Structural basis of multistationary quantum systems. II. Effective few-particle dynamics

W. G. Teich, K. Obermayer, and G. Mahler
Institut für Theoretische Physik, Universität Stuttgart, Pfaffenwaldring 57, 7000 Stuttgart 80, Federal Republic of Germany
(Received 12 August 1987)

The mutual coupling of electronic multistationary quantum systems realized, e.g., by semiconductor quantum dots is discussed. For local charge-transfer states such coupling can approximately be described in terms of dipole-dipole interactions: the respective renormalized transition frequencies allow for optically controlled conditional switching processes. Two coupled subsystems can in this way perform all the elementary logical operations, including microscopic information transfer. The appropriate architecture for such a quasimolecular computing system is shown to be the distributed computation based on local transition rules. As a simple example a microscopic realization of a one-dimensional one-way cellular automaton with two states per cell and nearest-neighbor coupling is discussed.

I. INTRODUCTION

In the preceding paper[1] referred to as part I in the following, we have shown that it is possible to construct multistationary quantum systems on the basis of semiconductor heterostructures (cf. also Refs. 2 and 3). Although the semiconductor is a typical many-body system, it can be decomposed into quasimolecular subsystems of effective single-particle transitions which are decoupled from the rest on an appropriate time scale. By driving the open quantum system via single modes of the electromagnetic field, a preparation of such metastable charge-transfer states is possible. Arranged in an array, these systems can be used as optically controlled storage devices in which individual electrons are used to code the stored information and the reading and writing of the memory is performed by applying suitable combinations of light pulses.

So far, this proposed system was but another version of optical storage based on persistent hole burning, which had to cope with a limited storage time and increased manufacturing problems. The aim of this paper is to extend this discussion to the case of locally interacting multistationary systems. A coupling of the microscopic subsystems is necessary in order to be able to construct elements which are capable of performing logical operations or, more generally, local transition rules.

On the microscopic level the number of interactions which could possibly be used for such a task is severely limited. Due to the low capture cross section a coupling via emission and absorption of single quasiparticles (i.e., photons or phonons) is impossible. Thus the two interactions we consider in this paper are the Pauli exclusion principle and the Coulomb renormalization of the (effective) single-particle energies.

This investigation of a specific submicroscopic realization of computing devices allows us to look for physical constraints and limitations which are due to this special (and idealized) implementation. While the early physical realization of computers has favored the sequential architecture—storing device (core memory) and central processing unit (vacuum tubes or transistors) have been made of different materials—a quasimolecular realization of a computing device calls for parallel computer architectures as will be shown below. For devices based on (conventional) highly integrated electronic circuits the necessity for new architectures which emphasize near-neighbor interactions has also already been recognized.[4]

In this sense our approach is somewhat complementary to the recent interest in novel computer architectures.[5–9] Here the motivation has been that the usual sequential architecture which goes back to the basic design of von Neumann[10] does not seem adequate for some classes of problems brought up recently. The sequential computer, as it is most widely used nowadays, was basically designed for "number crunching," and that is what it masters almost perfectly. But new tasks, like simulating complex hydrodynamic systems, image processing or pattern recognition, searches in data bases, or approaches to artificial intelligence, proved almost impossible to implement on sequential machines, and if implemented, they are very time and memory consuming. Instead, new kinds of computer architectures, the parallel-processing machines, are promising alternatives for problems of this kind. In this case the structure of the problem to be solved is, in some sense, represented by the structure of the computing device. Up to now, the most advanced realization of this kind is probably Hillis's connection machine[7,8] which, however, is still based on convential electronics.

Our paper is organized as follows: In Sec. II we describe multistationary systems with several electrons being influenced by the Pauli exclusion principle. In Sec. III we discuss the coupling of two bistable subsystems. In Sec. IV we give a model for a microscopic information transfer and we show that the basic logical operations (AND, OR, and NOT) can be performed with two coupled

subsystems. In Sec. V we extend this discussion to the case of a linear chain of subsystems, coupled via nearest-neighbor interactions. We show that a one-dimensional (1D) one-way cellular automaton can be simulated with our microscopic model. Finally, in Sec. VI, we give a short summary of our results.

II. MULTISTATIONARY SYSTEMS WITH SEVERAL ELECTRONS

Let us start from a discrete single-particle spectrum, corresponding to various localized valence- and conduction-band states, imbeded in the gap between the valence-band and conduction-band continuum. Depending on the position of the Fermi level, the low-temperature ground state corresponds to a specific distribution of holes (or electrons). We extend now the discussion of part I to a situation where there is more than one electron in the discrete valence-band states: a pure few-particle attractor state may then be characterized by the corresponding occupation numbers.

As a simple example we consider a cyclic eight-level spectrum (Fig. 1). Due to the spin-orbit interaction, as well as the spatial confinement of the quasiparticles, the spin degeneracy is lifted.[11] All levels are thus taken to be nondegenerate. If the Fermi level is pinned between states 5 and 7, the $T \rightarrow 0$ equilibrium ground state corresponds to one electron in state 1 and one electron in state 7 denoted by $\{1,7\}$. All possible six-electron distributions on the states 1, 3, 5, and 7 form the set of two-electron attractor states. Disregarding for the time being the Coulomb interaction, we can use the "contact interaction" of two electrons via the Pauli exclusion principle as a constraint to induce a conditional dynamics based on individual electrons. An electron can only be switched into another state if the new state is empty.

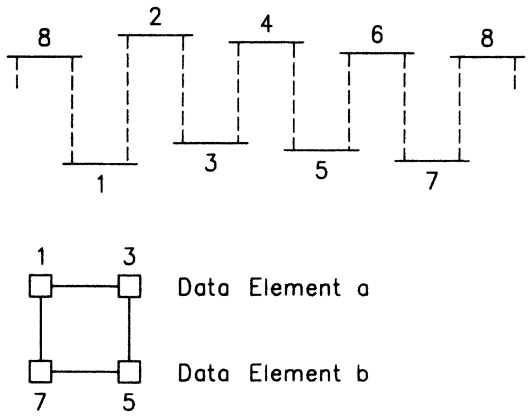

FIG. 1. Cyclic quantum optical model and coupling scheme of the attractor states for a multistable element capable of performing a combined AND/OR operation. The system consists of four single-particle attractor states (1,3,5,7), coupled via four transient states (2,4,6,8). The localization area of the different states is indicated. The binary variable a (b) is represented by the attractor states 1 and 3 (5 and 7).

