocation activation rate Γ. Alers, Buck, and Tittman[2] proposed that the activation energy might be changed during the superconducting transition. Hutchinson and McBride[3] suggested that there might be a difference in the local effective temperature that depended on the superconducting state. Natsik,[4] Estrin,[5] and Suzuki[6] each suggested that the frequency factor might vary inversely with the viscosity. All of these proposals have been decisively ruled out by the observation that the superconducting effect vanishes at a temperature below T_c for pure samples but for dilute alloys vanishes sharply at T_c.[7]

The enhanced plasticity must therefore be due to changes in the average distance d traveled by dislocations after thermal breakaway. The inertial model[8-11] suggests a simple mechanism for this effect. Underdamped dislocations can overshoot an equilibrium position sufficiently to overcome an otherwise reflective potential barrier. This process is sensitive to changes in the viscosity. However, this is a purely mechanical model and does not account for the temperature dependence. Landau[12-14] assumed that a fraction g of obstacles could be overcome athermally through inertial effects whereas the remainder would be overcome by thermal activation. His expression for g was obtained by an empirical fit to computer simulation and was not based on physical principles.

In this paper the rate-limited velocity is derived from basic principles. It is shown that Γ is not sensitive to the viscosity for typical values for dislocations but that d is strongly dependent on the viscosity due to inertial effects. We also show that a distribution of obstacle strengths is both expected and necessary to explain the data. In a subsequent paper, these results will be applied to internal friction experiments and experimental data on superconducting and normal lead will be interpreted in terms of this model. A brief summary of this theory together with a discussion of its application to plastic flow and also to internal friction has been presented elsewhere.[15,16] On the other hand, more specific discussion of some aspects of the considerations given here can be found in Ref. 17.

II. DISLOCATION MOTION THROUGH AN ARRAY OF OBSTACLES

The motion of a dislocation through an array of obstacles has been studied by computer simulations[18-27] or by analytic techniques with many simplifying assumptions.[28-39] Most of these studies are based on a geometrical approach where stable dislocation configurations are calculated for some array of obstacles. The breakaway stress and the activation rate are then determined for each configuration. By averaging over a large number of random arrays of obstacles, an average dislocation velocity as a function of the stress is obtained.

In this paper dislocation motion is considered in terms of the potential energy of the dislocation as it moves through an array of obstacles. A similar approach has been used in the Granato-Lücke model[40-45] where only segregated obstacles were considered. In principle, a knowledge of the potential in which the dislocation moves, of the viscous drag forces, and of the fluctuation forces is sufficient to determine the motion of a dislocation.

The potential energy of a dislocation is multidimensional with the number of dimensions N equal to the number of degrees of freedom of the dislocation minus the number of constraints on the dislocation. The motion of a dislocation can be described in terms of a reaction path in an N-dimensional configuration space that follows the lowest energy trajectory as the dislocation moves through the array of obstacles. The potential along the reaction path will have minima, corresponding to the stable configurations, with saddle points between the minima. The reaction path near each equilibrium position will correspond to the lowest frequency normal mode of the dislocation. The magnitude of the reaction path coordinate corresponds to the average displacement of the dislocation. The remaining normal modes of the dislocation determine the entropy of the system. The problem is then reduced to consideration of a system moving in a one-dimensional potential together with a determination of the normal modes of the system.

As a dislocation moves along the reaction path, its motion is opposed by a viscous force arising from collisions with electrons and phonons. These collisions also produce a fluctuation force that tends to restore thermal equilibrium. The motion of a dislocation along the reaction path therefore corresponds to the Brownian motion of a particle in an external force field.

