Negative minority-electron mobility in a nonequilibrium electron-hole plasma

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The recently observed phenomenon of negative minority-electron mobility in a GaAs-$Al_xGa_{1-x}As$ heterostructure is investigated theoretically, employing a balance-equation formulation of nonlinear carrier transport with dynamic carrier screening fully incorporated. Minority-electron mobilities limited by dynamically screened electron-hole interactions and phonon scatterings are calculated as functions of the applied electric field and lattice temperature, and are shown to be in good agreement with experimental data. Energy-transfer rates for the minority electrons and the electron and hole carrier temperatures are also calculated in this work.

Höpfel and collaborators[1-3] have recently reported their experimental observation of negative minority-carrier mobility in GaAs-$Al_xGa_{1-x}As$ quantum-well structures. Electron-hole interaction in an electric field, and the associated "carrier-drag" effect, is believed to be responsible for this interesting phenomenon. In this connection, it is important to have a full account of the role of dynamic screening of the electron-hole interaction (and of the phonon-scattering interaction) which moderates such carrier drag. As yet there is no report of an analysis of this problem taking account of dynamic screening in high electric fields, with attendant nonlinearity. With this in view, we discuss here such a theory of nonlinear hot-carrier transport in a nonequilibrium electron-hole plasma using a balance-equation approach,[4] and present the results of specific calculations. All aspects of carrier-carrier interactions are considered here, with electron-electron and hole-hole interactions treated in the random-phase approximation (RPA), while the dynamically screened electron-hole interaction is treated perturbatively to the lowest order. In addition, dynamically screened carrier-phonon scattering interactions are also included. Our evaluation of minority-electron mobility as a function of applied electric field and lattice temperature in a GaAs-$Al_xGa_{1-x}As$ heterostructure is in good agreement with the experimental results of Ref. 1. Furthermore, the energy relaxation mechanisms (electron-hole, electron-phonon, hole-phonon) are all considered in our determination of the carrier temperatures, along with the evaluation of the various energy-transfer rates.

The effects of carrier-carrier interactions in semiconductor transport differ considerably depending upon the carriers involved. Interactions among like carriers conserve total momentum, thus they cannot directly relax the carrier momentum. However, such interactions do affect the energy dependence of carrier interactions with phonons and impurities through dynamic screening, and this is especially important in the case of impurity-limited transport where plasmon effects can be significant.[5] On the other hand, interactions between different types of carriers, as in the present case of electron-hole interactions, do relax carrier momenta, and are therefore directly responsible for limiting carrier transport. In an external electric field, electrons and holes naturally drift in opposite directions, but the attractive scattering of a minority carrier by majority carriers tends to impart to it an average velocity compatible with that of the majority carriers. This is to say that the majority carriers tend to drag the minority carriers along with them, thus reducing the opposing drift velocity of the minority carriers. In extreme cases[1-3] of high-mobility majority carriers at low temperatures in relatively low electric fields, where the density of the majority carriers is much greater than that of the minority carriers, this drag effect is so pronounced that the minority carriers drift in the wrong direction following the majority-carrier transport, resulting in a negative absolute minority-carrier mobility. McLean and Paige[6] are apparently the first to have recognized the possibility of such a negative minority mobility due to carrier drag, and they also analyzed the role of electron-hole interactions in semiconductor transport employing coupled linearized Boltzmann equations. As indicated above, dynamic screening moderates this process, as does nonlinearity.

Our formulation of nonlinear balance equations is concerned with steady-state transport in a two-dimensional electron-hole plasma confined to the GaAs region of a GaAs-$Al_xGa_{1-x}As$ heterostructure, subject to an external electric field applied parallel to the heterolayer. Such nonlinear hot-carrier balance equations have already been derived for this kind of quantum-well structure in the presence of a single type of carrier (electrons),[7] and our considerations here parallel the earlier work of Ref. 7. The only exception is the inclusion of the second type of carrier (holes) in this work. The electrons are taken to have a two-dimensional (2D) sheet density N_e, effective mass m_e, and charge $-e$, while holes have a 2D density N_h, effective mass m_h, and charge e. The quantum well is taken to have width a, and we neglect tunneling out of it. The electrons and holes are assumed to populate their respective first subbands only, with wave-function envelopes across the well having the form

