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Sensitivity of the conductance to impurity configuration in the clean limit

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The sensitivity of the conductance of a metal to a short-range change in the scattering potential is calculated for an arbitrary value of the disorder. The universal-conductance-fluctuation result is reproduced when the elastic scattering rate is much larger than the inelastic scattering rate. In the opposite limit the change in the Born-approximation elastic scattering rate produces the largest effect. These formal results are applied to the case of impurities moving within a random distribution of impurities, and the estimated conductance fluctuation is compared to those observed in $(1/f)$-noise experiments on metal films.

I. INTRODUCTION

Quantum interference causes the low-temperature conductance of small metal samples to be very sensitive to the distribution of impurities and other elastic scatterers.[1–5] This sensitivity was first seen in magnetoconductance experiments, where the conductance oscillated aperiodically as a function of the applied magnetic field.[1] Similar oscillations have also been seen in the conductance as a function of the chemical potential.[6,7] Both kinds of experiments effectively change the distribution of impurities. Varying the magnetic field changes the phase of the electrons, while varying the chemical potential changes the electrons' wavelength. Because the change in the conductance is in many cases of order e^2/h, this effect has been called the universal conductance fluctuation. The most direct way to see this sensitivity, of course, would be to move impurities within the sample. It has long been postulated that elastic scatterers do move, causing the conductance to change as a function of time. According to recent theoretical estimates, moving just one impurity a few Fermi wavelengths should be observable.[8,9] Discrete switching in the conductance of metal samples as a function of time has been seen.[7,10]

As one increases the temperature, the aperiodic oscillations as a function of the magnetic field and the chemical potential as well as the discrete conductance switching disappear. This is caused by the sample effectively consisting of many different samples which are separated either in space (by an inelastic length L_{in}) or in energy (by an inelastic energy $\hbar\Gamma_{in}$). The conductance of each of these smaller samples oscillates independently, so that the total change in the conductance averages out to be very small. Even though the above effects are no longer visible, quantum interference may still be occurring. Feng, Lee, and Stone[9] have pointed out that the quantum-interference amplification of the change in the conductance due to the motion of defects might be seen in the power spectrum of the conductance fluctuations as a function of time at room temperature. In particular, they propose that the magnitude of room-temperature $1/f$ noise in disordered metals is due to the interference effect of the universal conductance fluctuations. If this were true, then a major problem in $1/f$ noise would be solved. Unfortunately, most, though not all, of the $(1/f)$-noise experiments in metals done to date have been in the regime where the inelastic scattering rate Γ_{in} is greater than the elastic scattering rate Γ_{el}. Universal conductance fluctuation calculations assume that $\Gamma_{el} \gg \Gamma_{in}$. Thus, the theory is in the wrong regime for most experiments.[11]

The purpose of this paper is to examine the role of interference effects on the sensitivity of the conductance to impurity configuration for all values of the disorder, Γ_{el}/Γ_{in}, not just for $\Gamma_{el}/\Gamma_{in} \gg 1$. The effect which is important in the clean limit is the correction to the Born-approximation scattering rate which enters into the Drude conductivity. Because the wave nature of the electron is implicit in the Born approximation, this is a quantum-mechanical effect. For the case of one impurity moving in a random distribution of impurities considered here this is particularly apparent, since classically there is no effect unless the total number of scatterers changes. Other cases within the same formalism, such as the charging of an interfacial state,[12] may be understood classically.

Since both the universal-conductance-fluctuation and Born-approximation-scattering effects have been treated in detail elsewhere, the emphasis here is on showing how the two effects come from the same formalism and on showing the similarities and differences between them. In Sec. II, where the formal results are presented, the universal-conductance-fluctuation result is shown to be due to a change in an effective dephasing rate by roughly the same amount as the change in the net scattering rate in the clean limit. Since this change depends on the microscopics of the scattering potential, the time-dependent universal conductance fluctuations are not universal. Estimates of the magnitude of the conductance change in Sec. III show that for the same mean-square magnetoconductance the time-dependent conductance fluctuations can vary by a factor of 10. In this same section the noise magnitudes produced by impurities moving within a random distribution of impurities is estimated and shown to be smaller than those typically observed in $1/f$ noise, but still larger than the universal-conductance-fluctuation result until $\Gamma_{el} > 10\Gamma_{in}$. In Sec. IV, I conclude and point to some recent work which may show why the $(1/f)$-noise magnitudes predicted here are too small.