The general equations of Brownian motion have been developed by Kramers[46] and by Chandrasekhar.[47] They begin with Langevin's equation which is the equation of motion for a particle in an external force field and in thermal contact with the surrounding medium:

$$m\ddot{x} = F(x) + Bv + A(t) , \qquad (1)$$

where x is the position of the particle, v is its velocity, m is the mass, and $F(x)$ is the external force. The influence of the surrounding medium is separated into two parts: a systematic part Bv representing the viscous force and a fluctuating part $A(t)$. The function $A(t)$ has only statistically defined properties so that the solution to Eq. (1)

must be in the form of a probability $\rho(x,v,t)$ that the system is at a position x with a velocity v at time t. The differential equation governing the behavior of ρ becomes

$$\frac{\partial\rho}{\partial t}+v\frac{\partial\rho}{\partial x}-\frac{1}{m}\frac{\partial U}{\partial x}\frac{\partial\rho}{\partial v}=\beta\rho+\beta v\frac{\partial\rho}{\partial v}+\beta\frac{kT}{m}\frac{\partial^2\rho}{\partial v^2}, \quad (2)$$

where $\beta=B/m$ and $\partial U/\partial x=-F(x)$. For dislocations, the mass m is replaced with the mass per unit length A, the viscosity is the force per unit length Bv, and $F(x)$ is the force per unit length.

If the potential along the reaction path is known, then the solution to Eq. (2) describes the probability distribution of a dislocation in phase space. For a random distribution of obstacles in the glide plane of the dislocation, a detailed calculation of the reaction path of the dislocation and the potential energy along that path is exceedingly complex. The problem becomes tractable if we assume that the detailed form of the potential energy between the positions of stable and unstable equilibria is of secondary importance. The potential along the reaction path can then be approximated as a series of quadratic potential wells and barriers connected by a linear potential representing the driving force on the dislocation. Such a potential is shown in Fig. 2.

Since Kramers's equation given in Eq. (2) can be solved for a linear and a quadratic potential, the problem of dislocation motion can be solved by matching boundary conditions at the edges of the linear and quadratic regions. Solutions must be obtained for both the linear potential and the quadratic potential.

(1) Linear. This is equivalent to an obstacle-free region. The steady-state solution is a Gaussian with a mean value moving at a velocity $v=b\sigma/B$ with a velocity spread of $2kT/m$, where b is the Burger's vector and σ is the resolved shear stress on the glide plane. If the initial velocity is much lower or higher than the steady-state value, the equilibrium velocity will be attained in a time constant β. If the potential barriers and wells are relatively weak, this describes the average dislocation velocity and applies to the drag-limited regime.

(2) Potential well. Near a position of stable equilibrium the potential has a quadratic dependence. The steady-state solution is a stationary Gaussian of width $kT/m\omega_A^2$ where ω_A represents the curvature of the potential well. There is a finite probability that the system will attain

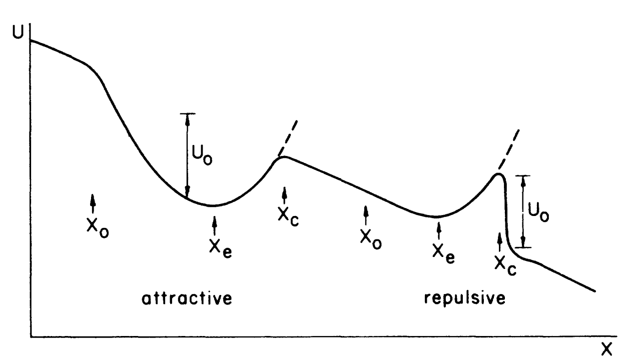

FIG. 2. Schematic of the potential energy along the reaction path of a dislocation for attractive and repulsive obstacles.

enough thermal energy to cross the potential barrier. If the system is not in equilibrium, but is initially displaced from equilibrium and is moving towards the equilibrium position, two very different solutions are obtained. First, the mean position of the probability distribution might come to rest at the equilibrium position in a time constant β either logarithmically or after a few oscillations. The steady-state solution then applies to determine the time before breakaway. Second, if the viscosity is sufficiently low, the mean position might overshoot the equilibrium position so that it actually reaches the top of the potential barrier. The dislocations that do reach the top of the barrier never come to rest at the equilibrium position in the potential well but cross the barrier and move to the next linear region. This is inertial overshooting of the barrier.