Thus if, e.g., a light-pulse combination, P, containing the frequencies ω_{43} and ω_{87} is applied to the attractor state $\{1,7\}$ ($\{3,5\}$) the electron of level 7 (3) is excited into the transient level 8 (4) but it cannot relax into the already occupied level 1 (5). After the end of the laser pulse it will recombine into its original state 7 (3). The complete machine table for such a process is given by

$$\begin{matrix}\{1,3\}\\ \{1,5\}\\ \{1,7\}\\ \{3,5\}\\ \{3,7\}\\ \{5,7\}\end{matrix} \rightarrow \begin{Bmatrix}\omega_{43}\\ \omega_{87}\end{Bmatrix} \rightarrow \begin{matrix}\{1,5\}\\ \{1,5\}\\ \{1,7\}\\ \{3,5\}\\ \{1,5\}\\ \{1,5\}\end{matrix}$$

If two binary variables a and b are locally represented by the one-electron attractor states 1,3 and 5,7, respectively (cf. Fig. 1), only four out of six possible attractor states are interpreted:

$$\{1,7\} \hat{=} a=0,\ b=0,$$
$$\{1,5\} \hat{=} a=0,\ b=1,$$
$$\{3,7\} \hat{=} a=1,\ b=0,$$
$$\{3,5\} \hat{=} a=1,\ b=1.$$

The transition rules for the light pulse P as given above, reduces to

a	b	a'	b'
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1

where a' and b' denote the values of the variables a and b after the laser pulses have been applied. One easily sees that a' contains the results of the AND operation and b' of the OR operation. The NOT (cf. part I) and this AND/OR element may be combined to more complex logical elements, e.g., to a semiadder.

The same AND/OR element could have been realized with the same level structure in a much simpler way, if the variables a and b were represented by the valence band states of a single electron (i.e., the Fermi level is pinned between states 1 and 7):

$$\{1\} \hat{=} a=0,\ b=0,$$
$$\{3\} \hat{=} a=0,\ b=1,$$
$$\{5\} \hat{=} a=1,\ b=0,$$
$$\{7\} \hat{=} a=1,\ b=1.$$

Then already a light pulse with a single frequency ω_{45} would perform the combined AND/OR operation. But this mapping of the variables onto the attractor states should be considered as "unfair," since most of the complexity of the operation is already anticipated by the coding: In contrast to the previous example, it is no longer possible to identify the value of each of the variables a or b independently of the other.

On the other hand, also the proposed logical elements based on the Pauli exclusion principle have a serious disadvantage: they destroy the input data. Since complex logical operations usually make use of the same data several times during a computational process, a mechanism is necessary which copies logical information from one data element into another. It can be shown that the correlation via the Pauli exclusion principle does not permit a copying mechanism. Thus it is necessary to invoke another interaction: the Coulomb interaction.

III. COUPLING OF MULTISTATIONARY SUBSYSTEMS

We return now to the "few-particle" attractor states as discussed in Sec. II. Restricting the frequencies of the optical control, one can hope to realize a situation in which the electrons are switched only within specific local subsets of states. If, due to appropriate barriers, the electrons are forced to relax only within these same states, we are led directly to the concept of multistable cells, for which the number of switching electrons remains constant. Contrary to the situation studied in Sec. II, the multistationarity is then a fixed property of each subsystem.

In the following we consider two bistable subsystems A and B which are separated by a distance R and coupled via the Coulomb interaction. Each of them is described by a three-level model with a spatial separation d of the two attractor states (Fig. 2). All of the transition frequencies ω_{31}, ω_{32}, ω_{64}, and ω_{65} have to be different in order for the preparation system to act selectively on subsystems A and B. This can be achieved by a suitable choice of the material parameters (e.g., the aluminum concentration in a $Ga_{1-x}Al_xAs$ realization) in each subsystem.

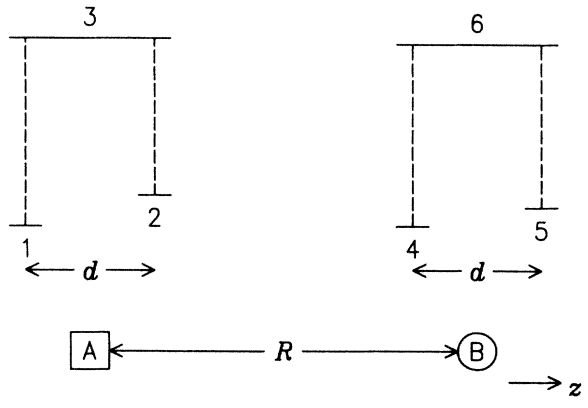

FIG. 2. Quantum optical model and coupling scheme for two interacting bistable subsystems A and B. The localization area of the different states is indicated. R is the distance between the two subsystems and d the separation of the two attractor states within one subsystem.

A. Matrix elements

Neglecting the electron spin for simplicity, the interaction matrix element between the two subsystems is given by[12]

$$W(ijkl)=\frac{e^2}{4\pi\epsilon\epsilon_0}\int\int d^3x\, d^3x'\,\Phi_i^*(\mathbf{x})\Phi_j^*(\mathbf{x}') \times\frac{1}{|\mathbf{x}-\mathbf{x}'|}\Phi_k(\mathbf{x}')\Phi_l(\mathbf{x})\,, \tag{3.1}$$

where Φ_i is the electronic one-particle wavefunction of state i and ϵ the relative dielectric constant of the embedding semiconductor material.

For large separation R of the two subsystems the overlap between the wavefunctions of subsystem A and those of subsystem B tends to zero. Therefore we can neglect the local exchange interaction. The matrix element (3.1) vanishes if the states i and l or j and k belong to different subsystems.

In order to calculate the remaining nonlocal Coulomb interaction matrix elements we expand $1/|\mathbf{x}-\mathbf{x}'|$ and find, up to dipole-dipole contributions

$$W(ijkl)=\frac{e^2}{4\pi\epsilon\epsilon_0 R}\delta_{jk}\delta_{il}+\frac{e^2}{4\pi\epsilon\epsilon_0 R^3}(\delta_{il}\mathbf{R}\cdot\mathbf{P}_{jk}-\delta_{jk}\mathbf{R}\cdot\mathbf{P}_{il}) +\frac{e^2}{4\pi\epsilon\epsilon_0 R^3}\left[\mathbf{P}_{il}\cdot\mathbf{P}_{jk}-\frac{3}{R^2}(\mathbf{R}\cdot\mathbf{P}_{il})(\mathbf{R}\cdot\mathbf{P}_{jk})\right]. \tag{3.2}$$

$\mathbf{R}=R\hat{\mathbf{e}}_z$ is the separation vector between the two subsystems, and the dipole (transition) matrix element is given by

$$\underline{P}_{il}=\int d^3x\,\Phi_i^*(\mathbf{x})\mathbf{x}\Phi_l(\mathbf{x})\,, \tag{3.3}$$

where the center of the coordinate system coincides with the center of subsystem A or B.

The most important matrix element for our purpose is $W(ijji)$. It describes the static Coulomb interaction between the two charge distributions in subsystem A and B and causes a renormalization of the energy levels. To assure charge neutrality, we suppose that a positive charge $+e$ is located at the center of each subsystem. The interaction of the electrons with these positive charges exactly cancels the monopole-monopole and the monopole-dipole part of $W(ijji)$ in (3.2), the only remaining term being the dipole-dipole interaction. To lowest order, the static dipole moments (3.3) can be approximated by (cf. Fig. 2)

$$\mathbf{P}_{ii}=\begin{cases}+(d/2)\hat{\mathbf{e}}_z & \text{for } i=2,5\\ 0 & \text{for } i=3,6\\ -(d/2)\hat{\mathbf{e}}_z & \text{for } i=1,4.\end{cases} \tag{3.4}$$

In doing so, we have assumed that the centers of the electronic charge distributions coincide with the spatial centers of the quantum dots and that the static dipole moments are oriented parallel or antiparallel to the sepa-

ration vector **R**. Thus we finally get for the matrix elements $W(ijji)=W(jiij)$

$$W(1441)=W(2552)=-\frac{e^2d^2}{8\pi\epsilon\epsilon_0R^3}\ ,$$
$$W(1551)=W(2442)=+\frac{e^2d^2}{8\pi\epsilon\epsilon_0R^3}\ . \tag{3.5}$$

All other matrix elements (involving the states 3 and 6) vanish.