II. FORMAL RESULTS

In this section the change in the conductance resulting from a short-range change in the potential is calculated. Short range here means that the potential only changes in a region much smaller than a mean free path. For example, one can imagine a point defect moving[9] or two defects rotating about one another.[13] In addition to depending on the change in the potential, the conductance change also depends on the environment around the potential. In this calculation the environment is put in by averaging over the positions of point scatterers, yielding the average or typical change in the conductance.

The linear-response expression for the conductance G of a sample of length L_z in the direction of the electric field and current is given by

$$G=\frac{1}{L_z^2}\int d^3\mathbf{x}\int d^3\mathbf{x}'\sigma_{zz}(\mathbf{x},\mathbf{x}') . \tag{2.1}$$

The nonlocal conductivity, σ_{zz}, is evaluated via perturbation theory. Although we are interested in finite temperatures with inelastic scattering, it is useful for the purpose of understanding the terms in the perturbation theory to consider the largest contribution to σ_{zz} for the special case of free electrons at zero temperature,

$$\sigma_{zz}(\mathbf{x}_1,\mathbf{x}_2)=2\frac{e^2}{h}\left[\frac{\nabla_1-\nabla_{1'}}{2mi}\right]_z\left[\frac{\nabla_2-\nabla_{2'}}{2mi}\right]_z \mathrm{am}(\mathbf{x}_{2'}\to\mathbf{x}_1)\mathrm{am}^*(\mathbf{x}_2\to\mathbf{x}_{1'})\Big|_{\mathbf{x}_1'=\mathbf{x}_1,\ \mathbf{x}_2'=\mathbf{x}_2} . \tag{2.2}$$

This expression for σ_{zz} is just the net amplitude squared for an electron at the Fermi energy to go from $\mathbf{x}_2$ to $\mathbf{x}_1$, $|\mathrm{am}(\mathbf{x}_2\to\mathbf{x}_1)|^2$, with two sets of derivatives to indicate the current and electric field directions. Since the net amplitude is the sum of the amplitudes for all possible paths to go from $\mathbf{x}_2$ to $\mathbf{x}_1$, each term in the perturbation theory represents a particular choice of an amplitude and a complex conjugate of an amplitude.[14] In Fig. 1, I have drawn schematically two different pairs of amplitudes and complex conjugates of amplitudes. The circles in this figure represent the region whose potential changes from δv to $\delta v'$, and the dots represent point defects whose positions will be averaged over in order to get the average potential change.

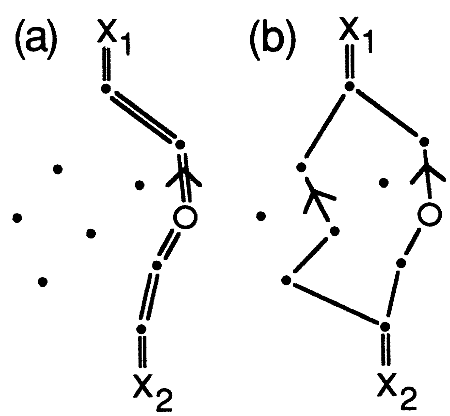

FIG. 1. Schematic of paths for the amplitudes in (a) the Born-approximation change in the scattering rate and (b) the universal-conductance-fluctuation effect.

The two kinds of paths shown in Fig. 1 yield two fundamentally different kinds of interference effects. In Fig. 1(a) the am and am* paths are identical. Summing these terms produces the "classical" result because we have summed the squares of the amplitude for each path instead of summing the amplitudes first and then squaring the result.[15] This result is not completely classical, however, because it still contains the information that it is an electron wave which is being scattered. Interference does take place on the length scales of δv. The terms of Fig. 1(b), on the other hand, involve interference between different paths . When one averages over the positions of the point scatterers, these terms average to zero because of the random-phase difference between the two paths, leaving only the terms of Fig. 1(a). Indeed defining G and G' to be the conductance with δv and $\delta v'$, respectively, for a particular configuration of the point scatterers and letting angular brackets denote an average over the positions of the point scatterers, the change in the conductance for the amplitudes of Fig. 1(a) is

$$(\delta G_1)^2=(\langle G\rangle-\langle G'\rangle)^2 . \tag{2.3}$$

The terms of Fig. 1(b) are, of course, not identically zero, but only zero on average. In order to estimate them we must first square the conductance change, $G'-G$, and then average over the positions of the point scatterers. Subtracting off the average result of Eq. (2.3), we get, for the amplitudes of Fig. 1(b),

$$(\delta G_2)^2 = \langle (G-G')^2 \rangle - (\langle G \rangle - \langle G' \rangle)^2 . \tag{2.4}$$

Summing the two conductance changes of Eqs. (2.3) and (2.4) yields the net conductance change, $\langle (G-G')^2 \rangle$.