$\psi \rightarrow \zeta(z)=(2/a)^{1/2}\cos(\pi z/a)$ for $-a/2<z<a/2$, and vanishing outside. Plane-wave functions are assumed for the $\bar{r}=(x,y)$ plane of the quantum-well sheets. Following the methodology of Ref. 7, we introduce center-of-mass (c.m.) momenta and coordinates $\mathbf{P}_1,\mathbf{R}_1$, and $\mathbf{P}_2,\mathbf{R}_2$ for electrons and holes, respectively, and obtain the Hamiltonian in the form

$$H=H_c+H_e+H_h+H_{\rm ph}+H_I \ , \tag{1}$$

where the c.m. Hamiltonian

$$H_c=\frac{\mathbf{P}_1^2}{2N_e m_e}+\frac{\mathbf{P}_2^2}{2N_h m_h}-N_e e\mathbf{E}\cdot\mathbf{R}_1+N_h e\mathbf{E}\cdot\mathbf{R}_2 \tag{2}$$

absorbs all direct effects of the electric field, and H_e is the "relative electron" Hamiltonian with its full complement of electron-electron Coulomb interactions, and H_h is the corresponding "relative hole" Hamiltonian with its hole-hole Coulomb interactions. $H_{\rm ph}$ is the free-phonon Hamiltonian. Furthermore,

$$H_I=H_{e\text{-ph}}+H_{h\text{-ph}}+H_{e\text{-}h} \ , \tag{3}$$

where $H_{e\text{-ph}}$ and $H_{h\text{-ph}}$ are the electron-phonon and hole-phonon interaction Hamiltonians, respectively, and $H_{e\text{-}h}$ describes the electron-hole Coulomb attraction. Proceeding with the development of steady-state dc balance equations for the electron c.m. momentum, hole c.m. momentum, and the associated energy-balance equations for electrons and holes, following the precepts of Ref. 7, our results take the form (electric field is taken in x direction)

$$\langle \dot{P}_{1x} \rangle = -N_e eE + f_1(v_1)+f_{12}(v_1-v_2)=0 \ , \tag{4}$$

$$\langle \dot{P}_{2x} \rangle = N_h eE + f_2(v_2)-f_{12}(v_1-v_2)=0 \ , \tag{5}$$

$$-\langle \dot{H}_e \rangle = v_1 f_1(v_1)+W_1(v_1)+W_{12}(v_1-v_2)=0 \ , \tag{6}$$

$$-\langle \dot{H}_h \rangle = v_2 f_2(v_2)+W_2(v_2) +(v_1-v_2)f_{12}(v_1-v_2)-W_{12}(v_1-v_2)=0 \ , \tag{7}$$

where we note that a more symmetric form of Eqs. (4) and (5) and (6) and (7) may be achieved with $f_{21}(v_1-v_2)=-f_{12}(v_1-v_2)$ and $W_{21}(v_1-v_2)=(v_1-v_2)f_{12}(v_1-v_2)-W_{12}(v_1-v_2)$. The details of derivation of these balance equations are lengthy and will be presented elsewhere. In the above equations, v_1 and v_2 are the average velocities of the centers of mass, i.e., the average drift velocities of the electrons and holes. The functions $f_1(v_1)$ and $f_2(v_2)$ are the forces exerted on the centers of mass of the electrons and holes, respectively, due to scatterings by phonons. We obtain these as ($j=1$ for electrons, $j=2$ for holes, $T_1=T_e$, $T_2=T_h$)

$$f_j(v_j)=2\sum_{\mathbf{q},\lambda} |M_j(\mathbf{q},\lambda)|^2 |I(iq_z)|^2 q_x \hat{\pi}_2^{(j)}(\bar{q},\Omega_{q\lambda}+q_x v_j)\left[n\left(\frac{\Omega_{q\lambda}}{T}\right)-n\left(\frac{\Omega_{q\lambda}+q_x v_j}{T_j}\right)\right] . \tag{8}$$

