Raman scattering from plasma excitations in a conducting double layer

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We have investigated the Raman-scattering amplitude from processes in which the radiation field excites longitudinal collective modes in a system comprised of two parallel quasi-two-dimensional conducting electronic layers embedded in a dielectric medium. Within the random-phase approximation we have derived explicit formulas for the density response and the Raman intensity as a function of the exchanged frequency. The Raman spectrum consists of two resonance peaks corresponding to the optical- and acoustic-plasmon branches which are peculiar to this system. Both the position and the width of the resonances have been studied and are shown to depend on the electronic elastic lifetime.

The Raman scattering amplitude from plasma modes in a layered electron gas has recently received a great deal of attention both theoretically[1,2] and experimentally.[3,4] The coupling of the collective excitations of these systems to external probes has been lately reviewed in Refs. 5 and 6. In this note we present the results of a study of the Raman scattering intensity from plasma excitations in a system comprised of two identical spatially separated conducting layers in which the electronic dynamics is for all practical purposes two-dimensional (2D). The layers are parallel and are embedded in a medium of average dielectric constant ϵ_0. This problem is of current experimental interest because the system at hand is a model for double-quantum-well heterojunctions and single inversion layers with more than one populated subband.[7] Recently the possibility of an electronic mechanism for superconductivity in a double well in which the effective interaction is mediated by plasma excitations has also been investigated.[8]

In the Raman scattering process at hand an incident photon of frequency ω_i, wave vector $\mathbf{k}_i$ (in the plane of the layers), and polarization vector $\boldsymbol{\epsilon}_i$ outside the material, is scattered by the collective excitations of the electronic system in the two layers. Outside the material the final photon has a frequency ω_f, in-plane wave vector $\mathbf{k}_f$, and polarization vector $\boldsymbol{\epsilon}_f$. At zero temperature the cross section for this process is found to be proportional to[1,5]

$$\frac{\partial\sigma}{\partial\omega\partial\Omega} \propto I(q,\omega)\,|\,\boldsymbol{\epsilon}_i\cdot\boldsymbol{\epsilon}_f\,|^2, \tag{1}$$

where

$$I(q,\omega)=-\int dz dz' Im\,[\Pi(q,\omega;z,z')] \times e^{-2ik(z-z')}e^{-(z+z')/\delta}, \tag{2}$$

with $\mathbf{q}=\mathbf{k}_i-\mathbf{k}_f$ and $\omega=\omega_i-\omega_f$ (ω positive). In Eq.(2) we have assumed that the two layers are located respectively at $z=0$ and $z=d$, and $\Pi(q,\omega;z,z')$ is the in-plane Fourier transform of $\Pi(\mathbf{x},z,t;\mathbf{x}',z',t')$, the density response function of the system. The two wave vectors $\delta^{-1}=(2\omega_i/c)\mathrm{Im}(\epsilon_0)^{1/2}$, and $k=(\omega_i/c)\mathrm{Re}(\epsilon_0)^{1/2}$, characterize the propagation of the light waves inside the polarizable medium.

The response function $\Pi(q,\omega;z,z')$ can be evaluated as follows. $\Pi(q,\omega;z,z')$ obeys the following Dyson's equation[1]

$$\Pi(q,\omega;z,z')=\Pi_0(q,\omega;z,z')+\int dz_1\int dz_2\,\Pi_0(q,\omega;z,z_1)V(q;z_1,z_2)\Pi(q,\omega;z_2,z'), \tag{3}$$

where, within the random-phase approximation (RPA), $\Pi_0(q,\omega;z,z')$ is the value of Π in the absence of Coulomb interactions. The in-plane Fourier transform of the Coulomb interaction between two electrons is given by $V(q;z,z')=v(q)e^{-q|z-z'|}$ where $v(q)=2\pi e^2/\epsilon_0' q$ and for simplicity we have assumed the layers to be strictly two-dimensional. From the fact that the electronic densities are localized at $z=0$ and $z=d$ one can readily infer that $\Pi(q,\omega;z,z')$ and $\Pi_0(q,\omega;z,z')$ have the following structure

$$\Pi(q,\omega;z,z') = \sum_{m,m'=0,1}\delta(z-md)\delta(z'-m'd)\Pi(q,\omega;m,m'), \tag{4a}$$

and

$$\Pi_0(q,\omega;z,z')=\sum_{m,m'=0,1}\delta(z-md)\delta(z'-m'd)\Pi_0(q,\omega). \tag{4b}$$