The lowest-order current-conserving approximation for the average conductance in the presence of impurity-averaged elastic scattering and electron-phonon scattering is the sum of the ladder graphs. One of the relevant diagrams is shown in Fig. 2(a). The dashed lines represent elastic scattering from the point defects, and the wavy lines represent electron-phonon scattering, which has been chosen because it is the dominant form of inelastic scattering at high temperatures. The conductance obtained by summing these diagrams is

$$\langle G \rangle = \frac{V}{L_z^2} \frac{ne^2}{m\Gamma_{\text{net}}} , \tag{2.5}$$

where the net scattering rate, Γ_{net}, is the sum of the elastic and inelastic scattering rates, and the volume of the sample is V. Now if we add to our system a scatterer represented by the local potential, δv, and average over its position, we will get another two-body interaction which must be summed in the ladder approximation. In Fig. 2(b), I show some of the new kinds of diagrams. The line with a circle in the middle represents scattering from δv. The resulting set of diagrams may be summed exactly in the absence of inelastic scattering. In the presence of inelastic scattering one can write a self-consistent equation to sum the diagrams, which in the case of electron-phonon scattering reduces to the linearized quasiclassical equation.[16] This equation can then be solved approximately.[17,18] If Γ_{net} is the scattering rate with δv then changing to $\delta v'$ can be accounted for by

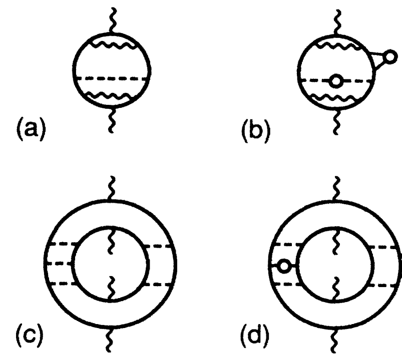

FIG. 2. Typical diagrams for computing (a) the mean conductance, (b) the mean conductance in the presence of δv or $\delta v'$, (c) the mean-square variation in the conductance, and (d) the mean-square conductance variation in the presence of δv and/or $\delta v'$.

$$\Gamma_{\text{net}} \to \Gamma_{\text{net}} + \delta\Gamma_1 , \tag{2.6}$$

$$\delta\Gamma_1 = 2\pi N(0) \int \frac{d\Omega_{\hat{\mathbf{k}}}}{4\pi} \int \frac{d\Omega_{\hat{\mathbf{k}}'}}{4\pi} \tfrac{3}{2}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')_z^2 \frac{|\delta v'(k_F(\hat{\mathbf{k}}-\hat{\mathbf{k}}'))|^2 - |\delta v(k_F(\hat{\mathbf{k}}-\hat{\mathbf{k}}'))|^2}{V} . \tag{2.7}$$

Using $\delta\Gamma_1 \ll \Gamma_{\text{net}}$ we can expand the denominator to obtain our final result:

$$(\delta G_1)^2 = \left[\frac{V}{L_z^2} \frac{ne^2}{m\Gamma_{\text{net}}} \right]^2 \left[\frac{\delta\Gamma_1}{\Gamma_{\text{net}}} \right]^2 . \tag{2.8}$$

This result for $\delta\Gamma_1$ can be obtained more simply using the relation that the inverse of the elastic mean free path, Γ/v_F, is the density of scatterers times their cross section. For the one scatterer the density is just V^{-1}. Using the Born-approximation cross section,[19] one obtains Eq. (2.7), except for the factor of $\frac{3}{2}(\hat{\mathbf{k}}-\hat{\mathbf{k}}')_z^2$ for the transport lifetime. In the case when δv is a function only of $|\hat{\mathbf{k}}-\hat{\mathbf{k}}'|$, this factor is the familiar $1-\cos\theta = 1-\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'$.

