Heisenberg-to-Ising crossover in a random-field model with uniaxial anisotropy

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(Received 23 September 1987)

Uniaxial anisotropy causes a crossover between Ising and Heisenberg behavior in a random-field model. At random-field strengths and dimensionalities, intermediate between those which give either the Ising and Heisenberg domain states, a new "boundary" domain state appears. A novel criterion is also derived for domain-boundary roughening in the Heisenberg limit. The two-dimensional version of this model can be applied to ferromagnetic-antiferromagnetic sandwiches with interface randomness and leads to a prediction of an exchange anisotropy effect *below* a critical thickness of order the domain-wall width, and an enhancement of this effect below a second critical thickness.

I. INTRODUCTION

The random-field model has been one of the most challenging and heavily studied problems in disordered magnetism.[1,2] Nevertheless most work (except Ref. 3) has focused on either the pure Ising or pure Heisenberg limits, and on the limit of a weak random field. By contrast this paper explores the crossover between Ising and Heisenberg behavior in a random-field model with finite uniaxial anisotropy and strong random fields.

This work is motivated by the recent suggestion[4] that an interfacial random-field effect could be responsible for the peculiar phenomenon of exchange anisotropy[5] in ferromagnetic-antiferromagnetic sandwiches. A variety of possible mechanisms, such as surface roughness or interfacial alloying, may generate a random field acting from the ferromagnetic into the antiferromagnetic layer, causing the latter to break up into domains analogous to those postulated by Imry and Ma.[1] In typical systems, this random field can be large, comparable in fact to the ferromagnetic or antiferromagnetic nearest-neighbor exchange fields. Furthermore, uniaxial anisotropy is present in many Heisenberg ferromagnets and antiferromagnets. As will be seen below, the large random field and the anisotropy together generate novel features not present in the conventional random-field problem. Of course, by considering the staggered order parameter, the antiferromagnetic problem can trivially be mapped into the ferromagnetic problem, which is treated below.

The problem is addressed with domain arguments of the type advanced originally by Imry and Ma,[1] and by Grinstein and Ma[2] for energy minimization and domain-boundary roughening, respectively. While not rigorous in themselves, such arguments have been substantiated by renormalization-group calculations in the Ising case,[2] and so it is reasonable to attempt similar arguments as a starting point for the new limits of interest here.

The paper is structure as follows: Section II introduces the model and carries out the Imry-Ma energy analysis, revealing a variety of novel domain states and culminating in a phase diagram as a function of dimensionality and random-field strength. Section III extends the Grinstein-Ma Ising roughening model to the Heisenberg limit and applies the roughening criteria to the phase diagram. Finally, Sec. IV draws the connection to exchange anisotropy. The results correct the earlier random-field model of exchange anisotropy[4] by showing that the effect occurs only below a critical antiferromagnetic layer thickness. An enhancement of the exchange field is predicted below a second characteristic thickness corresponding to the transition between a so-called "boundary" domain state and the Heisenberg domain state.

II. ANISOTROPY MODEL

Consider a homogeneous Heisenberg ferromagnet with dimensionality d, nearest-neighbor exchange-energy J, uniaxial anisotropy energy K per unit volume, and lattice constant a, subjected to a random field at each site. Let the direction of the random field be along the uniaxial anisotropy axis and its strength be fJ in units of energy. That is, f is a dimensionless factor expressing the ratio of the strength of the random field to the local exchange field of the ferromagnet. It is also useful to introduce[6] an exchange stiffness A, such that the energy per unit volume is $A(\Delta m)^2$, where m is the unit vector of the order parameter (the magnetization). From the squared gradient factor, it is apparent that the exchange stiffness A scales as J/a^{d-2}.

Following Imry and Ma,[1] let us assume the ferromagnet breaks up under the influence of the random field into a domain pattern of characteristic dimension L. Let us consider the energy terms arising from the random field, the exchange, and the anisotropy in such a system. The random-field energy per atom goes down statistically as the inverse square root of the number N of atoms in a domain of volume L^d. Taking $l=L/a$ as a reduced length, we have an energy per unit volume of $-fJ/a^dN^{1/2}$ or $-fA/a^2l^{d/2}$, ignoring numerical factors of order unity.

