Electron-hole scattering and the negative absolute mobility of electrons in a semiconductor quantum well

W. Cai and T. F. Zheng
Department of Physics, The City College of the City University of New York, New York, New York 10031

M. Lax
Department of Physics and the Graduate Center of the City University of New York, New York, New York 10031 and AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 21 October 1987; revised manuscript received 18 December 1987)

We study the transport of a quasi-two-dimensional electron-hole gas in a semiconductor quantum well. The screening in the presence of the electron-hole interaction is carefully considered. The method of nonequilibrium phonon wave packet, developed by us, is generalized to include the simultaneous presence of two-dimensional electrons and holes. The occurrence of negative absolute mobility for electrons is discussed. The mobility of minority electrons and majority holes are calculated by use of a drifted temperature model for both types of carriers. The mobilities of minority electrons (from negative to positive) as functions of lattice temperature and electric field are shown. Comparison is made with experiment.

I. INTRODUCTION

Recently there has been considerable interest in the two-dimensional (2D) semiconductor system for which electrons and holes coexist.[1] The photoexcited electron-hole (e-h) plasma in quasi-2D quantum wells have been extensively explored. The luminescence measurements have been used to derive informations about relaxation of this system. Recent development of spectroscopy techniques allows to directly study the effects of carrier-carrier scattering on the carrier relaxation in ultrafast processes ($<$ 1 psec).[2] On the other hand, the transport measurements of photoexcited electron-hole system have also provided some interesting results.[3,4] Recently, Höpfel, Shah, Wolff, and Gossard found that in a such system the minority electrons, which are injected by laser pumps on the *p*-modulation-doped quantum wells, can move along the positive direction of external electric field.[4] The negative absolute mobility of electrons occurs because of strong electron-hole drag and was first pointed out by McLean and Paige.[5]

The purpose of this paper is to theoretically study transport of an electron-hole system in a quasi-2D quantum well, with emphasis on the screening effect and the nonequilibrium optical phonon effect in the presence of electron-hole interactions. We calculate the mobility of minority electrons in a majority hole gas and show that the mobility of minority electrons is negative at low temperature and in a weak external electric field.

In a 2D electron-hole system the interaction between electrons and holes has a strength the same order as the electron-electron and hole-hole interactions. The following facts, therefore, should be taken into account in the calculation of the relaxation of a 2D e-h system: (1) The many body screening effect should be expressed by a more complex form than that for single type of carriers, especially, the electron-lattice interaction and the hole-lattice interaction are coupled with each other through e-h coupling. (2) The subband energies and subband wave functions for electrons and holes are determined under a common electrostatic potential, which in turn is produced by contributions from the density distributions of both electrons and holes. (3) The e-h scattering, which leads to the energy transfer and momentum transfer between electrons and holes, should be included in the dynamical equations of carriers. (4) Lattice excitations build up through both electron-phonon interaction and hole-phonon interactions. The emission and absorption of common nonequilibrium phonons by electrons and holes produce a special mechanism for exchange of momentum and energy between two types of carriers. Since in a quasi-2D system a better description of the nonequilibrium phonons is in the "phonon wave packet" representation related to the subband wave function of carriers,[6,7] a phonon wave packet representation involving both electron and hole subband wave functions should be introduced. In our following calculation (1), (3), and (4) are carefully considered, but (2) is treated by introducing some approximation for simplification.

We derive the condition for negative mobility of electrons. Our formula indicates that the negative absolute mobility for electrons occurs only in the case of minority electrons and is determined by competition between electron-hole drag and hole-lattice relaxation. Using the drifted temperature model for both electrons and holes and introducing a coordinate transform to the center-of-mass systems, separately, for electrons and holes, we obtain a set of coupled equations for drift velocities for electrons and holes, ($\mathbf{v}_e$, $\mathbf{v}_h$), and corresponding carrier temperatures, (T_e, T_h). This set of equations together with kinetic equations for nonequilibrium optical phonons have been numerically solved in the cases of weak electric field and strong electric field. Our results show that the mobility for minority electrons is negative at low lattice temperature and in a weak field. This mobility increases with increasing of lattice temperature or increasing of electric field and then becomes positive. These results are in reasonable agreement with the experimental measurements.[4]

The paper is organized as follows: In Sec. II the expressions for screened carrier-carrier and carrier-lattice (impurities) scattering matrix elements are given. In Sec. III we discuss the condition for negative absolute mobility for electrons and derive the set of balance equations for carriers and the kinetic equations for nonequilibrium optical phonons. In Sec. IV the numerical results are shown and a discussion is devoted.

II. SCREENED CARRIER-CARRIER AND CARRIER-LATTICE INTERACTION

The electrons and holes in a quasi-2D quantum well are quantized in the direction normal to the 2D plane (z direction), while they are essentially free to move in the plane [$\mathbf{r} \equiv (x, y)$]. The wave functions for electrons and holes can be described by

$$\Psi^{\mu}(\mathbf{r}, z) = \frac{1}{A^{1/2}} \exp(i\mathbf{k}\cdot\mathbf{r})\, \xi_j^{\mu}(z) , \tag{1}$$

where μ = e (for electrons), h (for holes) labels the types of carriers. $\xi_j^{\mu}(z)$ is the corresponding envelope function in z direction with $j=1,2,\ldots, J_{\mu}$ the corresponding subband indices. A is the area of the layer of the sample. $\mathbf{k}$ is the wave vector of the μ-type carriers in the 2D plane. The unscreened electron-electron, hole-hole, and electron-hole scatterings occur via Coulomb interactions, which can be described by $(J_{\mu})^2 \times (J_{\nu})^2$ matrices, $V^{\mu\nu}(q)$. Their elements are given by

$$[V^{\mu\nu}(q)]_{i'i,\, j'j} = \frac{2\pi e_{\mu} e_{\nu}}{\epsilon_0 q A} F^{\mu\nu}_{i'i,\, j'j}(q) , \tag{2}$$

with the form factor

$$F^{\mu\nu}_{i'i,\, j'j}(q) = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} dz'\, e^{-q|z-z'|}\, \xi^{\mu}_{i'}{}^{*}(z)\xi^{\mu}_{i}(z)\xi^{\nu}_{j'}{}^{*}(z')\xi^{\nu}_{j}(z') . \tag{3}$$

Here we denote that the first group of sub-indices of subbands ($i'i$) corresponds to the first sup-index of carriers (μ) and second sub-indices ($j'j$) to the second sup-index (ν); the same notation rule will be used hereafter. $e_{\mu} = \pm e$ for hole and electron, respectively, and ϵ_0 is the static dielectric constant. Here we denote $\mathbf{Q} \equiv (\mathbf{q}, q_z)$ as the 3D momentum exchange. The carriers are also scattering with lattice and impurities. The corresponding unscreened scattering matrix can be written as $M^{\mu-\mathrm{L}}_{i'i}(\mathbf{Q})$; here L = LO, TO, LA, TA, imp labels, respectively, longitudinal optical phonons (LO), transverse optical phonons (TO), longitudinal acoustic phonons (LA), transverse acoustic phonons (TA), and impurities (imp). The expressions for these matrices will be listed later.