The diagrams for computing $\langle G^2 \rangle - \langle G \rangle^2$ contain two loops which are averaged together.[3,4] Detailed analysis using current conservation shows that to a good approximation it is only necessary to keep diagrams such as that shown in Fig. 2(c). Since we are interested in zero magnetic field, both the diagrams with the directions of the Green's functions parallel and antiparallel must be kept. Summing these two sets of diagrams for $\langle G^2 \rangle - \langle G \rangle^2$ in the limit where the elastic scattering rate is much larger than the dephasing rate yields the universal-conductance-fluctuation result,

$$\langle G^2 \rangle - \langle G \rangle^2 = c_d \frac{V_d L_\varphi^{4-d}}{L_z^4} \left[\frac{e^2}{h} \right]^2 , \tag{2.9}$$

where $c_d = 4/\pi$, $8/\pi$, and 8 and V_d is volume, area, and length for three-, two- (or quasi-two-), and quasi-one-dimensional samples, respectively. Samples are considered to be quasi-two- or quasi-one-dimensional if one or two of the dimensions is much less than the dephasing length, $L_\varphi = (D/\Gamma_\varphi)^{-1/2}$. The dephasing rate, Γ_φ, is in many cases just the inelastic scattering rate, Γ_{in}, but it can also include other kinds of scattering, such as scattering from magnetic impurities. Since the inelastic scattering rate at room temperature is large, I will in many cases substitute Γ_{in} for Γ_φ when estimating the conductance fluctuations at room temperature. Also, at room temperature $k_B T$ is approximately $\hbar\Gamma_\varphi$, so that the factor, $\hbar\Gamma_\varphi/k_B T$, to account for energy averaging does not have to be included in Eq. (2.9).

Adding the scatterer represented by δv and/or $\delta v'$ to the system produces additional interactions or correlations, which are again denoted by lines with a circle in the center. As shown in Fig. 2(d) these must be inserted into Fig. 2(c) both as self-energy corrections and as correlations between the two measurements. When this is done, the dominant contribution to $(\delta G_2)^2$ of Eq. (2.4) can be accounted for by a change in the dephasing rate[8] in Eq. (2.9):

$$\Gamma_\varphi \to \Gamma_\varphi + \delta\Gamma_2 , \tag{2.10}$$

$$\delta\Gamma_2 = 2\pi N(0) \int \frac{d\Omega_{\hat{k}}}{4\pi} \int \frac{d\Omega_{\hat{k}'}}{4\pi} \frac{1}{2} \frac{|\delta v'(k_F(\hat{\mathbf{k}}-\hat{\mathbf{k}}')) - \delta v(k_F(\hat{\mathbf{k}}-\hat{\mathbf{k}}'))|^2}{V} . \tag{2.11}$$

That the change in the potential increases the effective dephasing rate is appealing because both changing the potential and increasing the dephasing rate reduce the correlations between the different measurements, G and G'. Expanding Eqs. (2.4) and (2.9) to get $(\delta G_2)^2$, we obtain as our final result

$$(\delta G_2)^2 = c_d \frac{V_d L_\varphi^{4-d}}{L_z^4} \left[\frac{e^2}{h}\right]^2 \left[1 - \left[1 + \frac{\delta\Gamma_2}{\Gamma_\varphi}\right]^{-(4-d)/2}\right] \approx c_d \frac{V_d L_\varphi^{4-d}}{L_z^4} \left[\frac{e^2}{h}\right]^2 (4-d) \frac{\delta\Gamma_2}{\Gamma_\varphi} . \tag{2.12}$$

This is precisely the result of Al'tshuler and Spivak[8] for samples greater in length than L_φ and within a factor of order unity of the result of Feng, Lee, and Stone.[9] The case when the sample is longer than a dephasing length is the only relevant case for experiments because even when the lithographical sample is smaller than L_φ, the effective sample length for the universal conductance fluctuations is still of order L_φ.[20,21] The first line of Eq. (2.12) explicitly includes the saturation of δG_2 at the static-conductance-fluctuation value of Eq. (2.9). This saturation is important for the noise not only because it limits the magnitude, but also because it affects the power spectrum.[22]

III. QUALITATIVE FEATURES AND ESTIMATES

There are several important differences and similarities between the Born-approximation result of Eq. (2.8) and the universal-conductance-fluctuation result of Eq. (2.12). First, the change in the conductance in the Born approximation comes from a change in the total scattering rate Γ_{net}, while for the universal conductance fluctuations it comes from a change in the dephasing rate Γ_φ. These two origins lead to different powers of $\delta\Gamma$ in $(\delta G_1)^2$ and $(\delta G_2)^2$. Since in either case $\delta\Gamma/\Gamma$ is a small number, this might lead one to believe that the Born-approximation result, which is proportional to $(\delta\Gamma_1/\Gamma_{\text{net}})^2$, is much smaller than the universal-conductance-fluctuation result, which is proportional to $\delta\Gamma_2/\Gamma_\varphi$. The prefactor in the Born-approximation result, $\langle G\rangle^2$, however, is much larger than the prefactor in the conductance-fluctuation result, $\sim(e^2/h)^2$, counteracting the different powers of the $\delta\Gamma$. As shown below, either effect can dominate, depending on the relative magnitudes of Γ_φ and Γ_{el}.