In the last equation $\Pi_0(q,\omega)$ is the density response function of a noninteracting 2D electronic system. Using Eqs. (4a) and (4b) in Eq. (3) we find a system of algebraic equations for the functions $\Pi(q,\omega;m,m')$ whose explicit solution leads to

$$\Pi(q,\omega;1,0)=\Pi(q,\omega;0,1)$$
$$=e^{-qd}v(q)\frac{\Pi_0^2(q,\omega)}{D(q,\omega)}\ , \tag{5a}$$

and

$$\Pi(q,\omega;0,0)$$
$$=\Pi(q,\omega;1,1)$$
$$=\frac{\Pi_0(q,\omega)}{D(q,\omega)}[1-v(q)\Pi_0(q,\omega)]\ , \tag{5b}$$

where

$$D(q,\omega)=[1-v(q)\Pi_0(q,\omega)]^2-e^{-2qd}[v(q)\Pi_0(q,\omega)]^2\ . \tag{6}$$

These expressions for $\Pi(q,\omega;m,m')$ substituted into Eq. (4a) lead to the required function $\Pi(q,\omega;z,z')$. We note in passing that $D(q,\omega)$ is the determinant of the dielectric tensor $\epsilon_{ij}(q,\omega)$ for the double layer system.[10]

The nonrelativistic spectrum of the longitudinal collective excitations is given in the present case by the poles of the response function $\Pi(q,\omega;z,z')$ [i.e. the zeros of $D(q,\omega)$]. In the absence of any processes leading to a finite electron lifetime this problem has been studied in Refs. 7, 9 and 10. All these theories are based on the RPA description of the electronic interactions. It is found that the spectrum consists of two plasmon branches, an optical plasmon (OP) and an acoustic plasmon (AP) with dispersion relation $\Omega_O(q)$ and $\Omega_A(q)$. In the long wavelength limit $\Omega_O(q)\approx q^{1/2}$ whereas $\Omega_A(q)\approx q$. In Ref. 10, in particular, the correct analytical expressions for the plasmon dispersion relation in the long wavelength limit were derived. Fig. 1 shows a numerical calculation of the plasmon spectrum for the case of two identical electronic layers separated by a distance

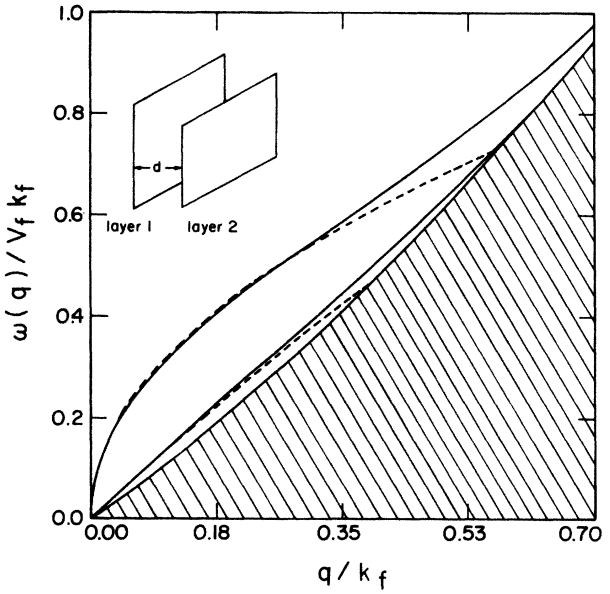

FIG. 1. Nonrelativistic longitudinal spectrum ω vs q for an electronic double layer in absence of impurity scattering. A schematic of this system is provided in the inset. The two continuous lines represent the optical (OP) and the acoustic (AP) branch of the plasmon spectrum. The shaded region is the electron-hole pair continuum. The layers are assumed to be identical and separated by a distance d. The parameters have been chosen as follows: $m^*=0.07m_e$, $n=7.3\times10^{11}$ cm^{-2}, $\epsilon_0'=13.1$, and $d=100$ Å. The two dashed lines are extrapolations of the exact long wavelength asymptotic formulas for the dispersion relations obtained in Ref. 10.