In a conventional ferromagnet, uniaxial anisotropy gives rise to domain walls[6] with width $\sqrt{A/K}$ and surface energy $\sqrt{AK}$, where again we have ignored numerical factors of order unity. Let us define a normalized domain-wall width $\delta \propto \sqrt{A/K}/a$. This is a characteris-

tic length in the problem, independent of the random-field strength. For domain sizes $l \gg \delta$, the domain-wall energy per volume goes as $A/l\delta a^2 \propto J/l\delta a^d$. This is an Ising-like limit in which the bulk spins point along the easy anisotropy axis and the domain boundary is described by a surface energy. By contrast, if the domain size l is smaller than δ, exchange energy dominates the anisotropy energy, and the gradient in the magnetic unit vector approaches $1/L$. Then the energy per unit volume is of order $A/l^2a^2 \propto J/l^2a^d$. This can be called a Heisenberg limit because anisotropy no longer plays a role.

Combining these energy terms, we obtain the energy per unit volume, normalized by $a^2/A \propto a^d/J$:

$$w \propto -\frac{f}{l^{d/2}} + \frac{1}{l\delta}, \quad l > \delta, \quad \text{Ising}, \tag{1}$$

$$w \propto -\frac{f}{l^{d/2}} + \frac{1}{l^2}, \quad l < \delta, \quad \text{Heisenberg}. \tag{2}$$

Near $l = \delta$ or $L \simeq \sqrt{A/K}$, these relationships will be modified because physically there must be a smooth transition between the two limiting regimes arising from the gradual crossover between interacting and noninteracting domain walls. Rather than attempt a more complex treatment which connects the two limits analytically, the following analysis uses only the simple limiting forms. The smoothing of the energy as a function of domain size near $l = \delta$ must then be kept in mind in interpreting the results.

The simple equations (1) and (2) lead to a remarkably diverse phase diagram. Taking derivatives, one obtains the domain sizes at energy extrema

$$l \propto (df\delta/2)^{2/(d-2)}, \quad l > \delta, \tag{3}$$

$$l \propto (df/4)^{2/(d-4)}, \quad l < \delta, \tag{4}$$

which can be reexpressed, using the definition

$$\eta \equiv \frac{df\delta^{(4-d)/2}}{2}, \tag{5}$$

as

$$l/\delta \propto \eta^{2/(d-2)}, \quad l > \delta, \tag{6}$$

$$l/\delta \propto (\eta/2)^{2/(d-4)}, \quad l < \delta. \tag{7}$$

The corresponding energies at the extrema are

$$w \propto (d-2)/2\eta^{2/(d-2)}, \quad l > \delta, \tag{8}$$

$$w \propto (d-4)/2(\eta/2)^{4/(d-4)}, \quad l < \delta. \tag{9}$$

At $l = \delta$ the energy is simply

$$w \propto -\eta + (d/2). \tag{10}$$

Figure 1 plots the extrema of Eqs. (6) and (7) as a function of dimensionality d and the parameter η, which is a normalized measure of the random-field strength. This figure, and also Eqs. (8) and (9), exhibit clearly the well-known critical dimensionalities 2 and 4 for the Ising and Heisenberg random-field models, respectively. Contact can be made with the Ising results of Imry and Ma[1] by considering the limit $f \to 0$ and $\delta \to 0$, hence $\eta \to 0$. Figure 1 shows the negative energy minima for $d < 2$ and $l/\delta > 1$, which represents the Imry-Ma domain state. For

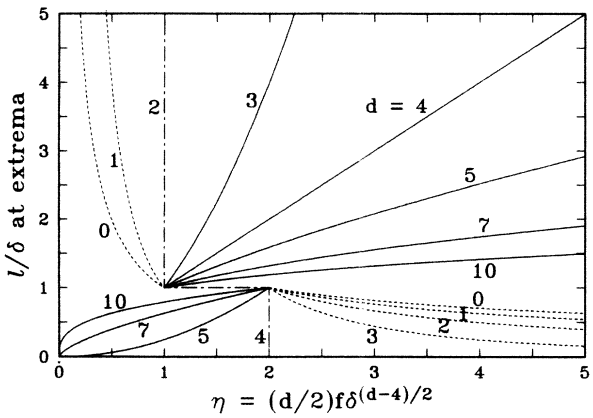

FIG. 1. Normalized domain size at energy extrema of a random-field Heisenberg model with uniaxial anisotropy, as a function of the normalized random field strength f, with dimensionality d as a parameter. Solid lines are minima, dotted lines are maxima, and dash-dotted lines are minima or zero energy states at the boundary between the Heisenberg and Ising limits. The normalized domain-wall width $\delta \equiv \pi\sqrt{A/K}/a$, where A is the exchange stiffness constant and K is the uniaxial anisotropy per unit volume.