The second term in (3.2) gives the leading order contribution to the matrix elements $W(ijki)$ and $W(ijjl)$ and describes a situation where the electron in one subsystem makes a virtual transition from state k to j (l to i) while the other electron stays in state i (j). This process violates the energy conservation within the one-particle picture and can therefore be neglected for small interaction energies.

Finally, the leading term for the dynamical part of the Coulomb interaction matrix element $W(ijkl)$ with $i\neq l$ and $j\neq k$ is given by the third term in (3.2). It describes the transition of an electron from state l to i, while the other electron makes a transition from state k to j. If l and j are excited states (e.g., $l=3$ and $j=6$) and i and k are not, this corresponds to a transfer of the electronic excitation from one subsystem to the other. This process is similar to the excitation transfer mechanism in molecular dynamics sometimes denoted the Foerster effect.[13–15] As long as all of the transition frequencies in the two subsystems are nondegenerate, the direct Foerster effect is not important since we have virtual transitions only. But if any two of the excitation energies in subsystems A and B happen to be the same, the Foerster process may lead to real transitions and can no longer be neglected in general.

B. Coulomb renormalization

Neglecting the dynamical part of the Coulomb interaction and applying first-order perturbation theory, the energy renormalization of, e.g., the levels $i=1$, 2, and 3 in subsystem A is given by (3.5). Due to the different localization areas of each state the energy renormalization is distinct for each level and depends on the occupied state j of subsystem B (Fig. 3). Therefore the transition frequencies in subsystem A depend on the state of subsystem B and vice versa (cf. also Fig. 2)

$$\omega_{3i}=\omega_{3i}(j);\quad i=1,2 \text{ and } j=4,5$$
$$\omega_{6j}=\omega_{6j}(i);\quad i=1,2 \text{ and } j=4,5\ . \tag{3.6}$$

For the splitting of the excitation energies we find from (3.5)

$$\hbar\Delta\omega=\hbar\omega_{31}(4)-\hbar\omega_{31}(5)=\frac{e^2d^2}{4\pi\epsilon\epsilon_0R^3}\ . \tag{3.7}$$

Taking $R=100$ nm and the material parameters for the semiconductor structure of part I (i.e., $d=16$ *nm* and $\epsilon=12$) we get $\hbar\Delta\omega\approx 0.03$ meV. Thus we can distinguish between several energy scales: transition energy, 10^3 meV; band-gap discontinuity, 10^2 meV; level splitting ($\hbar\omega_{31}-\hbar\omega_{32}$), 10^1 meV; Coulomb renormalization, 10^{-1} meV.

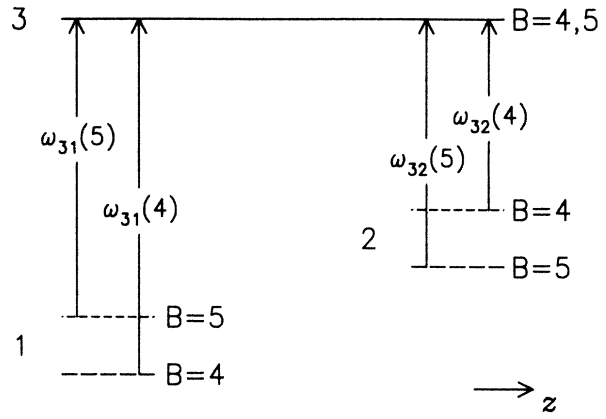

FIG. 3. Coulomb renormalization of the energy levels in subsystem A due to the presence of subsystems B (energy axis not to scale). The localization area of the different states is indicated and the conditional transition frequencies are given.

In part I we have seen that two three-level systems with a relative separation in frequency space of $\Delta\nu/\nu\geq 10^{-5}$ may be selectively switched by using light of moderate intensity. Since a typical laser bandwidth can be as small as $\Delta\nu/\nu\approx 10^{-7}$,[16] it is possible to use the splitting (due to the coupling to subsystem B) in the excitation energies of subsystem A to induce a conditional dynamics in subsystem A. Irradiation with light of frequency $\omega_{31}(4)$ and small enough bandwidth brings the electron in subsystem A into state 2 only if subsystem B is in state 4. If B is in state 5 the laser pulse will practically have no influence on the electron in subsystem A.

On the other hand, if we want to switch subsystem A from state 1 to 2 irrespective of the state of subsystem B, we have to use two pulses of frequencies $\omega_{31}(4)$ and $\omega_{31}(5)$ or one pulse with an average frequency of $\omega_{31}=\frac{1}{2}[\omega_{31}(4)+\omega_{31}(5)]$ and a bandwidth larger than the frequency splitting $\Delta\omega$.

C. Resonant excitation transfer

In order to estimate the influence of the dynamic part of the Coulomb interaction, we neglect the angular dependence and calculate an absolute upper bound $W_f\geq|W(ijkl)|$ for the dynamic Coulomb matrix element. From (3.2) we get

$$W_f=\frac{e^2}{4\pi\epsilon\epsilon_0}\frac{2}{R^3}|\mathbf{P}_{il}|\,|\mathbf{P}_{jk}|\ . \tag{3.8}$$

Rewriting the transition dipole matrix element (3.3) as[17]

$$\mathbf{P}_{il}=\frac{\hbar}{m\omega_{il}}\int d^3x\Phi_i^*(\mathbf{x})\nabla\Phi_l(\mathbf{x}) \tag{3.9}$$

we can estimate the magnitude of $\mathbf{P}_{il}$ by using the optical dipole transition rates calculated in part I.

Since the overlap between states 1 and 2 (or 4 and 5, re-

spectively) is small, we have neglected the optical dipole transition between these states compared to acoustic-phonon scattering (cf. part I). To the same extent we can neglect the matrix element $W(1542)$ on the time scale τ_d. The attractor states 1,2 and 4,5 are thus stable with respect to an excitation transfer from one subsystem to the other via the Foerster process.

For the other matrix elements we find (using $R=100$ nm and the material parameters of the semiconductor structure of part I) the upper bound $W_f \approx 10^{-5}$ meV. This corresponds to a time scale[15] for the excitation transfer from one subsystem to the other of the order of $\tau_f \approx 100$ nsec. This time is about an order of magnitude larger than the optical relaxation time for dipole transitions from the transient state 3 to the attractor states 1 or 2. But for a reliable preparation of an attractor state, the switching time T (that is, the length of the light pulse) has to be several times the optical relaxation time. Thus, if any two of the transition frequencies in the two subsystems are equal, the Foerster process will be important. It limits the switching time and thus gives a lower bound for the error probability of the system. This effect will be even large for smaller separations R of the subsystems. On the other hand, if the difference of the transition energies in the two subsystems is much larger than the matrix element W_f, the Foerster process can be neglected since it induces virtual transitions only.