In three dimensions both of the effects are independent of the volume of the sample once one has properly taken into account the factor of the V^{-1} hidden in the $\delta\Gamma$'s. For quasi-one- and quasi-two-dimensional samples the conductance-fluctuation result does have additional sample-size dependence, leading to an enhancement over the three-dimensional value by L_φ^2/wt and L_φ/t, respectively. The thickness and width of the samples are denoted here by t and w. Both of the effects are also proportional to L_z^{-4}.

The universal conductance fluctuations are strongly dependent on the magnetic field. For example, if one had a two-level system which switched between the potentials δv and $\delta v'$, then for some fields the state represented by δv would have the larger conductance while for other fields the state represented by $\delta v'$ would have the larger conductance. The Born-approximation result, on the other hand, has no such field dependence. One state would always have the larger conductance. When large numbers of switching events are happening at the same time, such as for room-temperature $1/f$ noise, the magnetic field dependence of the universal-conductance-fluctuation effect averages to zero because the magnetic field affects differently the δG associated with each microscopic change in the potential.

Both effects depend on the microscopics of the change in the potential. For the Born-approximation result this is not surprising, but it is also true for the universal conductance fluctuations. To illustrate this I contrast the change in the conductance due to the universal conductance fluctuations for an impurity moving in a typical metal wire of Webb *et al.*[1] and for the charging of an interfacial site in a silicon metal-oxide-semiconductor field-effect transistor (MOSFET).[7] In the MOSFET the fluctuation is of order e^2/h, while for one of the metal wires the effect is 1–2 orders of magnitude smaller.

From Eqs. (2.10) and (2.11) the motion of a defect with potential δv a few Fermi wavelengths increases Γ_φ by the scattering rate for that defect alone,

$$\delta\Gamma_2 \sim \Gamma_{\text{el}}/N_{\text{imp}} . \tag{3.1}$$

This rate can be estimated by the cross section for one scatterer. I use the cross section of a screened Coulomb impurity in gold, which is approximately $4\pi k_F^{-2}$. The mean-square conductance fluctuation, $(\delta G_2)^2$, is equal to the static conductance fluctuation of Eq. (2.9) times $\delta\Gamma_2/\Gamma_\varphi$. Using the smallest sample length of $L_\varphi = 2\ \mu\text{m}$ at 40 mK (see Ref. 20), an elastic mean free path of 170 Å obtained from the resistivity of Ref. 1, and an area of the wire of $(400\ \text{Å})^2$, this ratio is

$$\frac{\delta\Gamma_2}{\Gamma_\varphi} = 3 \left[\frac{L_\varphi}{L_{\text{el}}}\right] \left[\frac{a}{wt}\right] \sim \frac{1}{100} , \tag{3.2}$$

giving a fluctuation of order $0.1e^2/h$. At 4 K, δG_2 will be reduced by another order of magnitude due to energy averaging. Since the motion of defects due to thermal activation is more likely to be seen at 4 K than at 40 mK, the motion of one atom may be difficult to detect in this system.

Larger fluctuations are possible in silicon MOSFET's because of the longer screening length in semiconductors. Even at 100 K discrete switching due to the charging of an interfacial trap has been seen.[12] Since from Eqs. (2.7) and (2.11) $\delta\Gamma_2$ is approximately equal to $\delta\Gamma_1$ for the appearance of a scatterer ($\delta v=0$, $\delta v'\neq 0$), the change in the scattering rate at high temperatures can be used to predict the magnitude of δG at low temperatures. The high-temperature experiments on 0.1×1-μm^2 silicon $\langle 100\rangle$ MOSFET's with mobilities of 2000 $cm^2/V\,s$ saw fractional changes of the resistance from 0.1% to 0.7%. A fractional change in the resistance of 0.4% corresponds to a change in the scattering rate of $1.5\times 10^{10}\ s^{-1}$. The sample in which Skoçpol *et al.*[7] saw switching at 2 K was smaller than this ($0.06\times 0.3\ \mu m^2$), so that $\delta\Gamma_2$ is increased to $8.3\times 10^{-10}\ s^{-1}$. For the same sample the dephasing length was 0.7 μm, corresponding to an inelastic scattering rate of $3.4\times 10^{11}\ s^{-1}$,

$$\delta\Gamma_2/\Gamma_\varphi \sim \tfrac{1}{4} \ . \tag{3.3}$$

Substituting into Eq. (2.12) gives the observed magnitude, while the Born-approximation change in the conductance with the same $\delta\Gamma$ is a factor of 3.5 smaller. Thus, for MOSFET's it is possible to have δG of order e^2/h.