$d > 2$ there is no minimum, and inspection of Eq. (1) confirms that the ferromagnetic state has lowest energy. Equation (5) is also consistent with the result of Goldschmidt and Aharony.[3]

A novel feature of Fig. 1 is the line at $l = \delta$ between $1 \leq \eta \leq 2$. Here one finds a minimum at all dimensionalities. Figure 2(a) shows a set of energy contours illustrating this behavior for $d = 2$. As is evident from the $\eta = 1.5$ curve in the figure, the minimum is not characteristic of the extrema of either limit but occurs at their nonanalytic joining point. As discussed above, in the more complete physical picture, this nonanalytic behavior is rounded out. Nevertheless, since the long-range tails of domain walls are exponential,[6] the rounding will be small, and the minimum can be expected to persist and remain close to $l = \delta$.

As is evident from Fig. 1, below $d = 2$, this new "boundary" domain state evolves smoothly as a function of η into the conventional Imry-Ma domain states: into an Ising-like state for $\eta < 1$ and into a Heisenberg-like state for $\eta > 2$. For $2 < d < 4$ it can evolve into a Heisenberg-like state for $\eta > 2$. The boundary domain state can be either stable or metastable, that is, have positive or negative energy, according to Eq. (10).

At $d = 2$, which is of interest for the problem of exchange anisotropy to be discussed in Sec. IV, this simple model predicts the domain size l to be infinite (the ferromagnetic state) for $\eta < 1$. It drops to $l = \delta$ for $1 < \eta < 2$ in the new boundary domain state. Finally, for $\eta > 2$ it goes as $2\delta/\eta$ according to Eq. (7) in the Heisenberg Imry-Ma state.

However, it is by now well known[2] that for the critical $d = 2$ case, the simple model of Eq. (1) does not properly predict the equilibrium behavior. In the weak random-field limit (small η or f), the ferromagnetic state is bro-

ken up,[2] and the correlations scale as $\exp(1/f)$. Obviously, at small f, the domain size can be very large; so the state could be called "almost" ferromagnetic. The behavior for larger f is not known.

At dimensionalities greater than 4, as shown in Fig. 2(b), a metastable minimum appears at the boundary between the Heisenberg and Ising regions. Again, this minimum should persist in the presence of a moderate amount of rounding. Of course, this high-dimensionality behavior is not relevant to experiment.

All of these observations can be summarized in the diagram of Fig. 3. Here F denotes the ferromagnetic state, I the Ising Imry-Ma domain state, H the Heisenberg Imry-Ma domain state, B the new domain state at the boundary between the two limits, and L the state in which each spin follows the local exchange field, which implies the complete destruction of any short-range or long-range order. The parentheses indicate metastable states. Particularly the $d=2$ behavior will be of interest in Sec. IV below.

In many cases the lines in Fig. 3 should not be interpreted as phase boundaries. In the high-dimensionality limit, for example, Fig. 3 shows a fully disordered state, while conventionally one might expect a ferromagnetic state.[1] The resolution to this conceptual difficulty comes from recognizing that Eq. (2) is no longer valid when the domain size approaches atomic dimensions. Thus, in certain cases the disordered state of Eq. (2) (L in Fig. 3) is not physically relevant and the ferromagnetic state F becomes the only stable solution.