IV. LOCAL COMPUTATION

A. A model for microscopic information transfer

On the relevant time scale, each of the subsystems A and B of Sec. III is capable of storing and processing information. Again, this is based on a one-to-one mapping of the attractor states to values of suitably chosen information variables. For example subsystem A (B) can store the value of a binary variable a (b):

Attractor state	Information variable
$\{1\}$	$a=0$
$\{2\}$	$a=1$
$\{4\}$	$b=0$
$\{5\}$	$b=1$

This can be considered as a "fair" coding, since each variable is represented by a separate subsystem. The value of each variable can be inscribed independently of the value of all the other variables.

As we have already pointed out, in a computational sequence it is frequently necessary to copy the value of one variable into another. The truth table for a process where the content of variable b is copied into a variable a is given by

a	b	a'	b
0	0	0	0
0	1	1	1
1	0	0	0
1	1	1	1

Information which might have been present in variable a is obviously destroyed after this process.

Translated into the physical picture of our model system, it means that the electron in subsystem B has been prepared—by a suitable combination of light pulses—in one of the two attractor states 4 or 5. After the information transfer process (ITP) the electron in subsystem A has to be in the attractor state 1 or 2, depending on whether the electron in subsystem B is in state 4 or 5 and regardless of the state of subsystem A before the ITP. Thus, the information which is present in subsystem B will, after the ITP, also be available in subsystem A.

In order to realize such a process we use the fact that the information about the occupied level in subsystem B is already present in subsystem A through the nonlocal Coulomb interaction. That is we use the conditional dynamics of subsystem A in order to get a correlation between the states of subsystems A and B. After irradiation with two simultaneous laser pulses of duration T and frequencies $\omega_{32}(4)$ and $\omega_{31}(5)$ subsystem A will be in state 1 (2) if B is in state 4 (5). Thus we have copied the information content from subsystem B to A.

The time T, together with the optical relaxation time τ_r of the semiconductor material used, determines the reliability with which the copying process is performed. The larger T becomes, that is, the longer it takes to copy the information, the smaller is the error probability; however, T is constrained by $T \ll \tau_d$: Otherwise the original information in B may already be lost.

Similar ideas have been used by Haddon *et al.*[18] to propose information processing within single molecules. But, contrary to us, they did not consider a specific coupling mechanism and thus could give only formal results.

B. Channel capacity

Information theoretically,[19,20] subsystem A (and similarly subsystem B) can be viewed as an information source characterized through an alphabet $A=\{a_i;\ i=0,1\}$ and a probability distribution $P(a_i)$.

The information $I(a_i)$ is a measure for the degree of uncertainty which is removed by the knowledge of a special realization a_i of the information source. Using one "bit" as the unit of information, we have

$$I(a_i)=-\log_2 P(a_i)\ . \tag{4.1}$$

The entropy $I(A)$ of the information source is defined as the mean information gain per measured state

$$I(A)=\sum_i P(a_i)I(a_i)=-\sum_i P(a_i)\log_2 P(a_i)\ . \tag{4.2}$$

In the same way we can describe the two subsystems A and B together with the ITP as an information-theoretical channel which is characterized through the receiver alphabet $A=\{a_i;\ i=0,1\}$, the transmitter alphabet $B=\{b_i;\ i=0,1\}$, and a conditional probability distribution $P(a_i \mid b_j)$.

The transinformation $I(b_j;a_i)$ is a measure for the transmitted information. Since $P(b_j)$ is the *a priori* and $P(b_j \mid a_i)$ the *a posteriori* probability for finding b_j at the

transmitter when a_i is measured at the receiver, we have

$$I(b_j;a_i)=\log_2\frac{P(b_j \mid a_i)}{P(b_j)} . \tag{4.3}$$

The transinformation is symmetric with respect to interchanging transmitter and receiver and $I(b_j;a_i)=0$ if a_i is statistically independent of b_j.

The transentropy $I(B;A)$ of the channel is defined as the mean transinformation per ITP

$$\begin{aligned} I(B;A)&=\sum_{i,j} P(a_i b_j) I(b_j;a_i) \\ &=\sum_{i,j} P(b_j) P(a_i \mid b_j) \log_2 \frac{P(a_i \mid b_j)}{P(a_i)} , \end{aligned} \tag{4.4}$$

and the channel capacity is given as the maximum transentropy for all possible probability distributions $P(b_j)$:

$$C=\max_{P(b_j)} \{I(B;A)\} . \tag{4.5}$$

Neglecting the small difference between the optical relaxation rates from state 3 to 1 and from state 3 to 2 our coupled system can be characterized by the following conditional probabilities:

$$\begin{aligned} &P(a_0 \mid b_0)=1-p, \quad P(a_1 \mid b_0)=p , \\ &P(a_0 \mid b_1)=p, \quad P(a_1 \mid b_1)=1-p , \end{aligned} \tag{4.6}$$

where p is the noise probability of the channel, approximately given by

$$p=\tfrac{1}{2}\exp(-T/\tau_r) . \tag{4.7}$$

Thus we have a microscopic physical realization of a binary symmetric channel and find for the channel capacity[20]

$$C=1+p\log_2 p+(1-p)\log_2(1-p) \text{ bit/ITP} . \tag{4.8}$$

Using a typical relaxation time and choosing T in such a way that the noise probability $p=0.05$, we find for the channel capacity per unit time

$$C^*=C/T\approx 10^7 \text{ bits/sec} . \tag{4.9}$$

For the dissipative optical switching process the channel capacity per unit time is obviously limited by the optical relaxation rate. Thus, to be sure, the channel capacity (4.9) is about two orders of magnitude smaller than maximal transfer rates of macroscopic technical channels (e.g, $C^*\approx 10^9$ bits/sec for an optical transmission line[21]). However, the ITP we have described here serves a different purpose than usual transmission lines: It renders it possible that information processing units, realized on a quasimolecular level, are able to communicate with each other directly (and in parallel).

C. General transition rules for interacting pairs of bistable subsystems

The ITP we have described in the last chapter is only one example of a class of functions $a'=f_N(a,b)$ which map two variables onto one (two input, one output channel). Table I shows the complete set of the 16 possible functions $f_N(a,b)$ for binary variables a and b.

In order to realize such a function with two coupled subsystems A and B we have to change the state of one of them (e.g., of A) depending on the previous state of both subsystems A and B (transition rules). That means we have to perform a transformation $A\rightarrow A'=f_N(A,B)$ [or on the alphabet level $a\rightarrow a'=f_N(a,b)$]. Thus the original state of A (and in some cases, as we will see, also of subsystem B) is lost. The realization of the functions is irreversible. We can, in general, not reconstruct the original values a and b from the final ones a' and b.

The function $N=0$ ($N=15$) describes the resetting of the variable a to the value 0 (1) (WRITE operation) and can be performed through two simultaneous light pulses of frequencies $\omega_{32}(4)$ and $\omega_{32}(5)$ [$\omega_{31}(4)$ and $\omega_{31}(5)$]. $N=3$ is the identity operation (no light pulse necessary) and in $N=5$ we can identify the ITP described in the previous chapter. The function $N=1$ corresponds to a logical AND (conjunction) and the function $N=7$ to a logical OR (disjunction). They can be realized by one laser pulse of frequency $\omega_{32}(4)$ or $\omega_{31}(5)$, respectively.