$M_j(\mathbf{q},\lambda)$ is the matrix element of the electron ($j=1$) or hole ($j=2$) coupling with phonons with wave vector $\mathbf{q}$ (frequency $\Omega_{q\lambda}$ in the λth branch, in the 3D plane-wave representation), and the form factor $I(q)$ is given by $I(q)=\int dz\, e^{-qz}\zeta(z)^2$. $\hat{\pi}_2^{(j)}(\bar{q},\omega)$ are the imaginary parts of the equilibrium density-density correlation functions of the quantum-well system with only electrons ($j=1$) or holes ($j=2$) present. In the RPA we have [$\mathbf{q}=(\bar{q},q_z)$, $q=|\bar{q}|$]

$$\hat{\pi}^{(j)}(\bar{q},\omega)=\pi^{(j)}(\bar{q},\omega)/[1-V(q)\pi^{(j)}(\bar{q},\omega)] \ , \tag{9}$$

where

$$V(q)=(e^2/2\epsilon_0\kappa q)\int dz\, dz' e^{-q|z-z'|}\zeta(z)^2\zeta(z')^2 \ ,$$

κ being the background dielectric constant (here we have neglected image contributions due to a different background dielectric constant on either side of the heterojunction). In Eq. (9)

$$\pi^{(j)}(\bar{q},\omega)=2\sum_{\bar{k}}\frac{f((\varepsilon_{j,\bar{k}+\bar{q}}-\mu_j)/T_j)-f((\varepsilon_{j,\bar{k}}-\mu_j)/T_j)}{\omega+\varepsilon_{j,\bar{k}+\bar{q}}-\epsilon_{j,\bar{k}}+i\delta} \tag{10}$$

is the density-density correlation function of the 2D sheet of electrons ($j=1$) or holes ($j=2$) without Coulomb interactions. $f((\varepsilon_{j,\bar{k}}-\mu_j)/T_j)=\{\exp[(\varepsilon_{j,\bar{k}}-\mu_j)/T_j]+1\}^{-1}$ is the Fermi-Dirac distribution function for energy $\varepsilon_{j,\bar{k}}$ at the carrier temperature T_j ($T_1=T_e$ for electrons, $T_2=T_h$ for holes), and μ_j are the corresponding chemical potentials which are determined by the electron and hole sheet densities N_e and N_h. $n(x)=[\exp(x)-1]^{-1}$ is the Bose-Einstein distribution function.

The energy-transfer rates $W_j(v_j)$ for the electrons and holes due to carrier-phonon interactions are given by

$$W_j(v_j)=2\sum_{\mathbf{q},\lambda} |M_j(\mathbf{q},\lambda)|^2 |I(iq_z)|^2 \Omega_{q\lambda}\hat{\pi}_2^{(j)}(\bar{q},\Omega_{q\lambda}+q_x v_j)\left[n\left(\frac{\Omega_{q\lambda}}{T}\right)-n\left(\frac{\Omega_{q\lambda}+q_x v_j}{T_j}\right)\right] . \tag{11}$$

The force between electrons and holes $f_{12}(v_1-v_2)$ calculated to lowest order in $H_{e\text{-}h}$ is determined in our analysis as [$\omega_{12}=q_x(v_1-v_2)$]

$$f_{12}(v_1-v_2)=-\sum_{\bar{q}} q_x[V(q)]^2\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\left[n\left[\frac{\omega}{T_e}\right]-n\left[\frac{\omega-\omega_{12}}{T_h}\right]\right]\hat{\pi}_2^{(2)}(-\bar{q},\omega_{12}-\omega)\hat{\pi}_2^{(1)}(\bar{q},\omega)\ . \tag{12}$$

The corresponding energy-transfer rate between electrons and holes due to $H_{e\text{-}h}$ is

$$W_{12}(v_1-v_2)=-\sum_{\bar{q}} [V(q)]^2\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\omega\left[n\left[\frac{\omega}{T_e}\right]-n\left[\frac{\omega-\omega_{12}}{T_h}\right]\right]\hat{\pi}_2^{(2)}(-\bar{q},\omega_{12}-\omega)\hat{\pi}_2^{(1)}(\bar{q},\omega)\ . \tag{13}$$

The coupled momentum and energy-balance equations (4)–(7) can be solved numerically to yield the carrier-drift velocities v_1 and v_2, and the carrier temperatures T_e and T_h, in terms of the applied electric field. Moreover, $\mu_e=-v_1/E$ and $\mu_h=v_2/E$ are the associated carrier mobilities for electrons and holes, respectively.