For the Born-approximation effect, unlike the universal-conductance-fluctuation effect described above, it is essential that the moving defect lie near another defect for there to be a large change in the conductance. If two defects with potential $v(\mathbf{k})$ are separated by $\mathbf{R}$, then their net potential is

$$\delta v(\mathbf{k}) = v(\mathbf{k}) + v(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{R}} \ . \tag{3.4}$$

The structure factor causes the cross section to depend on the separation between the defects instead of just being the sum of their individual cross sections. Consequently, changing the separation between the defects from $\mathbf{R}$ to $\mathbf{R}+\delta\mathbf{R}$ causes the net scattering rate to change by

$$\delta\Gamma_1(\delta\mathbf{r}) = \Gamma(\mathbf{R}+\delta\mathbf{r}) - \Gamma(\mathbf{R}) \ , \tag{3.5}$$

$$\Gamma(\mathbf{R}) = 2\pi N(0)\int_{|\mathbf{p}|<2} d^3p \frac{3p_z^2}{8\pi p}\frac{|v(k_F\mathbf{p})|^2}{V}\cos(k_F\mathbf{p}\cdot\mathbf{R}) \ . \tag{3.6}$$

To get from Eq. (2.7) to Eq. (3.6), a change of variables from $\hat{\mathbf{k}}$ and $\hat{\mathbf{k}}'$ to $\mathbf{p}=\hat{\mathbf{k}}-\hat{\mathbf{k}}'$ has been made. Equation (3.6) is identical to an equation produced by Greene and Kohn[17] and applied to this particular problem by Martin,[23] who found that for divacancies the structure factor led to a change in the scattering rate between one and one-tenth the size of the scattering rate for one of the defects alone. This should not be too surprising since the unphysical situation of placing two atoms directly on top of each other produces an enhancement of twice the cross section of one of the scatterers. A similar argument has been made by Robinson[24] in the context of $1/f$ noise. He did not, however, get any of the results for noise magnitudes described here because he neglected the short-range nature of the interference effect described by Eq. (3.3). The averaging over all possible momentum transfers from 0 to $2k_F$ restricts the region where interference is important to R less than a few k_F^{-1}.

For an impurity moving in a random distribution of impurities, only the small fraction of the time that the impurity is near another impurity will there be a sizable change in the conductance. The $\delta\Gamma$ of Martin is reduced by the fraction of the volume of a sample which is within a nearest-neighbor distance of an impurity. Using a nearest-neighbor distance of $3.5k_F^{-1}$ and using k_F^{-1} as the width of an atom, the relevant volume for interference around one defect is $\sim 200k_F^{-3}$. Multiplying this by the total number of defects in the sample, N_{imp}, and dividing by the volume of the sample, we get as a crude estimate that $(\delta\Gamma_1)^2$ is reduced by a factor of $200n_{imp}k_F^{-3}$. To get a more accurate description the present author[25] has calculated the average effect of moving one impurity a distance δr:

$$\langle \delta\Gamma_1(\delta\mathbf{r})^2\rangle = n_{imp}\int d^3\mathbf{R}[\delta\Gamma(\mathbf{R}+\delta\mathbf{r})-\delta\Gamma(\mathbf{R})]^2 \ . \tag{3.7}$$

The integral in Eq. (3.7) was cut off for small R by the nearest-neighbor distance for gold, $3.5k_F^{-1}$, and a screened Coulomb potential in gold was used for δv. The resulting mean-square change in the conductance, $(\delta G_1)^2 \propto \langle \delta\Gamma_1^2\rangle$, is plotted as the solid line in Fig. 3. The mean-square change in the conductance due to the universal-conductance-fluctuation effect for the motion of a defect a distance δr is also shown in Fig. 4 for the cases of a δ-function potential[9] (dotted line) and a screened Coulomb impurity in gold (dashed line). With all three of these cases a motion of a few k_F^{-1} is sufficient to produce the full effect, which for the Born-approximation effect is

$$\langle \delta\Gamma_1(\infty)^2\rangle \sim 9.2(k_F^{-3}n_{imp})\left[\frac{\Gamma_{el}}{N_{imp}}\right]^2 \sim \frac{1}{k_F L_{el}}\left[\frac{\Gamma_{el}}{N_{imp}}\right]^2 \ . \tag{3.8}$$