Since the strength of electron-hole interaction is the same order as electron-electron interaction and hole-hole interaction, the dynamic screening effect should be expressed by a more complex form than that in the case of a single type of carrier. Also, the electron-lattice (impurity) interactions and hole-lattice (impurity) interactions are coupled with each other through the electron-hole interaction. These are shown in Fig. 1 in the random phase approximation (RPA). According to Fig. 1(a), the screened carrier-carrier scattering matrix, $\tilde{V}^{\mu\nu}$, satisfies the following equation:

$$\tilde{V}^{\mu\nu} = V^{\mu\nu} + \sum_{\eta} V^{\mu\eta}\Pi^{\eta}\tilde{V}^{\eta\nu} , \tag{4}$$

where Π^{η} is a diagonal matrix, its elements are the density-density correlation functions for 2D carriers, which are given by[8]

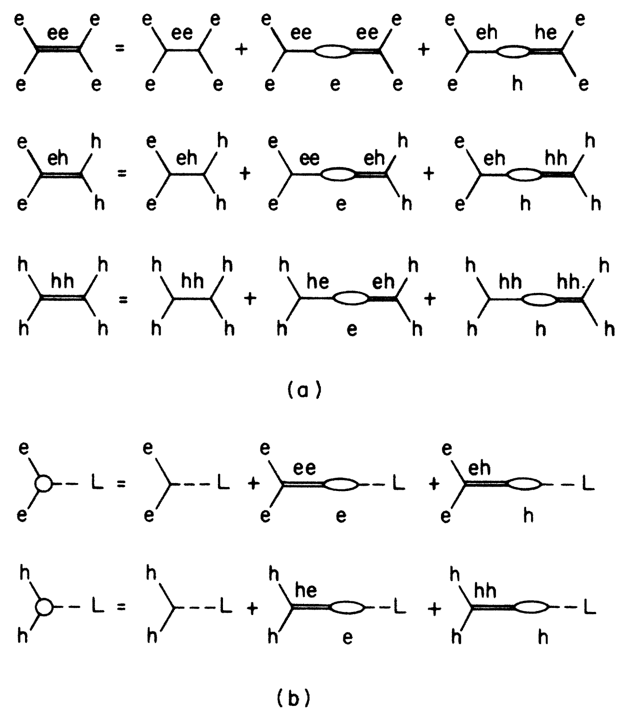

FIG. 1. (a) Diagrams for screening effect of carrier-carrier potential in the RPA. The single and double horizontal solid lines represent, respectively, the unscreened and screened carrier-carrier interaction, $V^{\mu\nu}$ and $\tilde{V}^{\mu\nu}$. The bubble represents the density-density correlation function, Π^{μ}. (b) Diagrams for screening effect of carrier-lattice (impurity) potential. The dashed line represents the phonon (impurity) line. The point and circle vertex represent, respectively, the unscreened and screened carrier-lattice scattering matrices, M and $\tilde{M}$.

$$\Pi^{\eta}_{i'i}(\mathbf{q},\omega) = 2\sum_{\mathbf{k}} \frac{f^{\eta}_{i',\mathbf{k}-\mathbf{q}} - f^{\eta}_{i,\mathbf{k}}}{\hbar\omega + E^{\eta}_{i',\mathbf{k}-\mathbf{q}} - E^{\eta}_{i,\mathbf{k}} + i\delta}, \quad (5)$$

with $f^{\eta}_{i,\mathbf{k}}$ the distribution function of η-type carriers in the $(i,\mathbf{k})$ state; $E^{\eta}_{i,\mathbf{k}} = E^{\eta}_i + \hbar^2k^2/2m_\eta$ is the energy of the state, with E^{η}_i the energy level of subband i (we use parabolic subband approximation). $\hbar\omega$ represents the energy exchange in the corresponding scattering process. If we define a $[(J_e)^2+(J_h)^2]\times[(J_e)^2+(J_h)^2]$ carrier-carrier potential matrix, $\tilde{V}$, as

$$\tilde{V} = \begin{bmatrix} \tilde{V}^{ee} & \tilde{V}^{eh} \\ \tilde{V}^{he} & \tilde{V}^{hh} \end{bmatrix}, \quad (6)$$

and define

$$\Pi = \begin{bmatrix} \Pi^{e} & 0 \\ 0 & \Pi^{h} \end{bmatrix}, \qquad V = \begin{bmatrix} V^{ee} & V^{eh} \\ V^{he} & V^{hh} \end{bmatrix}, \quad (7)$$

The screened carrier-carrier potential matrix can then be solved as

$$\tilde{V} = [\,1 - V\cdot\Pi\,]^{-1}\,V\,. \quad (8)$$

In the case that only the lowest band for electrons and holes are occupied, it can be straightforwardly written as

$$\tilde{V} = \frac{1}{\Delta}\begin{bmatrix} (1-V^{hh}\Pi^{h})V^{ee} + V^{eh}\Pi^{h}V^{he} & V^{eh} \\ V^{he} & (1-V^{ee}\Pi^{e})V^{hh} + V^{he}\Pi^{e}V^{eh} \end{bmatrix}, \quad (9)$$

with

$$\Delta = (1-V^{ee}\Pi^{e})(1-V^{hh}\Pi^{h}) - V^{eh}\Pi^{h}V^{he}\Pi^{e}\,. \quad (10)$$

According to Fig. 1(b), the screened matrices for carrier-lattice (impurity) interactions, $\tilde{M}^{\mu-L}$, are given by

$$\begin{bmatrix} \tilde{M}^{e-L} \\ \tilde{M}^{h-L} \end{bmatrix} = \begin{bmatrix} 1+\tilde{V}^{ee}\Pi^{e} & \tilde{V}^{eh}\Pi^{h} \\ \tilde{V}^{he}\Pi^{e} & 1+\tilde{V}^{hh}\Pi^{h} \end{bmatrix} \begin{bmatrix} M^{e-L} \\ M^{h-L} \end{bmatrix}. \quad (11)$$

Here we denote $\tilde{M}^{\mu-L}$ and $M^{\mu-L}$ as column vectors with component indices $(j'j)$. We notice that this kind of coupling between electron-lattice interaction and hole-lattice interaction must also be included even in the 3D electron-hole system.

In the case that electrons and holes coexist, the electrostatic potential, which induces a self-consistent heterojunction structure, is common for both electrons and holes and is produced by both electrons and holes. Therefore the electrostatic potential energy is repulsive for electrons, when it is attractive for holes. This situation appears in the p-modulation-doped quantum well, where electrons are a minority. In this case electrons can only be bound by a quantum well structure, whereas holes can be bound by the self-consistent heterojunction well. The existence of minority electrons, on the other hand, weakens the potential which binds holes. This difference between envelope function of electrons and that of holes, generally speaking, can affect the strength of electron-hole coupling through the form factor $F^{eh}(q)$ in Eq. (3). A detailed analysis depends on the parameters of carrier density, n_e, n_h, and the width of quantum well, d. In the recent experiments,[4] $d = 112$ Å, $n_e \sim 3\times10^{10}$ cm^{-2}, and $n_h = 1.6\times10^{11}$ cm^{-2} (for double heterojunctions in a quantum well). Using a variational function,[9] we estimate that the peak of $\xi^{h}_1(z)$ for the lowest subband is located near $0.5d$. In the following calculation, therefore, we assume that holes are also bound by a quantum well of width d. Under the assumption of infinite square well structure the form factors $F^{ee}(q)$, $F^{hh}(q)$, and $F^{eh}(q)$ have the same structure.

Finally, we list the unscreened matrices for carrier-lattice (impurities) scattering.[10] The electron-LO-phonon scattering occurs via polar Fröhlich interaction:

$$M^{e-LO}_{i'i}(Q) = i\frac{\alpha}{Q}H^{e}_{i'i}(-iq_z)\,, \quad (12)$$

with α the Fröhlich coupling constant, $\alpha = [2\pi e^2\hbar\omega_{LO}(1/\epsilon_\infty - 1/\epsilon_0)]^{1/2}$, with ω_{LO} the frequency of LO phonons and ϵ_∞ the high frequency dielectric constant. Here we define

$$H^{\mu}_{i'i}(p) = \int_{-\infty}^{\infty} dz\, \xi^{\mu\,*}_{i'}(z)e^{-pz}\xi^{\mu}_i(z)\,. \quad (13)$$

The hole-LO-phonon scattering occurs via both polar and deformation potential interactions:

$$M^{h-LO}_{j'j}(Q) = \left[-i(K_p)^{1/2}\frac{\alpha}{Q} + \frac{(D_tK)\hbar}{(2\rho\hbar\omega_{LO})^{1/2}} \right] H^{h}_{j'j}(-iq_z)\,, \quad (14)$$

where $D_tK = (\frac{3}{2})^{1/2}d_o/a_o$, with d_o the optical deformation potential constant and a_o the lattice constant. ρ is the mass density of the crystal, K_p the correction factor due to p-type wave functions for holes and existence of light holes.[11] The hole-TO-phonon (2 branches) scattering occurs via the deformation potential interaction:

$$M^{h-TO}_{j'j}(Q) = \frac{(D_tK)\hbar}{(2\rho\hbar\omega_{TO})^{1/2}}H^{h}_{j'j}(-iq_z)\,. \quad (15)$$