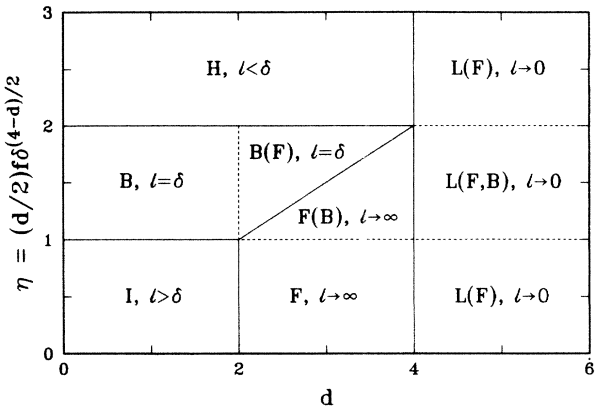

FIG. 3. "Phase" diagram for the random-field Heisenberg model with uniaxial anisotropy, as a function of dimensionality d and normalized random-field strength η. H denotes the Heisenberg domain state with normalized domain size l less than the characteristic domain-wall width. Similarly, B denotes the "boundary" domain state, I the Ising domain state, F the ferromagnetic state, and L the local state in which each spin follows the local random field. Parentheses indicate metastable states.

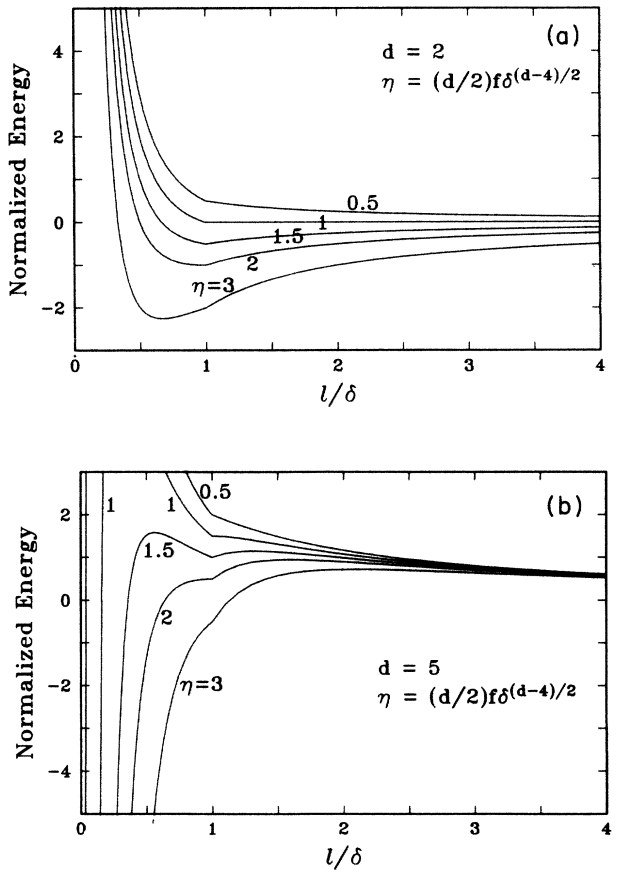

FIG. 2. Normalized energy vs normalized domain size for (a) a two-dimensional and (b) a five-dimensional random-field Heisenberg model with uniaxial anisotropy, with a set of different normalized random-field strengths. The curves in (a) for $d=2$ and $1<\eta<2$ exhibit the minimum which can be called the "boundary" domain state because it occurs at the boundary between the Heisenberg and Ising regions at $l=\delta$.

Another point is that with increasing random-field strength (η in Fig. 3), the Heisenberg domains in region H shrink and ultimately approach atomic dimensions. Then the H state goes smoothly into the L state.

Yet another effect arises from considering the kinetics of domain formation and growth.[7] In the three-dimensional random-field problem, studied in doped bulk antiferromagnets, it is well known that such kinetic effects impede the formation of the ferromagnetic state. Thus the F state in Fig. 3 may be difficult to observe experimentally; this will be relevant to our discussion of exchange anisotropy below.

III. DOMAIN BOUNDARY ROUGHENING

The simple energy analysis given above assumes that the domain boundary is smooth. If the boundary were rough, the boundary length or surface area could go to infinity even for finite domain diameter, invalidating the analysis. Grinstein and Ma[2] addressed the possibility of roughening for the Ising case, both with a full renormalization-group treatment and with a simple domain argument. Here an analogous simple domain argument is proposed for the Heisenberg case.