Special consideration needs the function $N=12$, which corresponds to the logical NOT (negation). In order to realize it, we first have to copy the variable a onto variable b; that is, we have to apply two laser pulses with frequencies $\omega_{64}(2)$ and $\omega_{65}(1)$. After the copying process, we can perform the negation by two light pulses of frequencies $\omega_{31}(4)$ and $\omega_{32}(5)$ (cf. Table II). In this case the input of both variables a and b is destroyed.

By using different combinations of laser pulses it is possible to generate all but two of the 16 functions in Table I. Table III gives the logical meaning of all the functions,

TABLE I. The 16 possible functions $a'=f_N(a,b)$ for binary variables a, b, and a' are shown. The different functions can be labeled by a decimal number N from 0 to 15 which is chosen in such a way that the four-digit binary representation of N corresponds to the entry in the a' column of the corresponding function.

a	b	$a'=f_N(a,b)$															
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
$N=$		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

TABLE II. Logical table and laser-pulse sequences [cf. Eq. (3.6)] for the negation process $a' = \bar{a}$.

a	b		a	b'		a'	b'
0	0		0	0		1	0
0	1	$\rightarrow \left\{ \begin{matrix} \omega_{64}(2) \\ \omega_{65}(1) \end{matrix} \right\} \rightarrow$	0	0	$\rightarrow \left\{ \begin{matrix} \omega_{31}(4) \\ \omega_{32}(5) \end{matrix} \right\} \rightarrow$	1	0
1	0		1	1		0	1
1	1		1	1		0	1

together with the combination of light pulses necessary to realize them. For some of them ($N = 4$, 8, 12, 13, and 14) the negation operation is needed and thus we do not only switch in subsystem A but also in subsystem B. Only the two functions $N = 6$ and $N = 9$, which cannot be represented as a simple binary sum (or product) of the two variables or their negation, cannot be realized with only two coupled bistable subsystems.

Since we can realize the elementary logical operations AND (conjunction), OR (disjunction), NOT (negation), and the copying process, it is possible to calculate any logic function by a suitable combination of coupled bistable systems. Especially it is possible to realize the function $N = 6$, the logic XOR (exclusive disjunction or binary addition modulo two). Haddon *et al.*[18] have shown, that the addition modulo-two function can be realized with three coupled binary subsystems only, when the dynamics of each subsystem depends on the state of the two others.

Thus it should, at least in principle, be possible to construct a universal computer based on adequately connected bistable subsystems. This leads to a sequential computational dynamics as it is used in most present day computing devices.

The sequential architecture implies the separation of the computing system into a complex processor and a passive memory. Thus it necessitates interactions between distant subunits which can hardly be realized by the special physical realization we are considering: The Coulomb renormalization can only be used for spatial separations of the order of 100 nm or less. For larger separations the interaction is too weak to allow a save selective switching of the subsystems. Since the number of possible interactions on a submicroscopic level is quite limited, the only possibility would be to use the interaction with the macroscopic preparation and measurement system in order to establish the necessary information flow between subsystems which are further apart. But this does not seem very reasonable since then most of the advantages of using a quasimolecular realization of a computing device would be lost.

V. DISTRIBUTED COMPUTATION

A. Linear arrays of interacting subsystems

A different computer architecture, which reduces the need for communications with the macroscopic environment, is based on a cellular structure and can be characterized as distributed computation. Mathematically, a cellular automaton (CA) consists of identical cells or processors, arranged in a regular lattice (1D, 2D, 3D, . . .). Each processor has a finite number of discrete states and the array evolves in time in discrete steps according to local transition rules, being the same for each cell.[22,23]

Thus we consider a cellular structure in which quasimolecular multistable elements are repeated many times and coupled via the Coulomb interaction. The time evolution is attained by repeatedly switching the elements according to local transition rules through appropriate light pulses. The locality of the interaction is achieved by the $1/R^3$ dependence of the frequency splitting (3.7). Different transition rules can be realized by different combinations of light pulses. Due to the finite relaxation time τ_r one time step has a finite duration and subsystems whose dynamics is coupled cannot be switched simultaneously.

As an example we consider a quasimolecular 1D CA

TABLE III. Logical meaning of the functions $N = 0$ to 15. Also given are the corresponding laser pulse sequences [cf. Eq. (3.6)] to have f_N realized by two coupled subsystems. The light-pulse combinations in [· · ·] can be applied simultaneously [symbols used: conjunction, ·; disjunction, +; and negation, $(\overline{\cdots})$].

N	$f_N(a,b)$	Laser-pulse sequences
0	0	$[\omega_{32}(4),\omega_{32}(5)]$
1	$a \cdot b$	$\omega_{32}(4)$
2	$a \cdot \bar{b}$	$\omega_{32}(5)$
3	a	no pulse necessary
4	$\bar{a} \cdot b = (\overline{a + \bar{b}})$	$\omega_{31}(4)$, $[\omega_{64}(2),\omega_{65}(1)]$, $[\omega_{31}(4),\omega_{32}(5)]$
5	b	$[\omega_{31}(5),\omega_{32}(4)]$
6	$(\bar{a} \cdot b) + (a \cdot \bar{b})$	not possible
7	$a + b$	$\omega_{31}(5)$
8	$\bar{a} \cdot \bar{b} = (\overline{a + b})$	$\omega_{31}(5)$, $[\omega_{64}(2),\omega_{65}(1)]$, $[\omega_{31}(4),\omega_{32}(5)]$
9	$(\bar{a} \cdot \bar{b}) + (a \cdot b)$	not possible
10	$\bar{b}$	$[\omega_{31}(4),\omega_{32}(5)]$
11	$a + \bar{b}$	$\omega_{31}(4)$
12	$\bar{a}$	$[\omega_{64}(2),\omega_{65}(1)]$, $[\omega_{31}(4),\omega_{32}(5)]$
13	$\bar{a} + b = (\overline{a \cdot \bar{b}})$	$\omega_{32}(5)$, $[\omega_{64}(2),\omega_{65}(1)]$, $[\omega_{31}(4),\omega_{32}(5)]$
14	$\bar{a} + \bar{b} = (\overline{a \cdot b})$	$\omega_{32}(4)$, $[\omega_{64}(2),\omega_{65}(1)]$, $[\omega_{31}(4),\omega_{32}(5)]$
15	1	$[\omega_{31}(4),\omega_{31}(5)]$

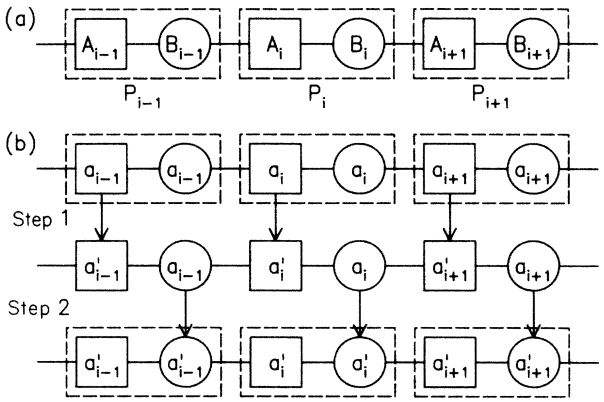

FIG. 4. (a) Linear arrangement of alternating subsystems A and B, coupled through nearest neighbor interaction. Two subsystems A and B form one cell P of a 1D one-way CA. (b) One time step of the CA, consisting of two switching processes: first all subsystems A are switched (step 1) and then all subsystems B are switched (step 2).