The electron-LA-phonon scattering occurs via both deformation potential and piezoelectric interactions:

$$M_{i'i}^{\mathrm{e-LA}}(\mathbf{Q}) = \left[\left[\frac{\hbar \Xi_{\mathrm{e}}^2 Q}{2\rho v_{\mathrm{L}}} \right]^{1/2} + i\frac{8\pi e e_{14}}{\epsilon_0 Q^2} \left[\frac{\hbar}{2\rho v_{\mathrm{L}} Q} \right]^{1/2} [q_x q_y \mathrm{e}_z^{\parallel}(\mathbf{Q}) + q_z q_x \mathrm{e}_y^{\parallel}(\mathbf{Q}) + q_y q_z \mathrm{e}_x^{\parallel}(\mathbf{Q})] \right] H_{i'i}^{\mathrm{e}}(-iq_z) . \quad (16)$$

The hole-LA-phonon scattering occurs also via the deformation potential and piezoelectric interactions:

$$M_{j'j}^{\mathrm{h-LA}}(\mathbf{Q}) = \left[\left[\frac{\hbar \Xi_{\mathrm{h}}^2 Q}{2\rho v_{\mathrm{L}}} \right]^{1/2} - i\frac{8\pi e e_{14}}{\epsilon_0 Q^2} \left[\frac{\hbar}{2\rho v_{\mathrm{L}} Q} \right]^{1/2} [q_x q_y \mathrm{e}_z^{\parallel}(\mathbf{Q}) + q_z q_x \mathrm{e}_y^{\parallel}(\mathbf{Q}) + q_y q_z \mathrm{e}_x^{\parallel}(\mathbf{Q})] \right] H_{j'j}^{\mathrm{h}}(-iq_z) . \quad (17)$$

The electron-TA-phonon scattering occurs via the piezoelectric interaction:

$$M_{i'i}^{\mathrm{e-TA}}(\mathbf{Q}) = \sum_{j=1}^{2} i\frac{8\pi e e_{14}}{\epsilon_0 Q^2} \left[\frac{\hbar}{2\rho v_{\mathrm{T}} Q} \right]^{1/2} [q_x q_y \mathrm{e}_{jz}^{\perp}(\mathbf{Q}) + q_z q_x \mathrm{e}_{jy}^{\perp}(\mathbf{Q}) + q_y q_z \mathrm{e}_{jx}^{\perp}(\mathbf{Q})] H_{i'i}^{\mathrm{e}}(-iq_z). \quad (18)$$

The hole-TA-phonon scattering occurs via the piezoelectric interaction and its scattering matrix elements can be written in a similar way. In Eqs. (16)–(18), v_{L} and v_{T} are the longitudinal- and transverse-sound speeds; Ξ_μ is the acoustic phonon deformation potential constant for μ type carriers; e_{14} is the piezoelectric constant; $\mathrm{e}^{\parallel}(\mathbf{Q})$ and $\mathrm{e}_{\mathrm{j}}^{\perp}(\mathbf{Q})$ are unit polarization vectors for longitudinal and transverse modes, respectively. The carrier-remote ionized doped impurities scattering can be described by[12]

$$M_{i'i}^{\mu-\mathrm{imp}}(q) = (N_I)^{1/2}(2\pi e_I e_\mu)\exp(-qs) H_{i'i}^{\mu}(q)/(\epsilon_0 q) , \quad (19)$$

with e_I the charge of impurity, N_I the density of impurities, and s the undoped spacer distance. The trapped holes on the $\mathrm{Ga}_{1-x}\mathrm{Al}_x\mathrm{As}$ side due to photoexcitation[13] can also be modeled as a kind of impurity with a density of trapped holes, N_H, and the effective distance of layer of trapped holes from a layer of carriers, s_H.

III. DYNAMICAL EQUATIONS FOR CARRIERS AND PHONONS

We first briefly discuss the condition for negative absolute mobility of electrons in the region of weak electric field where the conductivity is linear. In the steady state we have the force balance equation for electrons:

$$n_\mu e_\mu \mathbf{E} - \mathbf{F}^{\mu-\nu} - \mathbf{F}^{\mu-\mathrm{L}} = 0 , \quad (20)$$

where $\mathbf{E}$ is the external electric field, $\mathbf{F}^{\mu-\mathrm{L}}$ represents the frictional force due to the carrier-lattice interaction, and $\mathbf{F}^{\mu-\nu}$ represents the frictional force upon the μ-type carriers due to the carrier-carrier interaction with the ν-type carriers. It is obvious that $\mathbf{F}^{\mu-\nu} = -\mathbf{F}^{\nu-\mu}$. In the region of linear conductivity we have

$$\mathbf{F}^{\mu-\mathrm{L}} = n_\mu A^{\mu-\mathrm{L}} \mathbf{v}_\mu , \quad (21a)$$

$$\mathbf{F}^{\mu-\nu} = n_\mu n_\nu A^{\mu-\nu}(\mathbf{v}_\mu - \mathbf{v}_\nu) , \quad (21b)$$

where $\mathbf{v}_\mu$ is the drift velocity of μ type of carriers, $A^{\mu-\mathrm{L}}$ represents the contribution to the resistivity (per carrier) from μ-L scattering, and $A^{\mu-\nu}$ relates to the contribution to the resistivity (per carrier μ) from μ-ν scattering normalized to per carrier ν. Their expressions will be discussed later. From Eqs. (20) and (21) we immediately obtain the expression for the mobility for electrons, μ_{e}:

$$\mu_{\mathrm{e}} = |e| \frac{(n_{\mathrm{e}} - n_{\mathrm{h}}) A^{\mathrm{e-h}} + A^{\mathrm{h-L}}}{A^{\mathrm{e-L}} A^{\mathrm{h-L}} + n_{\mathrm{e}} A^{\mathrm{e-h}} A^{\mathrm{e-L}} + n_{\mathrm{h}} A^{\mathrm{e-h}} A^{\mathrm{h-L}}} . \quad (22)$$

Equation (22) indicates that for electrons a negative absolute mobility is only possible when $n_{\mathrm{e}} < n_{\mathrm{h}}$. (The corresponding statement for hole mobility could require $n_{\mathrm{h}} < n_{\mathrm{e}}$.) It is determined by the difference between the density of electrons and the density of holes, and the competition between electron-hole drag and hole-lattice scattering. At low temperature and under a weak electric field, the former dominates and the mobility of electrons is negative. When the lattice temperature or the electric field increase, the latter tends to dominate and the mobility of minority electrons becomes positive. Equation (22) reduces to Eq. (3) of Ref. 4 in the limit when $n_{\mathrm{e}} \ll n_{\mathrm{h}}$.

The carrier dynamics is described by a semiclassical nonlinear Boltzmann equation. We do not attempt a numerical solution of the kinetic equation, but simply make a standard ansatz on the form of the distribution functions of carriers. We assume that the distribution function of each type of carrier, $f_{i\mathbf{k}}^{\mu}$, can be described by a drifted Fermi-Dirac distribution at carrier temperature T_μ and with drift velocity $\mathbf{v}_\mu$. We also separate the center-of-mass motion of each type of carrier from its relative motion. The 2D momentum for μ-type carriers in the relative coordinates is defined as $\hbar\overline{\mathbf{k}} = \hbar\mathbf{k} - m_\mu \mathbf{v}_\mu$ and the distribution function in the relative coordinates, $\overline{f}_{i\overline{\mathbf{k}}}^{\mu}$, is a Fermi-Dirac function at temperature T_μ. The exchange of momentum and energy in the relative coordinates can then be written as

$$\overline{\mathbf{k}} - \overline{\mathbf{k}}' = \mathbf{q} , \qquad \overline{E}_{i\overline{\mathbf{k}}}^{\mu} - \overline{E}_{i'\overline{\mathbf{k}}'}^{\mu} = \hbar(\omega - \mathbf{q}\cdot\mathbf{v}_\mu) . \quad (23)$$