In the limit of weak applied electric field strength, the balance equations (4)–(7) can be expanded to linear order in the carrier-drift velocities. In this situation the energy-balance equations (6) and (7) mandate that $T_e=T_h=T$, while the momentum-force balance equations (4) and (5) take the form $[f'(0)=(\partial/\partial v)f(v)\,|_{v=0}]$

$$-N_e eE+v_1 f_1'(0)-v_2 f_{12}'(0)+v_1 f_{12}'(0)=0\ , \tag{14a}$$

$$N_h eE+v_2 f_2'(0)-v_2 f_{12}'(0)-v_1 f_{12}'(0)=0\ . \tag{14b}$$

This leads to linear mobilities which may be put into correspondence with the hydrodynamic results of Höpfel *et al.*[1,2] if we identify relaxation times which incorporate the role of screening as $\tau_1=-N_e m_e/f_1'(0)$, $\tau_2=-N_h m_h/f_2'(0)$, $\tau_{12}=-N_e m_e/f_{12}'(0)$, $\tau_{21}=-N_h m_h/f_{12}'(0)$. Thus we find

$$\mu_e=e(\tau_{12}/m_e+N_e\tau_2/N_h m_h-\tau_2/m_h)\times(1+\tau_{12}/\tau_2+N_e m_e\tau_2/N_h m_h\tau_1)^{-1}\ , \tag{15}$$

$$\mu_h=e(\tau_{21}/m_h+N_h\tau_1/N_e m_e-\tau_1/m_e)\times(1+\tau_{21}/\tau_2+N_h m_h\tau_1/N_e m_e\tau_2)^{-1}\ . \tag{16}$$

The conditions for negative minority-electron mobility have already been made clear in Refs. 1–3. Since $N_e\ll N_h$, and $\tau_{12}\ll\tau_2$ at low temperatures, $\mu_e/e\simeq\tau_{12}/m_e-\tau_2/m_h$ and this becomes negative when electron-hole scattering dominates over hole-phonon scattering. Since the former is most effective at low temperatures where the latter is weak, negative minority-electron mobility is most likely to occur at low temperatures. As the temperature rises, τ_{12}/m_e increases while τ_2/m_h decreases, such that μ_e goes through zero, and becomes positive at higher temperatures. These features are in evidence in our numerical evaluation of Eqs. (15) and (16), and our linear results, including dynamic screening, are presented in Fig. 1, where we plot the minority-electron mobility and majority-hole mobility (multiplied by -1) as functions of lattice temperature for a GaAs-$Al_xGa_{1-x}As$ quantum-well system. The lattice-scattering mechanisms considered here are the polar-optical-phonon coupling and acoustic deformation-potential coupling with electrons, and nonpolar-optical-phonon and acoustic deformation-potential couplings with holes. The physical parameters used in our calculations are electron-sheet density $N_e=3\times10^{10}$ cm^{-2}, hole-sheet density $N_h=1.5\times10^{11}$ cm^{-2}, quantum-well width $a=112$ Å, electron effective mass $m_e=0.07m_0$ (m_0 is the free-electron mass), hole effective mass $m_h=0.4m_0$, GaAs static dielectric constant $\kappa=12.9$, optical dielectric constant 10.8, optical-phonon frequency 35.4 meV, lattice density 5.31 g/cm^3, longitudinal sound speed 5.29×10^5 cm/s, transverse sound speed 2.48×10^5 cm/s, conduction-band deformation potential 8.5 eV, and valence-band deformation potential 9.0 eV.

To compare with experimental values of μ_e, data from Ref. 1 are reproduced in Fig. 1. Our calculated results are in good agreement with those of Ref. 1, except at the extremely-low-temperature limit, where impurity contributions, which we have omitted, would probably dominate.