A. Viscosity effects in thermal activation

To obtain an analytical solution for the rate-limited velocity, we first consider a dislocation at equilibrium in a potential well to determine the rate at which dislocation motion is initiated. The activation rate is obtained by solving for the steady-state solution to Eq. (2) and calculating the rate at which the system crosses the barrier at $x=x_c$. This is given by[46]

$$\Gamma=\int_{-\infty}^{\infty} v\rho(x=x_c,v)dv, \quad (3)$$

where we have assumed that the probability ρ is normalized by the total number of particles in the system. Extending the solutions of Kramers[46] and Chandrasekhar[47] to the case of dislocations by including the Vineyard entropy factor,[48] we obtain

$$\Gamma=\nu\exp(-U/kT), \quad (4)$$

where

$$\nu=2\pi\omega_A\left[\frac{\alpha-\beta}{a}\right]^{1/2}\prod_{i=2}^{N}\frac{\omega_i}{\omega_i'}, \quad (5)$$

$$a=\frac{\beta}{2}+\left[\frac{\beta^2}{4}+\omega_C^2\right]^{1/2}, \quad (6)$$

U is the stress dependent activation energy, ω_A (ω_C) is the lowest normal mode frequency of the dislocation in the stable (unstable) equilibrium position, and the unprimed (primed) ω_i refer to the N normal mode frequencies in the stable (unstable) equilibrium position. If the damping is well below a critical value $B_c^v=2A\omega_C$, Eq. (5) becomes the result obtained by the usual transition-state method where $(a-\beta)/a-1$. If $B\gg B_c^v$, the transition-state result is decreased by a factor $2\omega_C/\beta$. This decrease occurs when the potential barrier is relatively flat and the strong interaction with the environment can lead to fluctuations that slow the forward movement, thereby reducing the activation rate. If the damping is small or the barrier is sharply peaked, this effect is negligible.

Equation (4) is a general expression for the activation rate across a potential barrier. It does not depend on the form of the potential between the stable and unstable equilibrium positions. It is valid for all values of β except for $\beta<0.5$ in which case the fluctuation forces are too small to maintain thermal equilibrium near the potential well.[49] For a dislocation mass per unit length of 10^{-14} g/cm, this corresponds to a damping coefficient $B<5\times10^{-15}$ dyn sec/cm^2. This is many orders of magnitude below that expected for dislocations.

B. Thermal fluctuations during inertial effects

After dislocation motion is initiated by thermal activation, the potential is approximately linear and the solution to Eq. (2) represents drag-limited motion until it encounters the next obstacle. This time of flight is short compared with the time required for thermal activation in the rate-limited regime and can be ignored. Equation (2) must then be solved for a quadratic potential with the initial condition of initial displacement from equilibrium at x_0 and an initial velocity v_0. The solution for ρ is a Gaussian in x and v with the mean value being the solution to the well known damped harmonic oscillator problem with $T=0$. For $\beta>2\omega_A$, the system is overdamped and equilibrium is reached in a time constant β. For $\beta<2\omega_A$, the system will overshoot the equilibrium position and has a finite probability of reaching the top of the potential barrier at $x=x_c$ and crossing the barrier. This can occur only at the first oscillation since subsequent oscillations will be smaller in magnitude. The time $t_{\max}$ when the maximum overshoot occurs is the moment when the mean velocity first goes to zero. As in the familiar harmonic oscillator, this is the first solution to the equation

$$\tan t_{\max}=\frac{v_0(1-D^2)^{1/2}}{\omega_A x_0(1+Dv_0/\omega_A x_0)} . \tag{7}$$

This solution holds strictly only if the potential continues to be parabolic beyond x_c as shown in the dotted curve in Fig. 2. To calculate the probability of crossing the barrier, the potential is assumed to be parabolic and that the probability of reaching x_c is equivalent to the probability of crossing the barrier. If the system overshoots the position x_c by a large amount the probability of crossing the barrier is one and there is no effect of having assumed a parabolic potential. If the system does not reach x_c then a parabolic potential beyond x_c is also not important. Only for the case where the maximum displacement of a system is near x_c will the detailed shape of the potential affect the solution. However, since the potential barrier is sharply peaked, as is evidenced by the large values of ω_C discussed below, this approximation is acceptable except at high temperatures where the parabolic potential assumed to exist beyond x_c can cause reflections that do not occur in reality. The value of the probability is therefore slightly underestimated but the essential dependence on the applied stress and viscosity is retained.