First it is useful to review the Grinstein-Ma argument. As in Fig. 4(a), assume that the center of a domain wall bows out with an amplitude w and a wavelength b in $d-1$ dimensions, in response to the local random-field

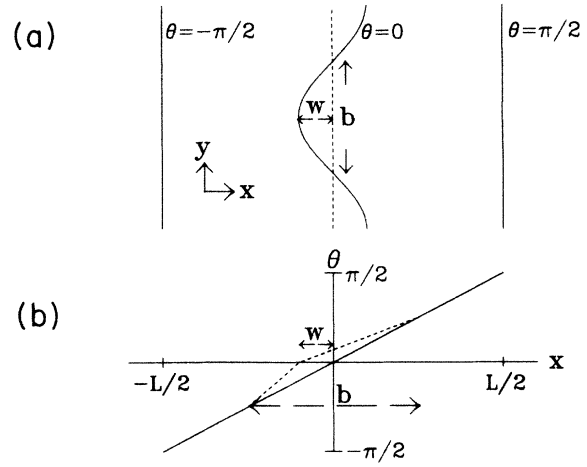

FIG. 4. Schematic roughening of a domain wall in a Heisenberg domain state. (a) shows a sinusoidal distortion to be understood as occurring in $d-1$ dimensions. (b) shows the distorted angle distribution, which extends over a thickness of order b.

potential. The volume of the bowed-out region scales as wb^{d-1}, so the number of atoms N in this region scales as wb^{d-1}/a^d, where a is the lattice parameter. The random-field energy gain per $(d-1)$-dimensional area scales as $fJ\sqrt{N}/b^{d-1}$, where fJ is taken as the local random-field energy per site. The bowing also entails an energy cost arising from the increased area of the domain wall, which in the Ising limit scales a Jw^2/b^2a^{d-1} per area. Minimizing these two energies, Grinstein and Ma found

$$w/b = f^{2/3}b^{(2-d)/3}a^{(d-2)/3} , \tag{11}$$

which exhibits the characteristic Ising critical dimensionality of 2. At $d=2$ the relative size of the bowing goes as $f^{2/3}$ and so is small for small f.

The Heisenberg problem differs from the Ising problem in several interesting ways. First, the exchange energy is nonlocal, since the spins are to a first approximation wound at a constant pitch through the domain, as illustrated in Fig. 4(b). The center of the wall may be defined as the point where the pitch angle θ is zero.

Spins a distance b away from the center of the distortion will experience little or no rotation because those spins integrate the canceling push and pull from different areas of the wall. Thus, it is apparent that the bowing of the wall center will cause spin rotations only in a region of thickness b, as illustrated schematically by the dashed line in Fig. 4(b). In the absence of the bowing, the angular gradient $d\theta/dx$ is π/L, but the bowing shifts the gradient to values of order $(\pi/L)[1\pm(w/b)]$ in a region of volume b^d. Thus, the change in exchange energy $A(\nabla m)^2$ per volume goes as $A(w/bL)^2$, where the linear terms in w/bL cancel out by symmetry. Per area b^{d-1}, this energy can be written Jw^2/bL^2a^{d-1}, in distinct contrast to the Grinstein-Ma Ising result.

A similar derivation can be given for the random-field energy in the Heisenberg case. In contrast to the Ising case, the Heisenberg spins in the middle of the wall are orthogonal to the field, and so the random-field energy gain per site from local bowing goes as $-fJ\Delta\theta/\sqrt{N}$, where the angular shift $\Delta\theta$ is to a first approximation simply $\pi w/L$, as is evident from Fig. 4(b). This energy gain occurs in a volume b^d, which determines $N=(b/a)^d$. Per area b^{d-1}, this energy thus becomes $-fJw/Lb^{(d-2)/2}a^{d/2}$.

Minimizing the energy of the combined exchange and random-field terms, one finds

$$w/L \propto f(b/a)^{(4-d)/2} , \tag{12}$$

which properly exhibits the Heisenberg critical dimensionality of 4. This result shows that for small f (weak random field) and $d>4$, large domains are regular in shape, because as $b\simeq L\to\infty$, $w/L\to 0$. Of course, in the model of the previous section, this limit is supplanted by the Ising limit as soon as the domain size expands beyond the characteristic domain-wall width.

Equation (12) can also be used to investigate the nature of the Heisenberg domain state in the region $d<4$ and $\eta>2$ in Fig. 3. One notes that b/a is always less than δ in the Heisenberg region. Furthermore $\eta>2$ implies $f\delta^{(4-d)/2}>4/d$. Thus, comparison with Eq. (12) shows that as the domain size goes to zero, w/L must also go to zero for a given strength f of the random field. This validates the Heisenberg model at small domain sizes. The degree of domain shape distortion at equilibrium increases with η, and, for example, at $d=2$ and $\eta=2$, $w/L\simeq 2$, indicating a moderately distorted domain shape. Since the Ising limit for $d=2$ and $\eta<1$ has a regular and presumably circular domain shape, one can extrapolate between these two limits to expect that in the boundary domain region $1<\eta<2$, the domain will be distorted but roughly circular.