A set of coupled equations for the rate of change of drift velocities of center-of-mass and the rate of change of energies in the relative coordinates for each type of carrier can be derived from Boltzmann equations for the carriers.[14] We obtain

$$\frac{\partial n_\mu m_\mu \mathbf{v}_\mu(t)}{\partial t} = e_\mu n_\mu \mathbf{E} - \sum_{\mathrm{L}} \mathbf{F}^{\mu-\mathrm{L}}(t) - \mathbf{F}^{\mu-\nu}(t) , \quad (24)$$

and

$$\frac{\partial \bar{E}^{\mu}(t)}{\partial t} = -\sum_{\mathrm{L}} \frac{\partial \bar{E}^{\mu-\mathrm{L}}(t)}{\partial t} - \frac{\partial \bar{E}^{\mu-\nu}(t)}{\partial t} . \quad (25)$$

$\mathbf{F}^{\mu-\nu}$, which represents the frictional force due to e-h scattering, with ν a different type of carrier than μ, is given by

$$\mathbf{F}^{\mu-\nu} = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} \sum_{i',i} \sum_{j',j} \sum_{\bar{\mathbf{k}}',\bar{\mathbf{k}}} \sum_{\bar{\mathbf{p}}',\bar{\mathbf{p}}} \hbar\mathbf{q}\, \bar{f}^{\mu}_{i\bar{\mathbf{k}}}(1-\bar{f}^{\mu}_{i'\bar{\mathbf{k}}'})\bar{f}^{\nu}_{j\bar{\mathbf{p}}}(1-\bar{f}^{\nu}_{j'\bar{\mathbf{p}}'})\, |\, \tilde{V}^{\mu\nu}_{i'i,\, j'j}(\mathbf{q},\, \bar{\omega})\,|^{2} \delta_{\bar{\mathbf{k}}',\bar{\mathbf{k}}-\mathbf{q}} \delta_{\bar{\mathbf{p}}',\bar{\mathbf{p}}+\mathbf{q}}$$

$$\times \delta[\bar{E}^{\mu}_{i\bar{\mathbf{k}}} + \bar{E}^{\nu}_{j\bar{\mathbf{p}}} - \bar{E}^{\mu}_{i'\bar{\mathbf{k}}'} - \bar{E}^{\nu}_{j'\bar{\mathbf{p}}'} + \hbar\mathbf{q}\cdot(\mathbf{v}_{\mu}-\mathbf{v}_{\nu})] , \quad (26)$$

where the term $\hbar\mathbf{q}\cdot(\mathbf{v}_{\mu}-\mathbf{v}_{\nu})$ appears in the last δ function because the energies $\bar{E}$ for electrons and holes are defined in different coordinates, and $\bar{\omega} = (\bar{E}^{\mu}_{i\bar{\mathbf{k}}} - \bar{E}^{\mu}_{i'\bar{\mathbf{k}}'})/\hbar$. The last term in Eq. (25) represents the energy loss rate in relative system due to e-h scattering and can be obtained from Eq. (26) by replacing $\hbar\mathbf{q}$ by $\bar{E}^{\mu}_{i\bar{\mathbf{k}}} - \bar{E}^{\mu}_{i'\bar{\mathbf{k}}'}$.

$\mathbf{F}^{\mu-\mathrm{L}}$ in Eq. (24) represents the frictional force due to carrier-lattice (impurity) interaction (L = LO, TO, LA, TA, imp) and is given by[15]

$$\mathbf{F}^{\mu-\mathrm{L}} = \sum_{\mathbf{q}} \hbar\mathbf{q}\Lambda^{\mu-\mathrm{L}}(\mathbf{q},\, t) . \quad (27a)$$

The rate of change of energy due to carrier-lattice (impurity) collisions, $\partial\bar{E}^{\mu-\mathrm{L}}/\partial t$ in Eq. (25), is given by

$$\frac{\partial \bar{E}^{\mu-\mathrm{L}}(t)}{\partial t} = \sum_{\mathbf{q}} (\hbar\omega_{\mathrm{L}} - \hbar\mathbf{q}\cdot\mathbf{v}_{\mu})\Lambda^{\mu-\mathrm{L}}(\mathbf{q},\, t) , \quad (27b)$$

with ω_{L} the frequency of corresponding phonons (and $\omega_{\mathrm{imp}} = 0$). For carrier-acoustic-phonon scattering (L = LA, TA), the nonequilibrium phonon effect is not important, and $\Lambda^{\mu-\mathrm{L}}$ in Eq. (27) is given by

$$\Lambda^{\mu-\mathrm{L}}(\mathbf{q},\, t) = \frac{2\pi}{\hbar} \sum_{i',\, i} \sum_{q_z} |\, \tilde{M}^{\mu-\mathrm{L}}_{i'i}(\mathbf{Q},\, \bar{\omega})\,|^{2} \{ I^{\mu(+)}_{i'i}(\mathbf{q},\, \bar{\omega})[1 + n^{\mathrm{L}}(T_L)] - I^{\mu(-)}_{i'i}(\mathbf{q},\, \bar{\omega}) n^{\mathrm{L}}(T_L) \} , \quad (28)$$

with $\bar{\omega} = \omega_{\mathrm{L}} - \mathbf{q}\cdot\mathbf{v}_{\mu}$ and $n^{\mathrm{L}}(T_L)$ the equilibrium occupation number of L-type phonons at the lattice temperature, T_L; $\omega_{\mathrm{LA}} = v_{\mathrm{L}}Q$ and $\omega_{\mathrm{TA}} = v_{\mathrm{T}}Q$ denote the frequency of longitudinal- and transverse-acoustic phonons, and

$$I^{\mu(\pm)}_{i'i}(\mathbf{q},\, \bar{\omega}) = \frac{2}{A} \sum_{\bar{\mathbf{k}}} \sum_{\bar{\mathbf{k}}'} \bar{f}^{\mu}_{i\bar{\mathbf{k}}}(t)[1 - \bar{f}^{\mu}_{i'\bar{\mathbf{k}}'}(t)] \times \delta_{\bar{\mathbf{k}}',\, \bar{\mathbf{k}}\mp\mathbf{q}} \delta(\bar{E}^{\mu}_{i\bar{\mathbf{k}}} - \bar{E}^{\mu}_{i'\bar{\mathbf{k}}'} \mp \hbar\bar{\omega}). \quad (29)$$

For carrier-impurity scattering (L = imp), we have

$$\Lambda^{\mu-\mathrm{imp}} = \frac{2\pi}{\hbar} \sum_{i',\, i} |\, \tilde{M}^{\mu-\mathrm{imp}}_{i'i}(\mathbf{q},\, -\mathbf{q}\cdot\mathbf{v}_{\mu})\,|^{2} \times I^{\mu(+)}_{i'i}(\mathbf{q},\, -\mathbf{q}\cdot\mathbf{v}_{\mu}) . \quad (30)$$

The screened scattering matrix elements $\tilde{M}$ in Eqs. (28) and (30) are calculated by Eq. (11), in which the unscreened elements have been listed in Sec. II. The same screening calculation is performed for carrier–optical-phonon scattering. For carrier–optical-phonon scattering (L = LO, TO), the effect of a nonequilibrium population of optical phonons is important and needs to be included.[16] We generalize the description of hot phonon dynamics in quantum well structures proposed by us[6] to the case that electrons and holes coexist.[17] In the plane wave representation the one-body density matrix is nondiagonal in the component q_z due to strong spatial inhomogeneity introduced by carrier confinement. The optical phonon density matrix is then expressed by $n^{\mathrm{L}}_{\mathbf{q}}(q_z,\, q_z',\, t)$ (L = LO, TO). The phonon kinetic equation is given by

$$\frac{\partial n^{\mathrm{L}}_{\mathbf{q}}(q_z,\, q_z',\, t)}{\partial t} = \left[\frac{\partial n^{\mathrm{L}}_{\mathbf{q}}(q_z,\, q_z',\, t)}{\partial t} \right]_{\mathrm{L-L}} + \left[\frac{\partial n^{\mathrm{L}}_{\mathbf{q}}(q_z,\, q_z',\, t)}{\partial t} \right]_{\mathrm{c-L}} . \quad (31)$$