$d=100$ Å . Here the effective mass of the electrons is taken to be $m^*=0.07m_e$, the areal electron density is $n=7.3\times10^{11}$ cm^{-2} and $\epsilon_0'=13.1$. These values are typical of GaAs/Al_xGa_{1-x}As heterostructures. For comparison the long wavelength limit asymptotic behavior derived in Ref. 10 is shown. Notice that the long wavelength limit results are approximately valid up to $q/k_F\approx0.2$.

We now turn to the evaluation of the Raman intensity $I(\omega)$. Using in Eq. (2) the explicit expression for the density response function $\Pi(q,\omega;z,z')$ [see Eqs. (4a), (5), and (6)], we find

$$I(q,\omega)=-\mathrm{Im}(\Pi_0(q,\omega)D(q,\omega)^{-1}\{1+e^{-2d/\delta}-v(q)\Pi_0(q,\omega)[1+e^{-2d/\delta}-2(\cos2kd)e^{-qd}e^{-d/\delta}]\}). \tag{7}$$

As is readily realized, if the electronic elastic lifetime is infinite the plasmon Raman spectrum consists of two delta functions centered at the resonance frequencies $\Omega_O(q)$ and $\Omega_A(q)$. This is at variance with the case of semiconducting superlattices where the width of the Raman peaks seems to be mainly determined by the presence in the spectrum of a plasmon continuum.[1] In what follows we will then investigate the effect of elastic impurity scattering on the plasmon Raman spectrum. Elastic impurity scattering leads in general to a finite width for the electronic states. As a consequence the collective modes are broadened and shifted to a lower frequency.[11,12] A microscopic evaluation of $\Pi_0(q,\omega)$ in the presence of impurities can be explicitly carried out in the regime of interest, (small q and ω), with the use of a diagrammatic approach. One obtains[11]

$$\Pi_0(q,\omega)\approx-N_0\left[1-\frac{\omega}{[(\omega+i\gamma)^2-v_F^2q^2]^{1/2}-i\gamma}\right]\ , \tag{8}$$

where $N_0=m^*/\pi\hbar^2$ is the density of states in a 2D electron gas and γ is the inverse electronic elastic lifetime. It should be mentioned here that as discussed in Ref. 11

weak localization effects are not important at the frequencies at hand. An alternative approach which allows to take properly into account the effect of the impurities is the one based on the well-known phenomenological expression first proposed by Mermin.[13] $\Pi_0(q,\omega)$ is in this case simply expressed in terms of $\chi_0(q,\omega)$, the noninteracting 2D electron gas Lindhard function,[14] and γ. As it turns out in the long wavelength and small frequency regime Mermin's expression for $\Pi_0(q,\omega)$ coincides with Eq. (8) so that either formula can be satisfactorily used in the context of the present calculation.[15]

At a fixed in-plane wave vector transfer q, and given γ, the Raman intensity $I(\omega)$, as a function of the photon frequency shift ω, has two peaks at frequencies $\Omega_O(q,\gamma)$ and $\Omega_A(q,\gamma)$ which, for realistic values of γ, are a few percent lower than the $\gamma=0$ values $\Omega_O(q)$ and $\Omega_A(q)$.[11] Fig. 2 shows the Raman intensity $I(\omega)$ for a system characterized by the parameter values used in Fig. 1. $I(\omega)$ is given here in arbitrary units. The actual absolute value, depending on transfer functions and enhancement factors, will vary for different experimental situations.[5,6] For illustration we have chosen here $q=0.02k_F$, $\gamma=0.3$ meV, $kd=0.25$, and $\delta=6000$ Å. For these values of the parameters the nonrelativistic approximation is very good. We also note that in the present case the effect of the photon decay length δ is negligible since δ is much larger then the interlayer separation d.

We find that a physical quantity of interest is the ratio $R(q)$ of the heights of the two Raman peaks (optical and acoustic) as a function of the in-plane wave vector transfer q.[16] It can be shown that for values of ω close to either one of the two resonances, and for small values of γ, $I(\omega)$ can be approximated with a Lorentzian. By evaluating the resulting expression at the two resonance peaks and taking the ratio we obtain