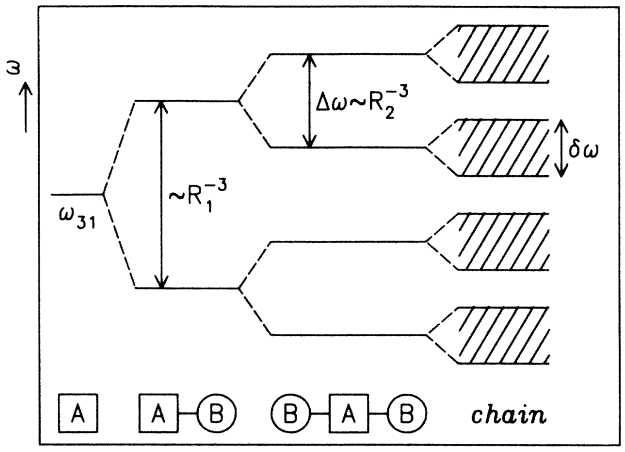

FIG. 5. Splitting of the transition frequency ω_{31} in the subsystem A due to the influence of other subsystems. One subsystem B, a distance R_1 away, yields a splitting proportional to $1/R_1^3$. A second subsystem B, a distance R_2 away, will further split the transition frequencies (proportional to $1/R_2^3$) and so on. For a conditional dynamics with nearest-neighbor coupling only, a frequency band of bandwidth $\delta\omega$ has to be applied in order to suppress the influence of subsystems which are further apart. $\Delta\omega$ is the separation of the four selective frequencies.

with nearest-neighbor interaction. The basic physical subunit is the bistable quantum-optical switch described in detail in part I. We need at least two subsystems A and B which can be switched with separate frequencies. They are arranged alternatingly on a linear chain [cf. Fig. 4(a)]. One time step consists of first switching all subsystems A_i, depending on the state of the neighboring subsystems B_{i-1} and B_i (which are kept constant while subsystems A_i are switched) and then switching all subsystems B_i, depending on the state of the neighboring subsystems A_i and A_{i+1} (which are kept constant while subsystems B_i are switched).

Thus we have to extend the considerations of Sec. III for two interacting subsystems to the case of a linear chain, with a conditional dynamics for each subsystem that depends on the nearest neighbors only. Equation (3.7) gives the splitting of the transition energies of a subsystem A_i due to the influence of a subsystem B_i, a distance R_1 apart from A_i. Another subsystem B_{i-1} (on the opposite side) a distance R_2 away from A_i will further split the transition energies, so that, in general, we will have four distinct frequencies for each transition (Fig. 5). Introducing the asymmetry parameter δ

$$\delta=\frac{|R_1-R_2|}{R_1+R_2}\ , \tag{5.1}$$

we can distinguish between several cases: for $\delta\to 1$ only the influence of the closest subsystem remains. We have two transition frequencies for each switching process as in the case of two coupled subsystems. For $\delta=0$, that is for a symmetric arrangement ($R_1=R_2$) of the three subsystems, two of the four transition frequencies are degenerate. Thus only a restricted conditional dynamics is possible. Nothing but totalistic transition rules[22] can be realized. For the special case

$$\delta=\frac{(2)^{1/3}-1}{(2)^{1/3}+1}\approx 0.115 \tag{5.2}$$

we get four equidistant transition frequencies with the separation in frequency space given by (3.7) and R being the larger of the two distances R_1 and R_2.

In an infinite linear arrangement of subsystems obviously not only the left and right nearest neighbor B_{i-1} and B_i will have an influence on the transition frequencies in A_i, but also all the other subsystems. Since we do want to switch subsystem A_i only dependent on the state of its nearest neighbors and independent of the state of all the other subsystems, we do have to apply a frequency band instead of a single transition frequency (cf. Fig. 5). The bandwidth $\delta\omega$ is given by the frequency splitting of all but the two nearest neighbors. For small asymmetry δ it can be approximated by

$$\delta\omega=\frac{e^2d^2}{2\pi\epsilon\epsilon_0R^3}\sum_{n=2}^{\infty}\frac{1}{n^3}\ , \tag{5.3}$$

where R is the average distance between two subsystems.

For $\delta\approx 0.115$ (equidistant transition frequencies) we find[24] $\delta\omega\approx 0.6\,\Delta\omega$. The bandwidth $\delta\omega$ of the irradiated laser pulse is smaller than the separation $\Delta\omega$ of the transition frequencies. Thus it is possible to switch subsystem A_i dependent on the state of the neighboring subsystems B_{i-1} and B_i and independent of the state of all the other subsystems.

Since we have subsystems with identical transition energies separated by a distance (R_1+R_2), we have to consider the effect of the resonant energy transfer on the dynamics of our CA. We can distinguish between two cases.

(1) The identical subsystems considered have the same local neighborhood. Then their transition frequencies will be equal within the range of the bandwidth $\delta\omega$. Thus

all subsystems are and ought to be switched if the corresponding frequency band is applied. The resonant energy transfer gives merely an additional channel for the relaxation process, but it does not alter the final single-particle attractor state, which is equal for all identical subsystems with the same local neighborhood.

(2) The subsystems have a different local neighborhood. Then their respective transition frequencies are out of resonance and the excitation energy transfer due to the Foerster process may be neglected.

B. Dynamics of a 1D one-way cellular automaton

By suitable programming the quasimolecular 1D cellular structure described in the previous chapter can be used to simulate any 1D one-way CA with two states per cell and nearest-neighbor coupling. Each cell P_i of the CA consists of two subsystem A_i and B_i, the states of which are correlated [Fig. 4(a)] so that we have only two states per cell. One CA time step consists of two switching processes [Fig. 4(b)].

(1) All subsystems A_i are switched, depending on the state of the neighboring subsystems B_{i-1} and B_i. Since the states of A_i and B_i are correlated, the new state of A_i depends only on the old state of subsystems A_i and A_{i-1}. But because, in contrast to only two coupled subsystems, B_i keeps the information about the old state of A_i during the switching process, all 16 transition rules (cf. Table I) with two input variables (old states of subsystems A_i and A_{i-1}) and one output variable (new state of A_i) can be realized by corresponding light-pulse combinations.

(2) All subsystems B_i are switched, depending on the state of the neighboring subsystems A_i and A_{i+1}. In order to get a new state of the CA we copy the state of A_i into B_i independent of the state of A_{i+1}. This can be performed by four simultaneous light pulses of frequencies $\omega_{64}(A_i=2,\ A_{i+1}=1)$, $\omega_{64}(A_i=2,\ A_{i+1}=2)$, $\omega_{65}(A_i=1,\ A_{i+1}=1)$, $\omega_{65}(A_i=1,\ A_{i+1}=2)$, and respective bandwidths $\delta\omega$.