The factor of 9.2 is smaller than the factor of 20–200 pre-

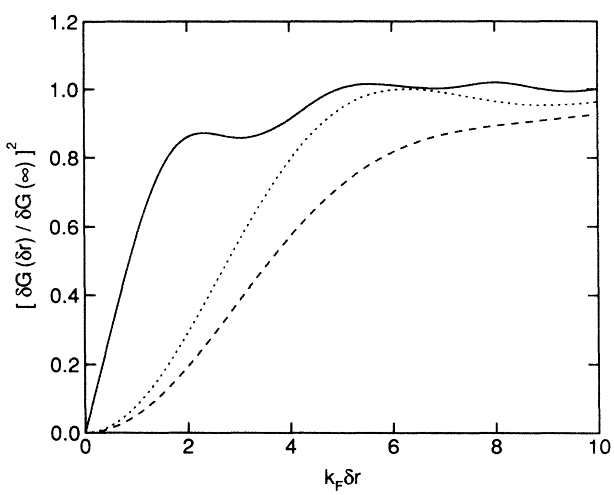

FIG. 3. The mean-square change in the conductance caused by moving one impurity a distance δr due to the Born-approximation effect (solid line) and the "universal"-conductance-fluctuation effect with constant potential (dotted) and a screened Coulomb impurity (dashed).

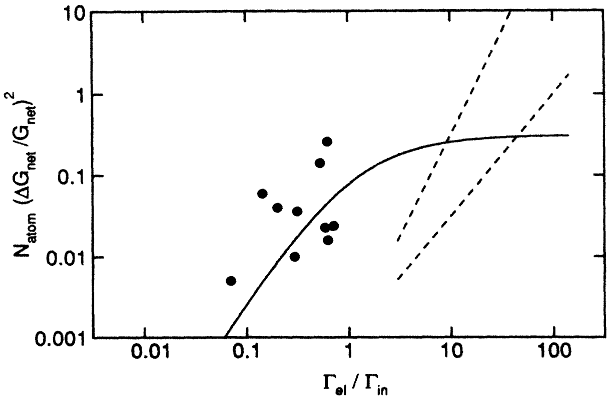

FIG. 4. The total conductance variation as a function of disorder assuming that all the impurities are active. The solid curve is the fluctuation due to the Born-approximation effect for impurities separated by more than a nearest-neighbor distance. The data for the noble metals of Ref. 27 assuming three decades of observed $1/f$ noise are plotted as solid circles. The universal-conductance-fluctuation results of Ref. 9 are shown by the upper dashed curve assuming no saturation, and by the lower dashed curve assuming saturation.

dicted by the crude estimate because not all movements produce the largest possible change in the scattering rate. In the second line of Eq. (3.8) the cross section of one of the defects has been estimated by $4\pi k_F^{-2}$ to show that $\delta\Gamma_1$ is reduced by approximately $(k_F L_{el})^{-1/2}$ over the "best" case of two defects lying close together. Sometimes, as in the case of hydrogen hopping in metals,[26] it is useful to distinguish between the moving defects and the static disorder. Denoting the scattering rate due to a total of N_{mov} defects moving by Γ_{mov} and the net elastic scattering rate by Γ_{el}, the mean-square change in the scattering rate of Eq. (3.8) becomes $0.3(\Gamma_{el}/N_{atom})(\Gamma_{mov}/N_{mov})$. To get the factor of 0.3, I have set the density of atoms equal to the free-electron density, $k_F^3/3\pi^2$.

Assuming that the motion of the defects is incoherent, these estimates of $\delta\Gamma$ and δG can be related to the total conductance fluctuation observed in $(1/f)$-noise experiments via

$$\Delta G_{net}^2 = \int S_G(f)df \sim N_{mov}(\delta G)^2 \ . \tag{3.9}$$

The power spectrum for the conductance fluctuations is $S_G(f)$, and N_{mov} is the total number of defects moving. Since very few experiments actually see the fall off in the excess noise at low frequencies, it is not possible to directly evaluate the integral in Eq. (3.9). In order to estimate ΔG_{net}^2 experimentally, one has two choices. The experimental value can be treated as a lower bound of the quantity that is calculated, or assumptions about the source of the noise can be made which allow the noise magnitude to be extended outside the observed range. Using the first method to obtain a lower bound, the data for the noble metals of Scofield, Mantese, and Webb[27] (assuming three decades in frequency of noise) are plotted as the solid dots in Fig. 4. As an example of the second method, Dutta, Dimon, and Horn[28] assume that the relaxation times of the objects causing the conductance fluctuations are caused by a distribution of activation energies. With this phenomenological model, they are able to estimate the noise magnitude outside the observed range. The resulting relative conductance fluctuations multiplied by the number of atoms for their silver and copper films are 0.9 and 0.3, respectively. They do not report the residual-resistivity ratio for these films; however, for any reasonably disordered film the values of 0.9 and 0.3 fall slightly above the other dots in Fig. 4.