IV. RANDOM-FIELD MODEL OF EXCHANGE ANISOTROPY

While the random-field Ising model has been extensively investigated experimentally in random antiferromagnets, the random-field Heisenberg-to-Ising crossover regime has not seen experimental exploration. In a recent paper[4] ferromagnetic-antiferromagnetic sandwiches exhibiting "exchange anisotropy" were proposed to be an experimental realization of this new kind of random-field model. In such sandwiches, a number of unusual phenomena are observed,[5] the most characteristic being a displacement of the ferromagnetic hysteresis loop by an internal effective field which appears after the sandwich has been cooled through its antiferromagnetic transition temperature in an applied field.

In the earlier work,[4] both the ferromagnetic and antiferromagnetic layers were postulated to have uniaxial anisotropy with easy axis in the plane and interfacial inhomogeneities of various sorts between the two layers which could give rise to an effective random field at the interface. When the ferromagnetic layer is single-domained, the random field in effect acts on the antiferromagnet at the interface and tends to break it up into antiferromagnetic domains. Accepting this model for our purposes here, let us consider how to relate this problem to the

earlier treatment of the generalized ferromagnetic random-field problem.

The first point is that the problem of an antiferromagnet in a random field is formally equivalent to that of a ferromagnet in a random field. This is because switching the direction of the random field at every second site does not affect the randomness of the distribution.

Let the thickness of the antiferromagnetic film be t_A. A cylindrical antiferromagnetic domain of diameter L occupies half the area of the square region $L\sqrt{\pi/2}$ on a side. Its wall energy[6] per area of interface is thus $8t_A\sqrt{AK}/L$, where A and K are now the antiferromagnet's exchange stiffness and anisotropy energies per unit volume. The random-field energy per interfacial area is thus approximately $-f_iJ/\sqrt{N}a^2$ or $-2f_iJ/\sqrt{\pi}La$, where f_iJ is the local random-field energy at the interface and $J=Aa$.

Then it can easily be shown that if the total energy per interfacial area is normalized by $a^2/8\pi t_A A$, and if $\delta\equiv\pi\sqrt{A/K}$, the total energy can be expressed in the Ising form of Eq. (1), but with

$$f=f_i a/4\pi^{3/2}t_A \ . \tag{13}$$

The coefficient here should be regarded as approximate, with factor-of-two accuracy, because the actual domain state will in general involve domains which are not perfectly circular and which enclose regions of either polarity. Of course, since f_i is not accurately known for actual experimental situations, it is mainly the thickness scaling of Eq. (13) which is of interest. Nevertheless, order of magnitude estimates are possible.

Equation (13) establishes the relationship of the random-field exchange anisotropy model to the two-dimensional ferromagnetic random-field model, and it has some interesting consequences. As seen in Sec. II, the two-dimensional random-field model exhibits a transition from an "almost" ferromagnetic to a "boundary domain" state above $\eta=1$, and a crossover to a Heisenberg domain state above $\eta=2$. Equations (5) and (13) thus imply critical thicknesses for the corresponding antiferromagnetic problem

$$t_{A,\mathrm{crit},1}=f_i\sqrt{A/K}/4\pi^{1/2} \ , \tag{14}$$

$$t_{A,\mathrm{crit},2}=f_i\sqrt{A/K}/8\pi^{1/2} \ . \tag{15}$$

For thicknesses greater than $t_{A,\mathrm{crit},1}$, the almost ordered antiferromagnetic state is favored, but below this thickness a boundary domain state becomes stable, changing to a Heisenberg domain state for thicknesses less than $t_{A,\mathrm{crit},2}$.