The first term on the right hand side of Eq. (31) denotes the contribution from phonon-phonon collisions. Here we use a single relaxation time ansatz,

$$\left[\frac{\partial n^{\mathrm{L}}_{\mathbf{q}}(q_z,q_z',t)}{\partial t} \right]_{\mathrm{L-L}} = - \frac{n^{\mathrm{L}}_{\mathbf{q}}(q_z,q_z',t) - \delta_{q_z,q_z'} n^{\mathrm{L}}(T_L)}{\tau^{\mathrm{L}}(T_L)} , \quad (32)$$

The relaxation time, $\tau^{\mathrm{L}}(T_L)$, is an experimental parameter assumed independent of wave vector.[18] The second term on the right hand side of Eq. (31) represents the contribution from carrier-phonon collisions, which includes the contributions from both electrons and holes, and is given by

$$\left[\frac{\partial n_{\mathbf{q}}^{\mathrm{L}}(q_z, q_z', t)}{\partial t}\right]_{\mathrm{c-L}} = \lim_{\epsilon\to 0^+}\frac{2}{\hbar A}\sum_{\rho}\sum_{i,\bar{\mathbf{k}}}\sum_{i',\bar{\mathbf{k}}'}\sum_{q_z''} f_{i\bar{\mathbf{k}}}^{\rho}(t)\,[\,1 - f_{i'\bar{\mathbf{k}}'}^{\rho}(t)]\,[\,1 + \hat{P}(q_z, q_z')]$$
$$\times\left[\delta_{\bar{\mathbf{k}}',\bar{\mathbf{k}}-\mathbf{q}}\frac{\widetilde{M}_{i'i}^{\rho-\mathrm{L}}(\mathbf{q}, q_z, \varpi)\,\widetilde{M}_{ii'}^{\rho-\mathrm{L}\,*}(\mathbf{q}, q_z'', \varpi)}{\epsilon - i(\bar{E}_{i\bar{\mathbf{k}}}^{\rho} - \bar{E}_{i'\bar{\mathbf{k}}'}^{\rho} - \hbar\varpi)}[\delta_{q_z'', q_z'} + n_{\mathbf{q}}^{\mathrm{L}}(q_z'', q_z', t)]\right.$$
$$\left. - \delta_{\bar{\mathbf{k}}',\bar{\mathbf{k}}+\mathbf{q}}\frac{\widetilde{M}_{i'i}^{\rho-\mathrm{L}}(\mathbf{q}, q_z, \varpi)\,\widetilde{M}_{ii'}^{\rho-\mathrm{L}\,*}(\mathbf{q}, q_z'', \varpi)}{\epsilon + i(\bar{E}_{i\bar{\mathbf{k}}}^{\rho} - \bar{E}_{i'\bar{\mathbf{k}}'}^{\rho} + \hbar\varpi)} n_{\mathbf{q}}^{\mathrm{L}}(q_z'', q_z', t)\right], \tag{33}$$

with $\varpi = \omega_{\mathrm{L}} - \mathbf{q}\cdot\mathbf{v}_\rho$. The operator $\hat{P}(q_z, q_z')$ acts on any function $F(q_z, q_z')$ as $\hat{P}F(q_z, q_z') = F^*(q_z', q_z)$. $\Lambda^{\mu-L}(\mathbf{q}, t)$ for L = LO or TO in Eq. (27) is then given by

$$\Lambda^{\mu-\mathrm{L}}(\mathbf{q}, t) = \frac{2\pi}{\hbar}\sum_{i',i}\sum_{q_z}\sum_{q_z'}\widetilde{M}_{i'i}^{\mu-\mathrm{L}\,*}(\mathbf{q}, q_z, \varpi)\widetilde{M}_{i'i}^{\mu-\mathrm{L}}(\mathbf{q}, q_z', \varpi)$$
$$\times\{I_{i'i}^{\mu\,(+)}(\mathbf{q}, \varpi)[\delta_{q_z,q_z'} + n_{\mathbf{q}}^{\mathrm{L}}(q_z, q_z', t)] - I_{i'i}^{\mu\,(-)}(\mathbf{q}, \varpi)n_{\mathbf{q}}^{\mathrm{L}}(q_z, q_z', t)\}\,. \tag{34}$$

As pointed in Ref. 5, the phonon kinetic equation for $n_{\mathbf{q}}^{\mathrm{L}}(q_z, q_z', t)$, is an integral equation in q_z and cannot be solved easily. We perform a change of representation by introducing the "phonon wave packet." Generalizing the procedure in Ref. 5, we define

$$N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t) = \frac{\sum_{q_z}\sum_{q_z'}\widetilde{M}_{i'i}^{\mu-\mathrm{L}\,*}(\mathbf{q}, q_z, \varpi)n_{\mathbf{q}}^{\mathrm{L}}(q_z, q_z', t)\widetilde{M}_{j'j}^{\nu-\mathrm{L}}(\mathbf{q}, q_z', \varpi)}{\sum_{q_z}\widetilde{M}_{i'i}^{\mu-\mathrm{L}\,*}(\mathbf{q}, q_z, \varpi)\widetilde{M}_{j'j}^{\nu-\mathrm{L}}(\mathbf{q}, q_z, \varpi)}\,. \tag{35}$$

The set of kinetic equations for $N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t)$ is given by

$$\frac{\partial N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t)}{\partial t} = \left[\frac{\partial N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t)}{\partial t}\right]_{\mathrm{c-L}} - \frac{N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t) - n^{\mathrm{L}}(T_L)}{\tau_{\mathrm{L}}}, \tag{36}$$

with

$$\left[\frac{\partial N_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, t)}{\partial t}\right]_{\mathrm{c-L}} = \frac{1}{\hbar}\sum_{\rho}\sum_{l',l}\frac{\widetilde{G}_{i'i,\,l'l}^{\mu\rho,\,\mathrm{L}}(\mathbf{q}, \varpi)\,\widetilde{G}_{l'l,\,j'j}^{\rho\nu,\,\mathrm{L}}(\mathbf{q}, \varpi)}{\widetilde{G}_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, \varpi)}$$
$$\times\{\,\pi\, I_{l'l}^{\rho\,(+)}(\mathbf{q}, \varpi)\,[\,2 + N_{i'i,\,l'l}^{\mu\rho,\,\mathrm{L}}(\mathbf{q}, t) + N_{l'l,\,j'j}^{\rho\nu,\,\mathrm{L}}(\mathbf{q}, t)]$$
$$-\,\pi\, I_{l'l}^{\rho\,(-)}(\mathbf{q}, \varpi)\,[\,N_{i'i,\,l'l}^{\mu\rho,\,\mathrm{L}}(\mathbf{q}, t) + N_{l'l,\,j'j}^{\rho\nu,\,\mathrm{L}}(\mathbf{q}, t)]$$
$$-\,i\,[\,J_{l'l}^{\rho\,(+)}(\mathbf{q}, \varpi) + J_{ll'}^{\;\rho\,(-)}(\mathbf{q}, \varpi)\,]\,[\,N_{i'i,\,l'l}^{\mu\rho,\,\mathrm{L}}(\mathbf{q}, t) - N_{l'l,\,j'j}^{\rho\nu,\,\mathrm{L}}(\mathbf{q}, t)]\}\,, \tag{37}$$

and

$$\widetilde{G}_{i'i,\,j'j}^{\mu\nu,\,\mathrm{L}}(\mathbf{q}, \varpi) \equiv \sum_{q_z}\widetilde{M}_{i'i}^{\mu-\mathrm{L}\,*}(\mathbf{q}, q_z, \varpi)\,\widetilde{M}_{j'j}^{\nu-\mathrm{L}}(\mathbf{q}, q_z, \varpi)\,. \tag{38}$$