As a result the new state of cell P_i depends on the previous state of itself and the left neighbor cell P_{i-1}

$$P_i(t+1)=f_N[P_i(t),P_{i-1}(t)]\ . \tag{5.4}$$

Since the minimum size of a laser spot is limited by its wavelength to about[25] 10 μm, individual subsystems with the same frequency spectrum cannot be addressed separately. But this was necessary for a preparation of the initial configuration of the CA. One can think of two alternative ways to accomplish this.

(1) By applying an inhomogeneous electric field the transition frequencies of identical subsystems are made different, so that each subsystem can be addressed individually in frequency space. Then the initial state of each subsystem can be prepared simultaneously (parallel input, selectivity in frequency space).

(2) We start the linear arrangement of subsystems A_i and B_i with a bistable subsystem C as the left neighbor of A_1 [Fig. 6(a)]. Subsystem C has to have transition frequencies which are well separated from the ones of A_i and B_i, so that C can be switched independently of all the other subsystems. Thus, by alternatingly performing a preparation of subsystem C and then copying the state of C into P_1 and of P_{i-1} into P_i, we can transform a temporal sequence of preparation states of subsystem C into a linear spatial arrangement of states of the CA (serial input, selectivity in time).

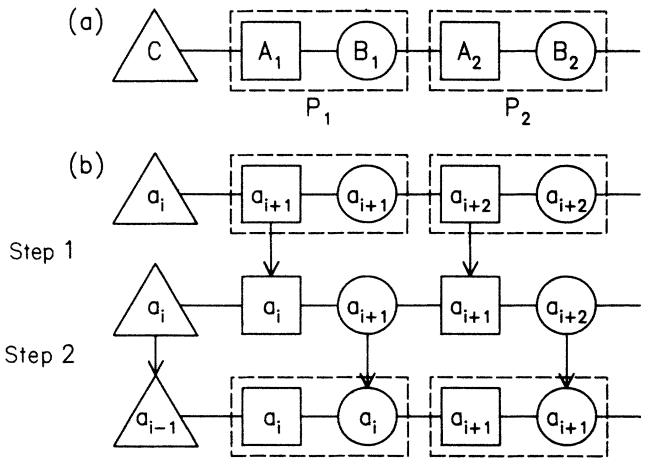

FIG. 6. (a) Arrangement of subsystems for a serial input. The linear chain of subsystems starts with a subsystem C, distinct from the subsystems A and B which form the chain. (b) One time step in the initial preparation of a CA: first the state of C is copied into A_1 and the state of B_{i-1} into A_i (step 1), then subsystem C is prepared in a new state and the state of A_i is copied into B_i (step 2).

The same procedure can be applied to measure the final configuration of the CA. On the right end of the linear arrangement we put a bistable subsystem D, the state of which can be measured as described in part I. Thus, by shifting the configuration of the CA one cell to the right each time step and measuring the state of subsystem D, the spatial sequence of CA states can be transformed into a temporal sequence of states of the macroscopic measurement apparatus.

Figure 7 shows the evolution of the 1D one-way CA for different transition rules as given in Table I. For the bidirectional 1D CA the evolution has been classified by Wolfram[23] into four classes: (1) The evolution leads to a homogeneous final state; (2) the evolution leads to a set of separated simple stable or periodic structures, which may also be shifted one cell to the left or right each time step; (3) the evolution leads to a chaotic pattern which (in some cases) shows self-similarities; (4) the evolution leads to complex localized structures which are sometimes long-lived.

Obviously the 1D one-way CA with two states per cell and nearest neighbor interaction is not complex enough to provide an example for a class-4 CA. One example for each of the remaining three classes is given in Fig. 7. The most complex behavior (class 3) is found for the addition modulo-two rule ($N=6$) and the complement thereof ($N=9$). Both rules lead to growing structures from a single seed and their spatio-temporal patterns show triangular self-similarities resembling those found by Wolfram[22] for the bidirectional 1D CA.

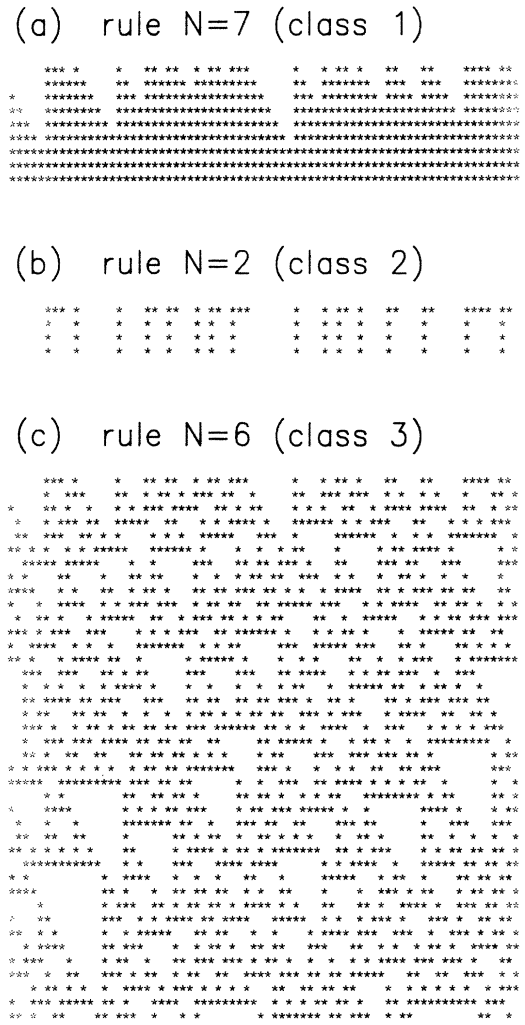

FIG. 7. Time evolution of a 1D one-way CA with two states per cell (a blank denotes 0 and * denotes 1) and nearest-neighbor coupling for different transition rules as given in Table I. The initial configurations are generated at random with equal probability for each state. For simplicity, periodic boundary conditions are used.

C. Extensions

The simple 1D one-way CA described above already shows a rather complex behavior, but not complex enough to be used as a universal computer. To increase the complexity (and fault tolerance), extensions in several directions are possible.

(1) The number of CA states per cell can be increased. Possibly the Pauli exclusion principle can then be used as a further interaction within a subsystem.

(2) The subsystems can be arranged and coupled within a 2D array.

(3) The interaction range can be extended to next-nearest-neighbor interaction or further.

(4) The inherent stochastic nature of the transition rules can be exploited (e.g., for stochastic simulations).

Albert *et al.*[26] have shown that a 1D one-way CA with 14 states per cell and nearest-neighbor interaction is capable of universal computation. This is not a lower bound, so that probably even fewer states per cell may be sufficient to perform the same task. 1D CA's with two states per cell and next nearest neighbor interaction already show the complex behavior of class-4 CA's. Wolfram[23] conjectures that CA of this class are capable of universal computation. For 2D CA's, e.g., the "game of life" by Conway, it was also shown that they are capable of universal computation.[27]

Alternatively, by relaxing the defining constraints of a CA, the behavior of our quasimolecular 1D cellular structure can be made more complex. Since the transition rules of the system are determined by the frequencies of the applied laser pulses, the rules can easily be changed after each time step, either systematically (e.g., periodically or dependent on the outcome of a measurement on the system) or at random. Thus a much more complex behavior of our system seems to be possible.