The probability of the system traveling beyond x_c is

$$P=\int_{-\infty}^{\infty}\int_{x_c}^{\infty}p(x,v,t=t_{\max})dx\,dv . \tag{8}$$

Using the expression for ρ obtained by Uhlenbeck[50,51] and substituting parameters applicable to dislocations, we have

$$P=\frac{1}{2}\left[1-\operatorname{erf}\left[\frac{\gamma-\sigma/\sigma_m}{\kappa}\right]\right] , \tag{9}$$

where

$$\kappa=\left[\frac{kT}{U_0}\right]^{1/2}(1-e^{-2\pi D'})^{-1/2}(1+e^{-\pi D'})^{-1} , \tag{10}$$

U_0 is the activation energy at zero stress, $D'=D(1-D^2)^{-1/2}$, $D=B/B_c$, and B_c is the critical damping for inertial overshoot given by $B_c=2A\omega_A$. The stress at which the potential well and barrier disappear is the mechanical breakaway stress σ_m. The value of γ depends on the initial velocity and the type of obstacle. For attractive obstacles there is a brief acceleration as the dislocation nears the obstacles. This is significant only for low initial velocities. If we assume that

$$v_0<0.01(1+\sigma/\sigma_m)c , \tag{11}$$

where c is the sound velocity, then

$$\gamma=\tanh(\pi D'/2) . \tag{12}$$

For high initial velocities and for repulsive obstacles at any velocity, the expression is

$$\gamma=(1+e^{-\pi D})^{-1} . \tag{13}$$

The probability P is a thermally broadened step function centered at $\sigma=\gamma\sigma_m$. For $\sigma>\sigma_m$ there is no potential barrier and P is one; in this case, the assumption of a parabolic potential is not valid. The probability P is plotted as a function of σ/σ_m in Fig. 3 for various values of the temperature and for a given value of underdamping. This is the probability that a dislocation will overcome an obstacle due to inertial overshooting of the equilibrium position. As the damping increases, the curves move to larger stresses until P is a step function at $\sigma=\sigma_m$ for the overdamped case. For periodic potentials, a similar

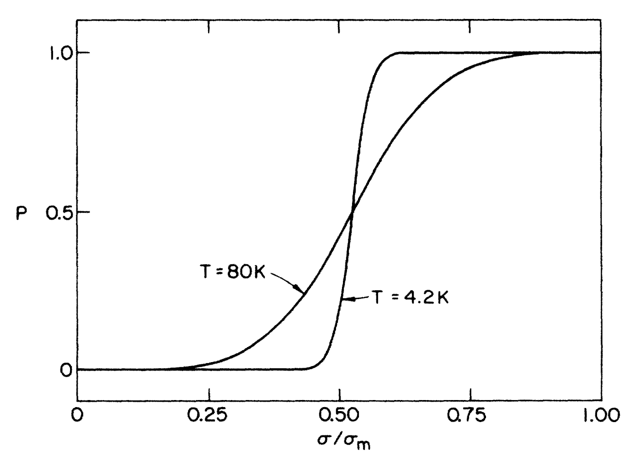

FIG. 3. The probability P of inertially overcoming an attractive obstacle of strength σ_m as a function of σ/σ_m for $D=0.35$ and zero initial velocity at two different temperatures.

broadening effect due to thermal fluctuations has also been discussed by Vollmer and Risken.[52]

The expressions obtained by Granato,[8,9] Suenaga and Galligan,[10] and by Kamada[11] for mechanical inertial effects correspond to a probability P that is a true step function at $\sigma=\gamma\sigma_m$ with γ given by Eq. (13). Their result therefore holds only for repulsive obstacles at zero temperature. For attractive obstacles and low initial velocities, the attractive part of the interaction must also be included to obtain Eq. (9).

The average distance a dislocation will travel is given by the mean distance d_0 between obstacles divided by the probability of being trapped by an obstacle so that

$$d=d_0(1-P)^{-1} . \quad (14)$$

As expected, if $P=0$ the dislocation is trapped at every obstacle and $d=d_0$, and if $P=1$ the dislocation will overcome every obstacle and $d=\infty$. In a real crystal the maximum distance a dislocation can travel is limited by the size of the crystal or by the interaction with other dislocations. If the average maximum distance is d_1 then Eq. (14) can be written as

$$d=d_0[r+(1-r)(1-P)]^{-1} , \quad (15)$$

where $r=d_0/d_1$.