In Eq. (37) $I_{l'l}^{\rho\,(\pm)}$ is given by Eq. (29), and

$$J_{l'l}^{\rho\,(\pm)}(\mathbf{q}, \varpi) = \frac{2}{A}\sum_{\bar{\mathbf{k}}}\sum_{\bar{\mathbf{k}}'}\bar{f}_{l\bar{\mathbf{k}}}^{\rho}(t)\,[\,1 - \bar{f}_{l'\bar{\mathbf{k}}'}^{\rho}(t)]\,\delta_{\bar{\mathbf{k}}',\bar{\mathbf{k}}\mp\mathbf{q}}\,\mathrm{P}\,\frac{1}{\bar{E}_{l\bar{\mathbf{k}}}^{\rho} - \bar{E}_{l'\bar{\mathbf{k}}'}^{\rho} \mp \hbar\varpi}\,. \tag{39}$$

$\Lambda^{\mu-L}(\mathbf{q}, t)$ in Eq. (34) is now replaced by

$$\Lambda^{\mu-\mathrm{L}}(\mathbf{q},\,t) = \frac{2\pi}{\hbar}\sum_{i'\,i}\tilde{G}^{\mu\mu,\,\mathrm{L}}_{i'i,\,i'i}(\mathbf{q},\,\varpi)\{I^{\mu,(+)}_{i'i}(\mathbf{q},\,\varpi)[1 + N^{\mu\mu,\,\mathrm{L}}_{i'i,\,i'i}(\mathbf{q},\,t)] - I^{\mu,(-)}_{i'i}(\mathbf{q},\,\varpi)N^{\mu\mu,\,\mathrm{L}}_{i'i,\,i'i}(\mathbf{q},\,t)\}\ . \tag{40}$$

We notice that $\Lambda^{\mathrm{e-L}}$, for example, in Eq. (40), which is related to the energy and momentum loss rate of electrons due to e-L scattering, is only concerned with the diagonal elements of matrix $N^{\mathrm{ee,\,L}}$ [we define a $(J_\mu)^2\times(J_\nu)^2$ matrix $N^{\mu\nu,\,\mathrm{L}}$, with elements $N^{\mu\nu,\,\mathrm{L}}_{i'i,\,j'j}(\mathbf{q},\,t)$]. The latter, however, couples with other matrices $N^{\mathrm{eh,\,L}}$, $N^{\mathrm{he,\,L}}$, and $N^{\mathrm{hh,\,L}}$ via Eqs. (36) and (37). Therefore, even in the case that only the lowest subbands for electrons and holes are occupied, a set of four coupled equations for L-type phonons needs to be solved. This reflects the fact that each type of phonon is emitted and reabsorbed by both electrons and holes, and exchange of nonequilibrium phonons provides a special mechanism for exchange of the energy and momentum between electrons and holes.

Equations (24) and (25) for electrons and holes and Eq. (36) for LO and TO phonons consist of a set of coupled equations for T_e, T_h, $\mathbf{v}_\mathrm{e}$, $\mathbf{v}_\mathrm{h}$, and distribution functions for nonequilibrium LO and TO phonons. In the following calculation we assume electrons and holes are nondegenerate (Fermi temperature $T_\mathrm{F} = 8.9$ K for holes when $n_\mathrm{h} = 1.6\times10^{11}\ \mathrm{cm}^{-1}$, and $T_\mathrm{e} = 11.6$ K for electrons when $n_\mathrm{e} = 3\times10^{10}\ \mathrm{cm}^{-1}$). The distribution function for each type of carrier in its relative coordinates then is Maxwellian:

$$\bar{f}^{\mu}_{i\bar{\mathbf{k}}} = \frac{\pi C_\mu n_\mu \hbar^2}{m_\mu k_B T_\mu}\exp\left[-\frac{\bar{E}^{\mu}_{i\bar{\mathbf{k}}}}{k_B T_\mu}\right], \tag{41}$$

with $C_\mu = [\ \sum_i \exp(-E^\mu_i/k_B T_\mu)\ ]^{-1}$. Under this assumption the following quantities have the analytical expressions:

$$\mathrm{Re}\Pi^{\mu}_{i'i}(\mathbf{q},\,\varpi) = \left[\frac{2m_\mu}{k_B T_\mu}\right]^{1/2}\frac{C_\mu n_\mu}{\hbar q}[\mathrm{sgn}(\eta_-)D(\eta_-)\exp(-E^\mu_{i'}/k_B T_\mu) - \mathrm{sgn}(\eta_+)D(\eta_+)\exp(-E^\mu_i/k_B T_\mu)], \tag{42}$$

$$\mathrm{Im}\Pi^{\mu}_{i'i}(\mathbf{q},\,\varpi) = -\left[\frac{\pi m_\mu}{2k_B T_\mu}\right]^{1/2}\frac{C_\mu n_\mu}{\hbar q}[\exp(-\eta_-^2)\exp(-E^\mu_{i'}/k_B T_\mu) - \exp(-\eta_+^2)\exp(-E^\mu_i/k_B T_\mu)]\ . \tag{43}$$

Here $D(x)$ is Dawson's integral:[19]

$$D(x) = \exp(-x^2)\int_0^{|x|} dt\ \exp(t^2)\ , \tag{44}$$

and

$$\eta_\pm = \left[\frac{m_\mu}{2\hbar^2 q^2 k_B T_\mu}\right]^{1/2}\left[\hbar\varpi + (E^\mu_{i'} - E^\mu_i) \pm \frac{\hbar^2 q^2}{2m_\mu}\right]. \tag{45}$$

We also have

$$I^{\mu,(\pm)}_{i'i}(\mathbf{q},\,\varpi) \approx \left[\frac{m_\mu}{2\pi k_B T_\mu}\right]^{1/2}\frac{C_\mu n_\mu}{\hbar q}\exp(-\gamma_\pm^2)\exp(-E^\mu_i/k_B T_\mu)\ , \tag{46}$$

with

$$\gamma_\pm = \left[\frac{m_\mu}{2\hbar^2 q^2 k_B T_\mu}\right]^{1/2}\left[\hbar\varpi \pm (E^\mu_{i'} - E^\mu_i) \pm \frac{\hbar^2 q^2}{2m_\mu}\right]. \tag{47}$$

and

$$J^{\mu,(+)}_{i'i}(\mathbf{q},\,\varpi) + J^{\mu,(-)}_{ii'}(\mathbf{q},\,\varpi) = \mathrm{Re}\Pi^{\mu}_{i'i}(\mathbf{q},\,\varpi)\ . \tag{48}$$

The following integral in Eq. (26) is given by

$$\sum_{\bar{\mathbf{p}}',\bar{\mathbf{p}}} \bar{f}^{\nu}_{j\bar{\mathbf{p}}}(1-\bar{f}^{\nu}_{j'\bar{\mathbf{p}}'})\delta_{\bar{\mathbf{p}}',\,\bar{\mathbf{p}}+\mathbf{q}}\,\delta[\bar{E}^{\mu}_{i\bar{\mathbf{k}}}+\bar{E}^{\nu}_{j\bar{\mathbf{p}}}-\bar{E}^{\mu}_{i'\bar{\mathbf{k}}'}-\bar{E}^{\nu}_{j'\bar{\mathbf{p}}'}+\hbar\mathbf{q}\cdot(\mathbf{v}_\mu-\mathbf{v}_\nu)] \approx \left[\frac{m_\nu}{2\pi k_B T_\nu}\right]^{1/2}\frac{C_\nu n_\nu}{\hbar q}\exp(-\chi_\nu)\exp(-E^\nu_j/k_B T_\nu)\ , \tag{49}$$

with

$$\chi_\nu = \frac{m_\nu}{2\hbar^2 q^2 k_B T_\nu}\left[E^\mu_i + E^\nu_j - E^\mu_{i'} - E^\nu_{j'} + \frac{\hbar^2\bar{\mathbf{k}}\cdot\mathbf{q}}{m_\mu} - \hbar^2 q^2\left[\frac{1}{m_\mu}+\frac{1}{m_\nu}\right] + \hbar\mathbf{q}\cdot(\mathbf{v}_\mu - \mathbf{v}_\nu)\right]^2. \tag{50}$$

In the case of a weak electric field applied, we can expand Eqs. (46) and (49) to first order in the drift velocity and substitute them into Eqs. (26) and (27a). The expressions for $A^{\mu-L}$ and $A^{\mu-\nu}$ in Eq. (21) are then obtained directly.