The presence of a domain state, coupled with a coercivity which anchors the domain walls in place, then leads to the exchange anisotropy effect. For, as argued elsewhere,[4] if one measures the hysteresis loop of the ferromagnetic layer, a field offset H_E in the center of this hysteresis loop arises from the difference $\Delta\sigma$ in interfacial energies for the two opposite orientations of the ferromagnetic layer of the sandwich. For a ferromagnetic magnetization M_F and thickness t_F (not to be confused with the ferromagnetic random-field system discussed earlier in the section), one has

$$H_E=\Delta\sigma/2M_F t_F \ . \tag{16}$$

Changing the ferromagnetic layer orientation corresponds to changing the sign of the random-field energy term, which is the first term in Eqs. (1) and (2). Thus, two times the random-field energy term gives $\Delta\sigma$. Using the coefficients of the simple model given above, one finds

$$H_E=2f_i A/\sqrt{\pi}M_F t_F L \ , \tag{17}$$

where L is in turn given by $\pi\sqrt{A/K}$ in the boundary region between $t_{A,\mathrm{crit},1}$ and $t_{A,\mathrm{crit},2}$, and, according to Eq. (4) or (7), by

$$L=8\pi^{3/2}t_A/f_i \tag{18}$$

for $t_A<t_{A,\mathrm{crit},2}$. Substituting Eq. (18) into Eq. (17), one finds

$$H_E=2f_i\sqrt{AK}/\pi^{3/2}M_F t_F, \quad t_{A,\mathrm{crit},2}<t_A<t_{A,\mathrm{crit},1} \ , \tag{19}$$

$$H_E=f_i^2 A/4\pi^2 M_F t_F t_A, \quad t_A<t_{A,\mathrm{crit},2} \ . \tag{20}$$

H_E as a function of thickness is plotted as the solid line in Fig. 5. Above a critical antiferromagnetic film thickness, the effect is predicted to vanish in the simple theory [Eq. (1)]. This thickness depends on f_i, which is not accurately known but which can be expected to be of order one for transition-metal exchange interactions which are primarily local or short range. Then the critical thickness scales as the characteristic domain-wall parameter $\sqrt{A/K}$, which is typically several hundred Angstroms in transition metals.

However, as mentioned above, a more complete theory of random-field phenomena[2] predicts a large, but finite, domain size or correlation length, corresponding to a small but finite offset field. Moreover, kinetic effects[7] are expected to impede the formation of large domain sizes, especially when the material is cooled through its transition temperature in the presence of the random field.

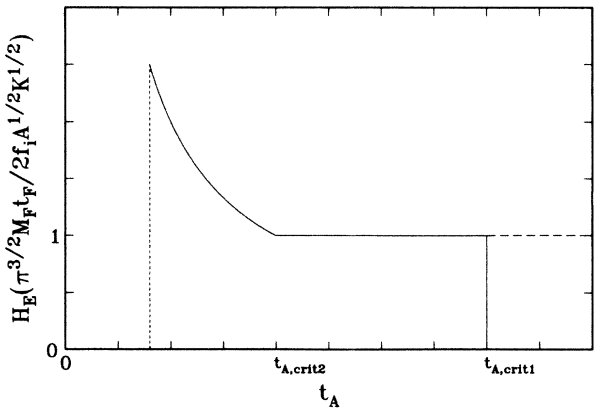

FIG. 5. Predicted exchange anisotropy offset versus antiferromagnetic film thickness. The dotted line indicates schematically a low-thickness instability due to insufficient coercivity for stabilizing the domain structure during the hysteresis loop experiment. The dashed line indicates a hypothetical metastable exchange anisotropy coming from domains which are kinetically impeded from attaining their equilibrium size (see text).

Then the upper critical thickness may be difficult to observe experimentally because metastable domains would give a metastable exchange field as suggested by the dashed horizontal line in the large thickness region of Fig. 5.

In the ultrathin film limit, one can expect a lower critical thickness for exchange anisotropy, as indicated by the dotted line in Fig. 5. The stability of the domain configuration depends on the interplay of (a) destabilizing forces generated at the interface when the ferromagnetic layer is reversed and (b) the stabilizing coercive force. The relative strength of the coercivity will decline with decreasing antiferromagnetic layer thickness if the coercivity arises from bulk inhomogeneities in the layer. Therefore, below a critical thickness, the domain walls will depin during the hysteresis loop experiment, annihilating the domain structure and so reducing $\Delta\sigma$ and H_E to zero.