When the applied electric field is not very weak, the set of nonlinear balance equations (4)–(7) must be solved simultaneously for the determination of the carrier-drift velocities v_1 and v_2 and the carrier temperatures T_e and T_h. Results of our nonlinear calculations incorporating dynamic screening are presented in Figs. 2 and 3, where carrier mobilities and carrier temperatures are plotted as functions of the applied electric field strength. The physi-

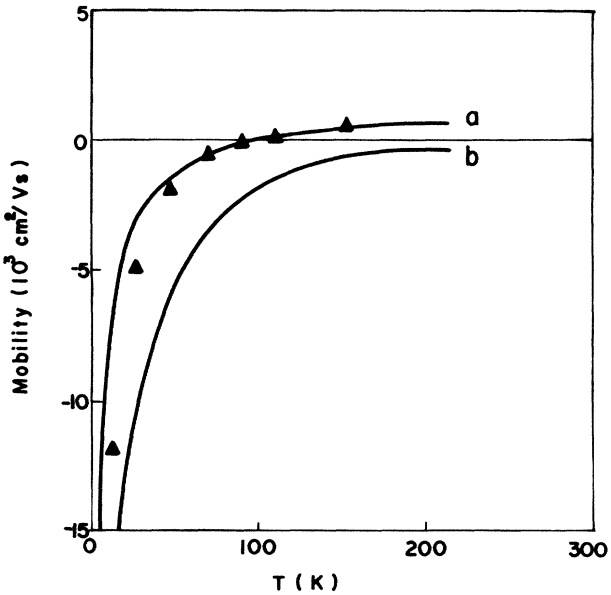

FIG. 1. Carrier mobilities as functions of lattice temperature: curve *a*, minority-electron mobility μ_e; *b*, majority-hole mobility multiplied by -1. Solid triangles are experimental values of μ_e taken from Ref. 1.

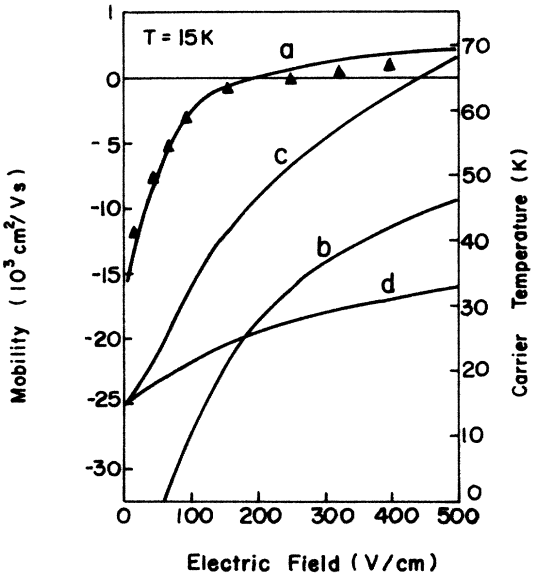

FIG. 2. Carrier mobilities and temperatures as functions of the applied electric field at lattice temperature 15 K: *a*, minority-electron mobility; *b*, majority-hole mobility multiplied by -1; *c*, electron temperature; *d*, hole temperature. Solid triangles are experimental values of μ_e taken from Ref. 1.

cal parameters are those used in Fig. 1. In Fig. 2, the lattice temperature is taken to be 15 K, and experimental values of μ_e from Ref. 1 are reproduced for comparison. Again, reasonably good agreement is obtained between our calculated minority-electron mobility and those measured experimentally.[1] Another set of calculations is shown in Fig. 3, which is obtained with a lattice tempera-

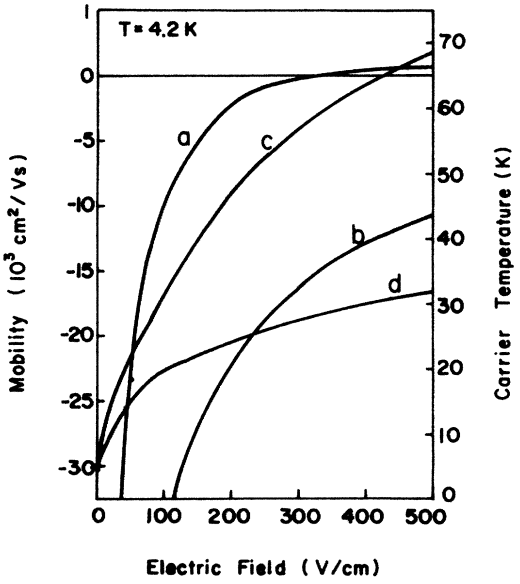

FIG. 3. Carrier mobilities and temperatures as functions of the applied electric field at lattice temperature 4.2 K: *a*, minority-electron mobility; *b*, majority-hole mobility multiplied by -1; *c*, electron temperature; *d*, hole temperature.