Finally, we can drop the restriction that the states of A_i and B_i have to be linked together: Then both subsystems A_i and B_i carry information independently of each other. First, all subsystems A_i are switched, depending on their previous state and the state of their nearest neighbors

$$A_i(t+1)=F_a[A_i(t),B_{i-1}(t),B_i(t)] . \tag{5.5a}$$

Then all subsystems B_i are switched (possibly with a different transition rule)

$$B_i(t+1)=F_b[B_i(t),A_i(t+1),A_{i+1}(t+1)] . \tag{5.5b}$$

By starting with switching A_i, the two subsystems A_i and B_i are not treated symmetrically, since the new state of B_i depends on the already switched state of the neighboring subsystems A_i and A_{i+1}. This corresponds to two interwoven 1D symmetric CA's with two states per cell and nearest-neighbor coupling, where two CA's are switched alternatingly. In some cases this arrangement might be favorable as, e.g., for simulations of the 1D Ising model.[28]

Similar to the case of two coupled subsystems, not all of the possible 256 transition rules (three input, one output channel) can be realized with the special physical realization of a CA we are considering here. The reason for this is the finite relaxation time, so that rules, which differ only in the previous state of the subsystem to be switched, cannot be applied simultaneously.

VI. SUMMARY

We have demonstrated that multistationary quantum systems can be coupled together either via the Pauli exclusion principle or—probably more promising—via the Coulomb interaction. Using the Coulomb interaction, we have shown that the basic elements (AND, OR, NOT, and COPY), necessary to build a universal computer, can be realized by means of semiconductor heterostructures. In this sense, there seem to be no fundamental physical laws which might invalidate the concept of quantum computa-

tion. But in order to reduce the necessary interactions with the macroscopic preparation and measurement system, it seems more appropriate to use a parallel instead of the usual sequential computer architecture. The subsystems are then coupled locally and resemble the structure of a cellular automaton. In this respect it does not seem reasonable to build a quantum computer as a universal machine, but rather as a special purpose computer, which utilizes the inherent physical properties of a quasimolecular realization of an information processing machine, such as parallelism and the inherent stochastic nature of the microscopic dynamics. The quantum computer might be complemented and controlled by a conventional computer, since both systems and the optical control (semiconductor laser) could be fabricated of the same material, e.g., $Ga_{1-x}Al_xAs$, and might such be integrated on the same chip.

Other physical systems might as well be used as a submicroscopic realization of a computing system, such as in the molecular electronics based on organic molecules as proposed by Carter.[29–31] In such a realization, the Coulomb interaction might as well be used as a coupling mechanism between different subunits (molecules). The requirements are that the subunits show multistationarity on a certain time scale and that the switching process is accompanied by a charge transfer, so that the dipole moments of the molecules are different in different metastable states.

It is obvious that in any case the structural basis of the required complex dynamics needs more detailed theoretical and experimental investigations. Though biological systems are working examples of molecular complex systems, it presently remains an open question whether manufacturing processes of the near future can really be controlled down to the molecular level in all three dimensions.

ACKNOWLEDGMENTS

We would like to thank Professor H. Haken for valuable discussions. Financial support by the Deutsche Forschungsgemeinschaft (SFB 329, Physikalische und chemische Grundlagen der Molekularelektronik) is gratefully acknowledged.

[1]K. Obermayer, W. G. Teich, and G. Mahler, preceding paper, Phys. Rev. B **37**, 8096 (1988).
[2]K. Obermayer, G. Mahler, and H. Haken, Phys. Rev. Lett. **58**, 1792 (1987).
[3]G. Mahler and K. Obermayer, in *Computational Systems—Natural and Artificial,* Proceedings of the International Symposium on Synergetics, Schloss Elmau, Bavaria, 1987, *Springer Series in Synergetics,* edited by H. Haken (Springer-Verlag, Berlin, 1987).
[4]D. K. Ferry and W. Porod, Superlatt. Microstruct. **2**, 41 (1986).
[5]B. A. Hubermann and T. Hogg, Phys. Rev. Lett. **52**, 1048 (1984).
[6]T. Toffoli, Physica D **10**, 195 (1984).
[7]W. D. Hillis, Physica D **10**, 213 (1984).
[8]W. D. Hillis, Sci. Am. **256**(6), 86 (1987).
[9]P. Wiley, IEEE Spectrum **24**(6), 46 (1987).
[10]J. von Neumann, *Collected Works,* edited by A. H. Taub (Pergamon, Oxford, 1963), Vol. V.
[11]R. Lassnig, Phys. Rev. B **31**, 8076 (1985).
[12]H. Haken, *Quantenfeldtheorie des Festkörpers* (Teubner, Stuttgart, 1973), p. 156.
[13]Th. Förster, Ann. Phys. (Leipzig) **2**, 55 (1948).
[14]V. M. Kenkre and P. Reinecker, *Exciton Dynamics in Molecular Crystals and Aggregates,* Vol. 94 of *Springer Tracts in Modern Physics* (Springer-Verlag, Berlin, 1982).
[15]A. S. Davydow, *Quantum Mechanics* (Pergamon, Oxford, 1965), p. 522.
[16]P. Pokrowsky, W. E. Moerner, F. Chu, and G. C. Bjorklund, Opt. Lett. **8**, 280 (1983).
[17]C. Cohen-Tannoudji, B. Diu, and F. Laloe, *Quantum Mechanics* (Wiley, New York, 1977), p. 1310.
[18]R. C. Haddon and F. H. Stillinger, in *Molecular Electronic Devices,* edited by F. L. Carter (Dekker, New York, 1982), p. 19.
[19]C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (University of Illinois Press, Champaign, IL, 1949).
[20]R. M. Fano, *Transmission of Information* (MIT Press/Wiley, New York, 1961).
[21]R. S. Tucker, J. M. Wiesenfeld, P. M. Downey, and J. E. Bowers, Appl. Phys. Lett. **48**, 1707 (1986).
[22]S. Wolfram, Rev. Mod. Phys. **55**, 601 (1983).
[23]S. Wolfram, Physica D **10**, 1 (1984).
[24]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), p. 811.
[25]W. E. Moerner and M. D. Levenson, J. Opt. Soc. Am. B **2**, 915 (1985).
[26]J. Albert and K. Culik II, Complex Syst. **1**, 1 (1987).
[27]E. Berlekamp, J. Conway, and R. Guy, *Winning Ways for Your Mathematical Plays* (Academic, New York, 1982), Vol. 2.
[28]G. Y. Vichniac, Physica D **10**, 96 (1984).
[29]*Molecular Electronic Devices,* edited by F. L. Carter (Dekker, New York, 1982).
[30]F. L. Carter, Physica D **10**, 175 (1984).
[31]S. Roth, Z. Phys. B **64**, 25 (1986).