At first glance experiments[5,8–12] would appear to support some, but not all, of these predictions. The early experiments on Co-CoO layers[5] appeared to show an exchange field independent of antiferromagnetic layer thickness down to a lower cutoff of a few tens of Angstroms. Such a lower cutoff has also been observed in more recent experiments on the FeNi-FeMn system.[8] However, the experiments show neither the higher thickness cutoff nor the rise at low thicknesses, predicted in Fig. 5.

A closer look at the experiments reveals that the thickness dependence was probed only out to a few hundred Angstroms, which is not significantly larger than the expected critical thickness, of order the antiferromagnetic domain-wall width. Most other experiments[9,10] have been on antiferromagnetic layers less than 150 Å thick. One possible exception is experiments[11] on relatively thick single crystal NiO layers with Ni on top. However, while a unidirectional anisotropy term was observed in torque measurements, direct hysteresis loop offsets were not reported. Other work on thick FeMn layers[12] is complicated by the fact that the antiferromagnetic γ phase is apparently only stable near appropriate fcc interfaces like copper or FeNi; thicker FeMn films show formation of the nonmagnetic (at room temperature) α phase further away from the interface.

In summary, there has been insufficient probing up to now of the high-thickness regime of Fig. 5. Experiments are needed to test the onset of metastability above the upper critical thickness. Existing data also does not show the predicted rise at low thicknesses, but this could be the result of some smearing as a function of thickness, an effect difficult to control on this almost-atomic scale. Some other considerations important in comparison to experiment will be reviewed elsewhere.[13]

The prediction of an upper critical antiferromagnetic film thickness [Eq. (14)] above which exchange anisotropy disappears could be questioned in view of the cylindrical domain approximation. As the thickness increases above a domain wall width, one must also consider the possibility of a horizontal domain wall which "caps" the cylindrical domain and limits its height. Such an analysis was performed in Ref. 4, where the total energy was, in fact, calculated to be positive. Therefore, in contrast to the implications of that paper, thick antiferromagnets are not likely to give the exchange anisotropy effect through the random-field mechanism, unless metastable states are obtained.

Further insight into the physics of the domain state can be obtained by returning to the formal equivalence of a ferromagnet and an antiferromagnet in the presence of a random field. Consider a region where, by chance, the local fields acting on the antiferromagnet are all ferromagnetically aligned in the same direction. It is well known that above a threshold field, the antiferromagnet "spin-flops" into a configuration in which the staggered magnetization points primarily orthogonal to the field direction, but with a certain degree of canting. In the corresponding ferromagnetic problem, this would be the case of a ferromagnet under the influence of a staggered field, where the ferromagnet takes up a canted configuration with the moments primarily orthogonal to the staggered field.

By a classic spin-flop energy analysis, it can be shown that the canted state is favored over the easy axis antiferromagnetic state provided $f^2J > Ka^3$, where, as before, fJ is the local field energy per site. Substituting Eq. (13), one immediately predicts that the canted phase is only stable below a critical thickness of order $f_i\sqrt{A/K}$, the same result as in Eq. (14).

This coincidence can be understood by recognizing that in the ferromagnetic Imry-Ha Heisenberg model, the canted ferromagnetic structures in which the moments point orthogonal to an antiferromagnetically staggered local field are just the domain walls separating the domain regions. In other words, regions with a tendency to regular alternation of the local field are regions where domain walls congregate, while regions with ferromagnetic alignment of local fields attract the centers of the domains. Similarly, the spin-flopped regions of the antiferromagnet will form the antiferromagnetic domain walls, while the easy-axis regions form the centers of the antiferromagnetic domains. When the canted or spin-flopped regions are no longer energetically stable, the entire domain structure disappears.

There are many other interesting aspects of this problem, particularly the topological structure of the circular domains considered above, which can be characterized by winding numbers, much as in the well-studied case of magnetic bubbles.[6] Also of interest is the cubic anisotropy and unusual spin structure of FeMn, which is not a simple linear antiferromagnet, as assumed here, but rather has a spin structure in which the four sublattice moments point toward the four faces of a tetrahedron. These effects also influence the predictions of the theory and its comparison to experiment. These issues will be discussed in a separate publication.[13]

ACKNOWLEDGMENT

The author thanks J. C. Slonczewski, G. Grinstein, Y. Imry, D. Mauri, E. Kay, and J. K. Howard for key discussions.

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