C. Viscosity effects on the velocity

The frequencies ω_A and ω_C are important parameters in this theory. They not only appear quantitatively in the activation rate but represent critical criteria for two viscosity effects. First, for $B<B_c=2A\omega_A$ the dislocation is in the underdamped condition in the potential well and inertial effects can occur. Second, $B>B_c^v=2A\omega_C$ the frequency factor is reduced by the viscosity.

These two normal mode frequencies can be calculated as a function of the loop length by analyzing a dislocation with evenly spaced pinnors.[17] Each pinnor binds the dislocation with a force constant f. Defining the dislocation line tension as μ and the loop as l, the equation of motion for a dislocation can be solved for an equilibrium value and oscillations about equilibrium. Two equilibrium positions are obtained: the stable equilibrium position at the bottom of the potential well and the unstable position at the top of the potential barrier. In each case, N normal modes are obtained where N is the loop length divided by the lattice constant. The value of ω_A corresponds to the lowest frequency in the stable equilibrium case and is the first solution to the transcendental equation[43]

$$\tan\theta=-\frac{2\mu}{fl}\theta , \quad (16)$$

where $\theta=\omega l/c$. The value of ω_C corresponds to the lowest frequency in the case of unstable equilibrium and is the only imaginary value, corresponding to a potential barrier rather than a well. Its magnitude is given by the single imaginary solution to the transcendental equation

$$\cot\frac{\theta}{2}=\frac{2\mu}{fl}\theta . \quad (17)$$

An imaginary solution exists only for those values of the pinning constant where an unstable equilibrium is possible. The real solutions to Eq. (17) contribute to the entropy factor as shown in Eq. (5).

Inserting typical values for dislocation parameters, these solutions are used to evaluate the critical damping criteria which are plotted in Fig. 4 as a function of the loop length. Three distinct regions occur. For high damping, the activation rate, but not the distance traveled, depends on B. For intermediate damping, the velocity does not depend on B. For low damping, the distance traveled, but not the activation rate, depends on B through inertial effects.

The lowest damping a dislocation can have is limited by radiation damping and is shown as the lower limit in the bottom of Fig. 4. The phonon damping increases linearly with temperature and even near the melting temperature is well below B_c^v as indicated by the upper dashed line. From this figure it is clear that the activation rate will normally not depend on the damping for dislocations. The inertial effect on the distance a dislocation travels will occur only if the loop lengths are sufficiently short or if the damping is low enough.

III. DISTRIBUTION OF OBSTACLE STRENGTHS

A system of a large number of dislocations in a random array of obstacles would be expected to exhibit a range of activation energies rather than a single value as indicated in Eq. (4). The mechanical breakaway stress σ_m depends not only on the strength of the dislocation-point defect interaction but also on the dislocation loop length and the degree of collinearity. Internal friction measurements clearly demonstrate such a range in the amplitude dependence of the hysteresis loss. With increasing stress amplitude, more dislocations with smaller loop lengths are broken away.

The experimental observation of enhanced plasticity in the superconducting state also implies the existence of a range of breakaway strengths. In the limit $T\to 0$ dislocation motion can be initiated only if $\sigma\to\sigma_m$, independent

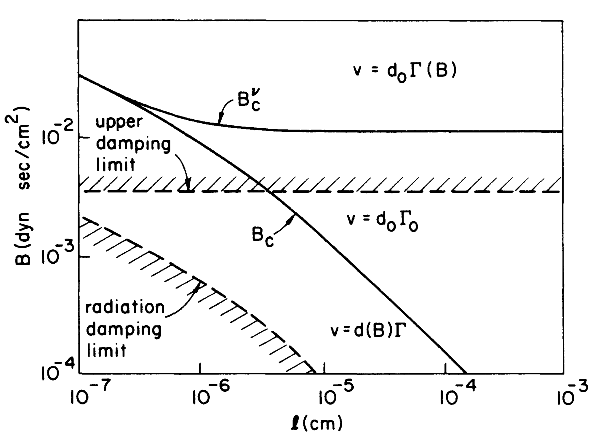

FIG. 4. Diagram showing the three regimes of velocity dependence on viscosity, depending on the value of the damping coefficient and the dislocation loop length.

of the damping. However, it is observed that plastic strain occurs over a wide range of the applied stress before and after the superconducting transition even at very low temperatures. It is therefore important to consider a distribution of activation energies.