The results for both the Born-approximation and the universal-conductance-fluctuation effects are also plotted in Fig. 4. Using Eq. (3.8) and assuming that there is one electron per atom, the predicted total noise magnitude for the Born-approximation effect is given by

$$N_{atom}\left[\frac{\Delta G_{net}}{G_{net}}\right]^2 \sim 0.3\left[\frac{N_{mov}}{N_{imp}}\right]\left[\frac{\Gamma_{el}}{\Gamma_{net}}\right]^2 , \tag{3.10}$$

which is drawn as the solid curve in Fig. 4 for $N_{mov}=N_{imp}$ using the inelastic scattering rate for gold at room temperature $(k_F v_F/\Gamma_{in}=500)$. Considering that the dots are only a lower bound for ΔG_{net}^2 of Eq. (3.9), Eq. (3.10) is too small by at least an order of magnitude. Using Eqs. (2.12) and (3.1) the corresponding result for the universal conductance fluctuations is

$$N_{atom}\left[\frac{\Delta G_{net}}{G_{net}}\right]^2 \sim \frac{\Gamma_{in}}{2k_F v_F}\left[\frac{N_{mov}}{N_{imp}}\right]\left[\frac{\Gamma_{el}}{\Gamma_{net}}\right]^{2.5} , \tag{3.11}$$

which is drawn as the upper dashed line in Fig. 4 again for $N_{mov}=N_{imp}$ and the inelastic scattering rate for gold at room temperature. The lower dashed line in this figure denotes the e^2/h saturation,

$$N_{atom}\left[\frac{\Delta G_{net}}{G_{net}}\right]^2 \sim \frac{\Gamma_{in}}{2k_F v_F}\left[\frac{\Gamma_{el}}{\Gamma_{net}}\right]^{1.5} , \tag{3.12}$$

which is the upper bound for the universal conductance fluctuation ΔG_{net}^2. Roughly 10% of the impurities must be active to cause saturation. The important point to notice is that the universal-conductance-fluctuation effect is in the wrong regime to apply to these experiments, as well as to most $(1/f)$-noise experiments done to date.[11,29] Since the upper bound for the universal-conductance fluctuation-effect only becomes larger than the Born-approximation effect for mean free paths less than 10 Å in Fig. 4, it is doubtful even for extremely disordered films that the universal conductance fluctuations will be the dominant effect.[29] There is no upper bound for the Born-approximation effect, which is in contradiction to the frequently stated version of the ergodic hypothesis[4] used in universal-conductance-fluctuation calculations that averaging over impurity configurations is equivalent to averaging over magnetic field. The Born-approximation effect does not have any magnetic field dependence.

IV. CONCLUSION

In this paper the role of interference effects on the sensitivity of the conductance to impurity configuration has been examined for all values of the disorder. The dominant contribution in the clean limit comes from the change in the net scattering rate of the Drude conductivity, as opposed to an effective dephasing rate for the case of the universal conductance fluctuations. The length scales involved for interference in the clean limit are a few inverse Fermi wavelengths, much smaller than those involved in the universal conductance fluctuations. The universal-conductance-fluctuation is more important for $\Gamma_{el} \gg \Gamma_{in}$, while the Born-approximation effect is larger for $\Gamma_{el} < \Gamma_{in}$. Both effects depend on the microscopics of the potential change. In particular, for the universal conductance fluctuations this can lead to different values for the change in the conductance even when the mean-square magnetoconductance variation for static disorder is the same. The estimated noise magnitudes for the Born-approximation effect for impurities moving within a random distribution of impurities is too small to account for the experimentally observed noise. This does not mean, of course, that the rearrangement of atoms cannot be the cause of $1/f$ noise, but only that the motion of atoms within this particular model cannot account for the noise magnitudes observed. Recently, using estimates similar to those made here, Pelz and Clarke[13] have shown that if defects are paired it is possible to account for room-temperature ($1/f$)-noise magnitudes with less than 10^{-3} of the defects being active. It is not clear whether this is a reasonable number or not. Other work to try to determine the microscopic change in the potential has been and continues to be done.[30,31,26]

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