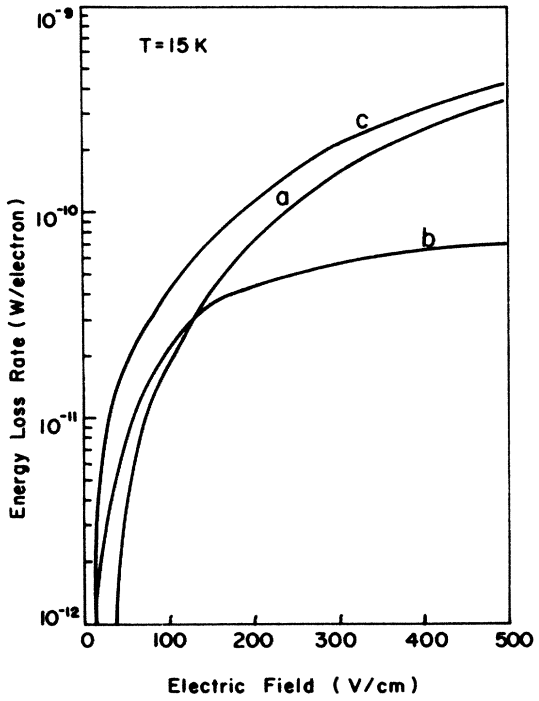

FIG. 4. Electron-energy-transfer rates as functions of the applied electric field at lattice temperature 15 K: *a*, electron-phonon; *b*, electron-hole; *c*, total electron-energy-transfer rate.

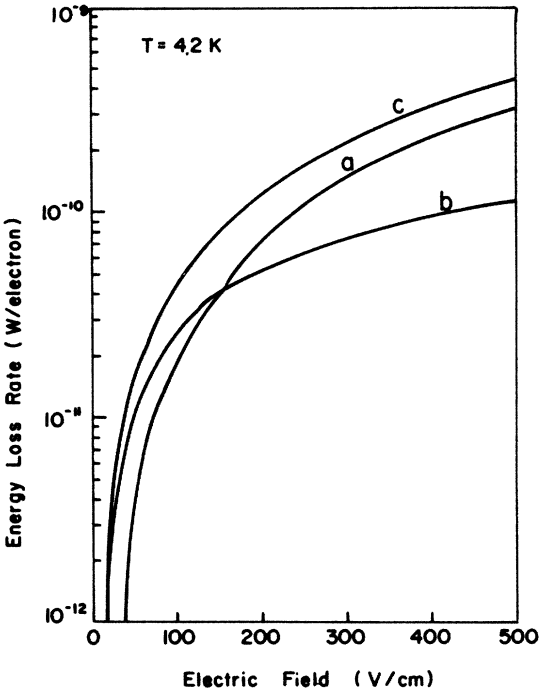

FIG. 5. Electron-energy-transfer rates as functions of the applied electric field, at lattice temperature 4.2 K: *a*, electron-phonon; *b*, electron-hole; *c*, total electron-energy transfer rate.

ture of 4.2 K. These calculations confirm that the negative minority-electron mobility decreases (that is, the electron mobility increases) both as the lattice temperature increases and the electric field increases: μ_e becomes positive for $E > 200$ V/cm at $T = 15$ K, and it becomes positive for $E > 350$ V/cm at $T = 4.2$ K. It is of interest to note the dependence of the carrier temperatures on the electric field. While the electron temperature rises rapidly with increasing electric field, the holes stay relatively cool, reflecting a more efficient coupling between the holes and the lattice.[8]

Finally, we have also evaluated energy-transfer rates as functions of applied electric field in the course of solving the energy and momentum balance equations. The results for the minority electrons in the quantum well under consideration are given in Figs. 4 and 5 for lattice temperatures 15 and 4.2 K, respectively.

In conclusion, we have performed numerical calculations of carrier transport for a two-component electron-hole plasma, taking full account of dynamic screening and nonlinearity. Our theoretical results agree with experimental observations quantitatively insofar as negative minority-electron mobility is concerned, in both the linear case and the nonlinear case.

Finally, we should emphasize the importance of employing the temperature-dependent RPA density-density correlation function in connection with dynamic screening, without which accurate calculation would not be possible.

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