A distribution of dislocation breakaway strengths is important in this theory in two different places. First, the rate at which dislocations are broken away must be summed over all possible activation energies. If we define the function $N(\sigma_m)d\sigma_m$ as the fraction of the mobile dislocation density Λ which are in a stable configuration having a mechanical breakaway stress between σ_m and $\sigma_m + d\sigma_m$, then the rate of plastic strain is

$$\dot{\epsilon} = \Lambda b v = \Lambda b d \int_0^{\infty} N \Gamma d\sigma_m \ . \tag{18}$$

The distribution N is a strong function of the applied stress and of time as well as the concentration and type of impurities. No dislocation can be found in a configuration for which $\sigma_m < \sigma$ so that N shifts to higher values as the applied stress increases. Three factors which influence the rate of change of $N(\sigma_m)$ are the rate at which dislocations are unpinned from configurations of strength σ_m, the rate at which dislocations are repinned by configurations of strength σ_m, and the possible change of σ_m due to a shift in the internal stress arising from the interactions with other dislocations. This distribution function is complicated and is not known; no attempt will be made to derive it here. To illustrate the effect of the width of the distribution, a very crude approximation will be made in this article in which $N(\sigma_m)$ is characterized by a mean value σ_m^0 and by a relative width η.

The second factor that depends on a distribution of pinning strengths is the average distance traveled. The relevant distribution function is $P_e(\sigma_m)d\sigma_m$ which is the probability that a potential barrier encountered by a dislocation after breakaway has a mechanical breakaway strength between σ_m and $\sigma_m + d\sigma_m$. The average distance traveled is then

$$d = d_0 \left[r + (1-r) \int_0^{\infty} (1-P) P_e(\sigma_m) d\sigma_m \right]^{-1} . \tag{19}$$

If $P_e(\sigma_m)$ is a δ function we again obtain Eq. (15). The function $P_e(\sigma_m)$ is essentially independent of the applied stress since σ_m depends only on the dislocation configuration and the obstacle interaction energy. If P_e varies slowly near the breakaway stress, then thermal broadening of the trapping probability P is effectively canceled in the integral in Eq. (19). Although thermal fluctuations can assist in the inertial overcoming of stronger obstacles, they can also reduce the probability of overcoming weaker obstacles. These two effects essentially cancel and thermal broadening effects can be ignored. Equation (19) is plotted in Fig. 5 as a function of the stress where $1-P$ is taken to be a step function at $\sigma = \gamma \sigma_m$ and P_e is a Gaussian function with a mean value σ_m^0 and a width γ.

The importance of the distribution of obstacle strengths can be illustrated at very low temperatures where Γ can be taken to be a step function at $\sigma = \sigma^*$ where σ^* is the thermal breakaway stress.[44,53] If the applied stress is constant, then the integral in Eq. (18) is zero since $N(\sigma)$ is zero for $\sigma^* < \sigma_m$ and Γ is zero for $\sigma^* > \sigma_m$. However, if the stress is increased from $\sigma + d\sigma$ in the time interval dt, then the strain rate becomes

$$\dot{\epsilon} = \Lambda b \, dN(\sigma^*) \dot{\sigma}^* \ . \tag{20}$$

If the elastic strain is added to the plastic strain in Eq. (20), then integration yields the stress-strain relation

$$\epsilon = \sigma / G + \Lambda b \int_0^{\sigma^*} dN(\sigma') d\sigma' \ , \tag{21}$$

where G is the shear modulus. For tensile stresses a resolved shear stress factor must also be included. Taking $N_0(\sigma_m)$, the initial value of $N(\sigma_m)$ at zero stress, and $P_e(\sigma_m)$ to be a Gaussian function with a mean value σ_m^0 and a relative width η, Eq. (21) is plotted in Fig. 6 for both overdamped and underamped dislocations for $r = \frac{1}{10}$. Equation (21) can be understood as the total num-