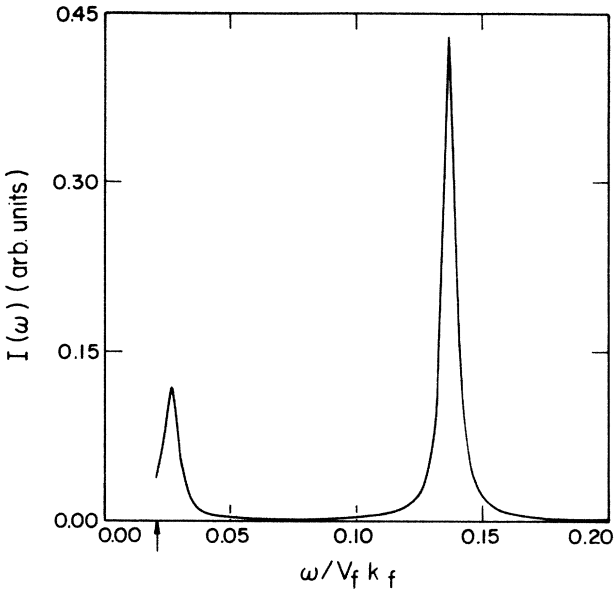

FIG. 2. Raman scattering intensity as a function of the frequency ω for the case of finite electron elastic lifetime. The physical parameters of the system are the same as in Fig. 1. We have chosen here $q=0.02k_F$, $\gamma=0.3$ meV, $kd=0.25$, and $\delta=6000$ Å. The arrow at low frequency indicates the onset of the electron-hole pair continuum in RPA.

$$R(q)=\frac{\chi_0(q,\Omega_O(q))\partial D(q,\Omega_A(q))/\partial\omega}{\chi_0(q,\Omega_A(q))\partial D(q,\Omega_O(q))/\partial\omega}\,\frac{\{1+e^{-2d/\delta}-v(q)\chi_0(q,\Omega_O(q))[1+e^{-2d/\delta}-2(\cos 2kd)e^{-qd}e^{-d/\delta}]\}}{\{1+e^{-2d/\delta}-v(q)\chi_0(q,\Omega_A(q))[1+e^{-2d/\delta}-2(\cos 2kd)e^{-qd}e^{-d/\delta}]\}}\,, \tag{9}$$

where we have made use of Mermin's formula. In this limit we can simply use in Eq. (9) $\Omega_{O,A}(q)$ instead of the actual values $\Omega_{O,A}(q,\gamma)$. Furthermore for consistency the derivative $\partial D(q,\omega)/\partial\omega$ appearing in the expression for $R(q)$ must be evaluated using the 2D Lindhard function $\chi_0(q,\omega)$. Fig. 3 shows the behavior of the peak intensity ratio $R(q)$. The values of the relevant parameters are the same as in Fig. 2. It is interesting to notice that the acoustic branch leads to the largest peak for small wave vectors while the optical branch is the prominent feature in the Raman spectrum at intermediate and larger values of q. Because the actual γ is finite the value of $R(q)$, as evaluated directly from Fig. 2 for instance, differs by a few percent from that calculated using the approximated expression of Eq. (9) which however becomes increasingly more accurate as q increases.

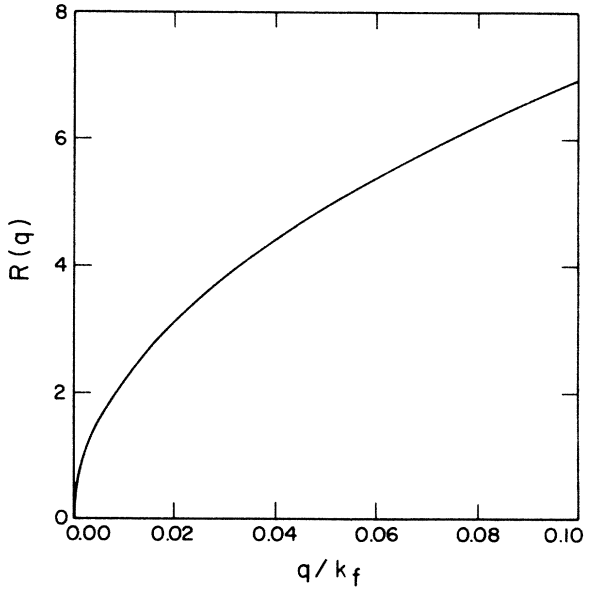

FIG. 3. Plot of the peak intensity ratio $R(q)$, Eq. (9), as a function of the plasmon wave vector q. The parameters are the same as in Fig. 2.

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