IV. RESULTS AND DISCUSSION

We have solved numerically the above set of coupled equations describing the dynamics of carriers and optical phonons in nonequilibrium steady state in a GaAs-$Ga_{1-x}Al_xAs$ quantum well. The subband wave functions for electrons and holes are obtained by assuming that the carriers are bound in an infinite high square quantum well with well width $d = 112$ Å. For simplicity, we only consider here the case where the carriers occupy the lowest subband, separately, for electrons and holes. The parameters used in calculation are listed in Table I.[20] The density of carriers is chosen as $n_h = 1.6\times10^{11}$ cm^{-2} and $n_e = 3\times10^{10}$ cm^{-2}. The density of impurities (with undoped spacer distance $s = 294$ Å) $n_{imp} = 3\times10^{12}$ cm^{-2} is chosen so the hole mobility at 4.2 K is $\mu_h = 5.38\times10^4$ cm^2/V sec (a Fermi-Dirac distribution function for holes is assumed at 4.2 K).[4] The effect of trapped holes in the $Ga_{1-x}Al_xAs$ side is not included in the present calculation.

The mobilities for both carriers, μ_e and μ_h, in a weak electric field are shown in Fig. 2 as functions of lattice temperature, T_L. The mobility of minority electrons is negative at low temperature. With an increase of lattice temperature this mobility increases, except in a region of very low temperature, until it reaches a positive value. The value of this mobility is determined by the competition of e-h scattering and hole-lattice (impurity) scattering as shown in Eq. (22). In the region of very low temperature contribution to A^{h-L} in Eq. (22) mainly comes from hole-impurity scattering, which is unsensitive to the change of temperature. With increasing temperature the hole-phonon scattering incorporates and overcomes the effect of e-h scattering; it causes the mobility of electrons to increase and become positive. The presence of hot optical phonons induces an increasing of hole mobility in a weak electric field[15] and, hence, a decreasing of mobility of minority electrons.

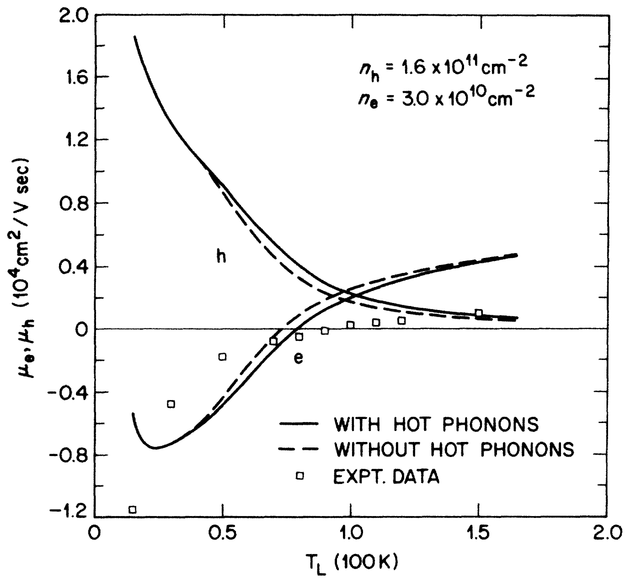

FIG. 2. Mobility of electrons, μ_e, and mobility of holes, μ_h, as functions of lattice temperature, T_L, in a weak electric field. Data for electron mobility come from Ref. 4.

Figure 3 shows the weak field mobility of electrons (solid curves) and holes (dashed curves) as functions of

TABLE I. Parameters for carrier-lattice interaction.

Symbol	Value
$\hbar\omega_{LO}$ (meV)	36.2
$\hbar\omega_{TO}$ (meV)	33.3
m_e	$0.067m_0$
m_h [a]	$0.6m_0$
ϵ_∞	10.91
ϵ_0	12.91
d_o (eV)	41
a_o (Å)	5.65
ρ (g cm^{-3})	5.36
K_p [b]	1
v_L (cm/sec)	5.29×10^5
v_T (cm/sec)	2.48×10^5
Ξ_e (eV)	7
Ξ_h (eV)	3.5
e_{14} (V/cm)	1.41×10^7

[a]m_0 is the free electron mass. Only heavy holes are considered.
[b]For a better match the value of hole mobility (Ref. 4) at 77 K, μ(77 K) = 3700 cm^2/V sec. Also, the effect of the overlap integral for polar optical phonon scattering should be weaker than that of the deformation potential.

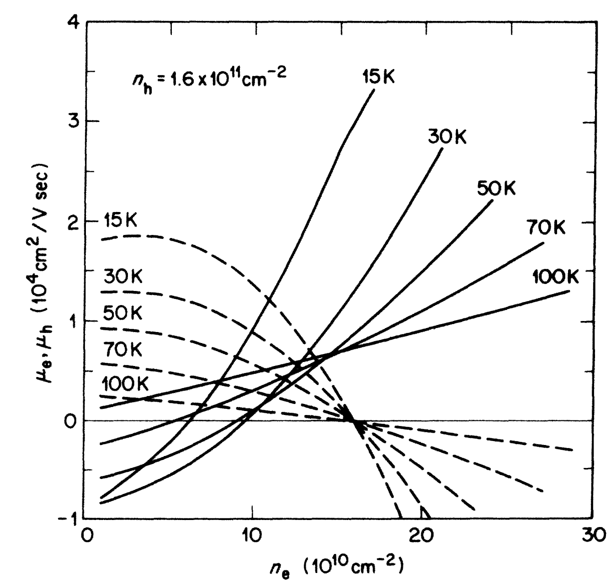

FIG. 3. Mobility of electrons, μ_e (solid curves), and mobility of holes, μ_h (dashed curves), as functions of density of electrons, n_e, at different lattice temperature, T_L (in a weak electric field).

electron density n_e at different lattice temperature. The mobility of electrons increases with an increase of electron density. The slope of the electron mobility $\partial\mu_e/\partial n_e$ decreases with an increase of lattice temperature. The mobility of holes can also become negative when the electron density is larger than the hole density.[21] At low temperatures, when $A^{e-L} \ll A^{e-h}(n_e - n_h)$, the crossover points for hole mobility are close to $n_h = n_e$ as shown in Fig. 3. With an increase of temperature, A^{e-L} will be enhanced due to electron-LO-phonon scattering, it is predicted that the crossover points will vary with temperature.

The non-ohmic mobilities of electrons and holes are shown in Fig. 4 as functions of electric field. The carrier velocity and the temperature of hot carriers are also depicted, respectively, in Fig. 5 and Fig. 6 as functions of electric field. The hot hole temperature and nonlinear velocity of majority holes with electric field behaves as the usual result of nonlinear transport. The minority electrons, however, move along the positive direction of the field in a weak electric field and reach negative maximum velocity at a certain value of the field, then slow down and turn back in the opposite direction upon increasing the field. With increase of the electric field the hole temperature increases and then enhances the resistivity due to hole-lattice scattering, which overcomes the hole-electron drag. The electron temperature increases faster than the hole temperature with field. Reabsorption of nonequilibrium phonons enhances carrier heating and the hole mobility decreases faster than obtained when phonons are in equilibrium, so the electron mobility increases faster with field in the presence of hot phonons. The different effects of hot phonons between strong and weak fields, as anticipated,[15] are due to the competition of reabsorption of energy and momentum from hot phonons.

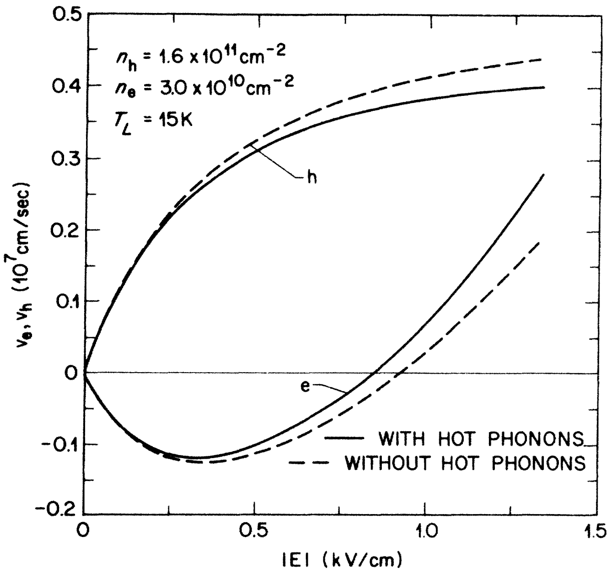

FIG. 5. Velocity of electrons, v_e, and velocity of holes, v_e, as functions of electric field.