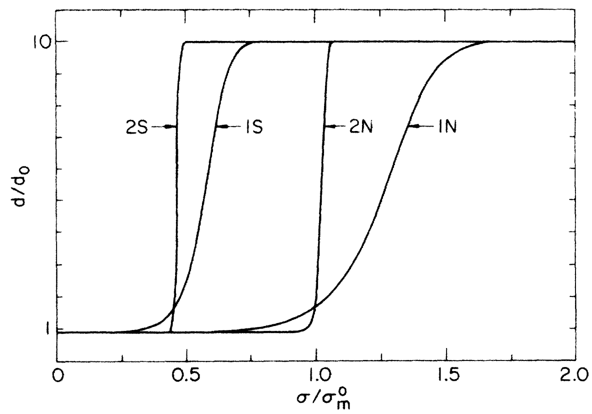

FIG. 5. The average distance traveled by a dislocation as a function of the applied stress for different values of the damping parameter D and of the width η of the distribution of obstacle strengths for $r = 0.1$. 1N: $D = 1.0$, $\eta = 0.2$; 1S: $D = 0.3$, $\eta = 0.2$; 2N: $D = 1.0$, $\eta = 0.02$; 2S: $D = 0.3$, $\eta = 0.02$.

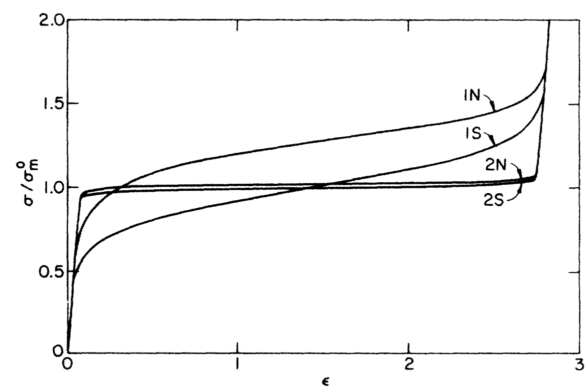

FIG. 6. Calculated stress-strain curve for the normal and superconducting states for two different widths η of the distribution of obstacle strengths. 1N: $\eta = 0.2$, overdamped case; 1S: $\eta = 0.2$, underdamped case; 2N $\eta = 0.02$, overdamped case; 2S: $\eta = 0.02$, underdamped case.

ber of dislocations that break away as the stress is increased from 0 to σ times the distance each dislocation travels. Inertial effects are evident because of the impact on the average distance traveled. Thermal effects are evident because the total number of dislocations broken away is increased at higher temperatures.

IV. SUMMARY

Dislocation motion has traditionally been understood as being either drag limited or rate limited. The observation of enhanced plasticity in superconductors provided clear evidence that viscosity and thermal effects are both important at the same time. Although this enhanced plasticity has been described phenomenologically in terms of the inertial model, there was no fundamental derivation of both viscous and thermal effects. In this paper the basic equations of Brownian motion were applied to dislocation motion. Solutions to Kramers's equation were shown to describe drag-limited motion, viscous effects in thermally-activated rate-limited motion, and thermal fluctuations in the viscosity-dependent inertial effects. Three regimes of behavior were observed. For high values of the damping the frequency factor is reduced, for low values inertial effects are important, and for intermediate values there is no dependence on the viscosity. For dislocations a "phase diagram" can be drawn (see Fig. 4) showing the ranges of viscosity and loop length for each type of behavior. For typical values for dislocations, the dominant viscosity effect is the change in the average distance traveled due to inertial effects. The theory developed here is not restricted to dislocation motion but applies generally to any system subject to these types of viscous interactions. Dislocation motion is a favorable system for studying motion in a viscous medium since the damping is known and can be changed in the case of superconducting materials. However, due to the large number of dislocations in a crystal, statistical effects of a distribution of loop lengths and obstacle strengths must be considered. Internal friction measurements of dislocation motion have the additional benefit of reproducibly and nondestructively probing the nature of this motion. In a subsequent paper, internal friction measurements in lead will be discussed in terms of this theory.

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