Although the order of magnitude and the trend of electron mobility with T_L and **E** obtained by our calculation are in agreement with experimental measurements, a quantitative discrepancy between both results, however, exists. One reason is that in the experiments the determination of drift velocity of electrons is based on measuring the luminescence image spatial profiles. The density of electrons inside the image spot is not uniform. Also, the concentration of electrons decays in an exponential manner during the measurement. As shown in Eq. (22) and Fig. 3, the drift velocity of electrons is sensitively related to the density of electrons. Höpfel

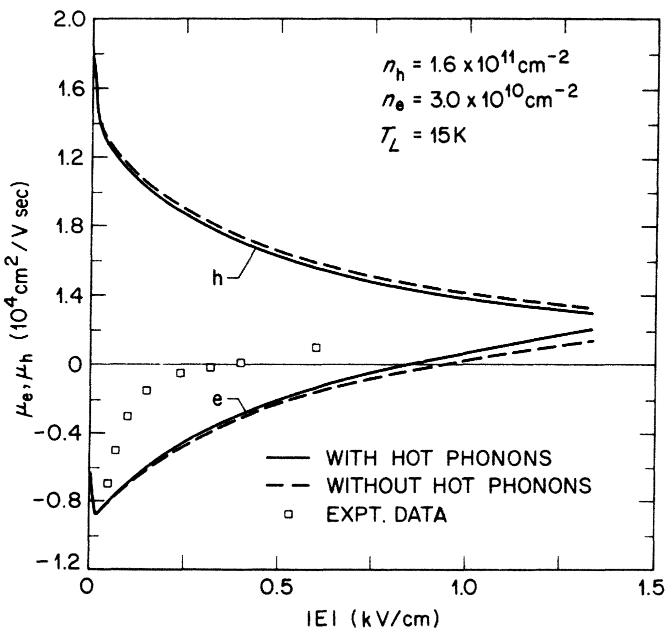

FIG. 4. Mobilities of carriers as functions of electric field, |**E**|. Data for electron mobility come from Ref. 4.

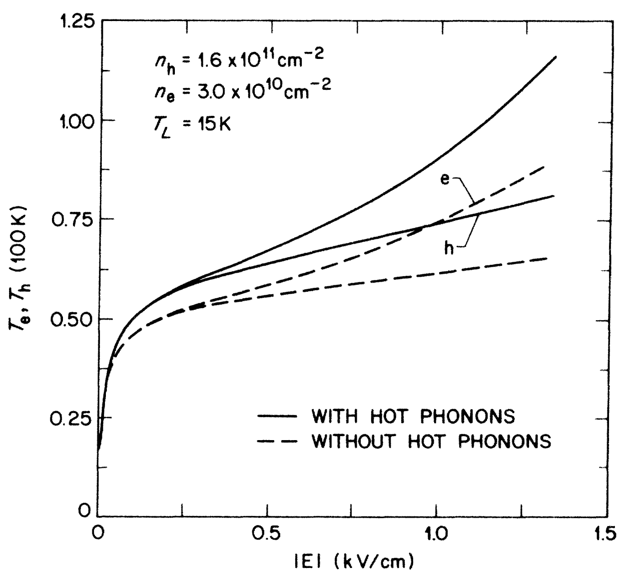

FIG. 6. Electron temperature, T_e, and hole temperature, T_h, as functions of electric field.

et al. analyze their photoexcitation data by assuming a constant drift velocity. This is a reasonable approximation in the case that $n_e \ll n_h$; however, it is not easy to quantitatively estimate the effect on the mobility μ_e due to the spatial and time change of electron density. On the other hand, some crude approximations and a large number of parameters, whose values are only approximately known, are included in our calculation. In view of the approximations in our theory, as well as the possible uncertainties in the experimental results, agreement between theory and experiment is reasonable.

In summary, we have studied the transport of a quasi-2D electron-hole gas. The formula for the screening effect and the effect of hot phonons, derived in this paper, are also useful for treatment of various relaxation phenomena in 2D e-h gas, such as time dependent processes in photoexcited electron-hole plasma. We have calculated the mobility of minority electrons and majority holes. The negative absolute mobility of electrons occurs in the low temperature and the weak electric field. This result is in reasonable agreement with the experiments by Höpfel *et al.*[4] For simplifying the numerical computation, we use the drifted temperature model and nondegenerate assumption for carrier distribution functions. We also assume carriers only occupy the lowest subband in an infinite square quantum well. The above assumptions seem to apply in the region of the present parameters of temperature, density of carriers, and strength of electric field, although they certainly induce some quantitative deviations. In the 2D quantum structure, the e-h coupling should sensitively depend on the density of carriers and width of the quantum well, as we discussed in context. The subject of the effect of electron-hole interaction on the carrier relaxation needs to be further explored.

ACKNOWLEDGMENT

We would like to thank Dr. M. C. Marchetti for fruitful discussion. The work at the City College of the City University of New York was partly supported by the U.S. Army Research Office and the U.S. Department of Energy.

[1]D. K. Ferry, M. A. Osman, and R. Joshi, Proceedings of the Fifth International Conference on Hot Carriers in Semiconductors (unpublished).

[2]J. A. Kash, J. C. Tsang, and J. M. Hvam, Phys. Rev. Lett. **54**, 2151 (1985).

[3]R. A. Höpfel, J. Shah, and A. C. Gossard, Phys. Rev. Lett. **56**, 765 (1986).

[4]R. A. Höpfel, J. Shah, P. A. Wolff, and A. C. Gossard, Phys. Rev. Lett. **56**, 2736 (1986).

[5]T. P. McLean and E. G. S. Paige, J. Phys. Chem. Solids. **16**, 220 (1960).

[6]W. Cai, M. C. Marchetti, and M. Lax, Phys. Rev. B **34**, 8573 (1986).

[7]M. C. Marchetti, W. Cai, and M. Lax, Proceedings of the Fifth International Conference on Hot Carriers in Semiconductors (unpublished).

[8]See, for example, T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[9]F. Stern and W. E. Howard, Phys. Rev. **163**, 816 (1967).

[10]See, for example, *Physics of Nonlinear Transport in Semiconductors,* edited by D. K. Ferry *et al.* (Plenum, New York, 1980).

[11]J. D. Wiley, in *Semiconductors and Semimetals,* edited by R. K. Willardson *et al.* (Academic, New York, 1975), Vol. 10, Chap. 2.

[12]F. Stern, Appl. Phys. Lett. **43**, 974 (1983).

[13]M. J. Chou, D. C. Tsui, and G. Weimann, Appl. Phys. Lett. **47**, 609 (1985).

[14]H. Fröhlich and B. V. Paranjape, Proc. Phys. Soc. London, Sect. B **69**, 21 (1956); T. F. Zheng, W. Cai, and M. Lax, Phys. Rev. B (to be published).

[15]W. Cai, M. C. Marchetti, and M. Lax, Phys. Rev. B **37**, 2636 (1988).

[16]P. J. Price, Physica B + C **134B**, 164 (1985).

[17]M. C. Marchetti (private communication).

[18]D. Von der Linde, J. Kuhl, and H. Klingenberg, Phys. Rev. Lett. **44**, 1505 (1980).

[19]*Handbook of Mathematical Functions,* No. 55 of *Applied Mathematics Series,* National Bureau of Standards, edited by M. Abramowitz and I. A. Stegun (U.S. GPO, Washington, D.C., 1964).

[20]P. Kocevar, Physica B + C **134B**, 155 (1985); J. S. Blakemore, J. Appl. Phys. **53**, R123 (1982).

[21]R. A. Höpfel, J. Shah, P. A. Wolff, and A. C. Gossard, Appl. Phys. Lett. **49**, 572 (1986).