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Random fields and orientational order in models with a continuous-energy minimum set in q space and in magnetic systems favoring an incommensurate order

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We study the influence of quenched random fields on the ground-state properties of a Ginzburg-Landau-Wilson model with a continuous set of energy minima corresponding to modulated phases with a fundamental wave vector on a ring embedded in d-dimensional reciprocal space. Thus, at low temperatures and in the absence of random fields, there is both orientational and translational order. Arbitrarily weak random-field breaks translational, i.e., long-range modulated order for $d<4.5$. However, for $d<d^*$, for some d^*, long-range orientational order is also unstable. We argue that $d^*>3$. Therefore, in three-dimensional realizations of the model the modulated state breaks into domains having random orientations of their local wave-vectors. Some magnetic systems favoring modulated order can be approximately described by models with a continuous set of energy minima, e.g., magnetic superconductors, such as $ErRh_4B_4$, $HoMo_6S_8$, and $HoMo_6Se_8$. However, spatial anisotropy, due to a crystal lattice, breaks the degeneracy of a continuous energy minimum down to a discrete set of energy minima in q space. Thus, the stability of the orientational order is determined by a competition, which is studied here, between random fields and spatial anisotropy. We speculate that orientationally disordered structures, observed in the coexistence phase of magnetic superconductors, which are composed of domains with different orientations of local wave vectors, could be a random-field effect.

I. INTRODUCTION

Inhomogeneously ordered states, such as mass- and charge-density waves or modulated spin states, break a continuous symmetry of the system-translational symmetry. In 3D, arbitrarily weak random fields (RF) destroy long-range modulated order.[1] This is well-known not only to physicists studying modulated structures but also to those interested in RF problems since incommensurate charge density wave systems had offered an experimental confirmation that an RF in 3D prevents a continuous symmetry from being broken.[1] In fact, a modulated structure exhibiting large, but finite translational coherence length, can be appropriately interpreted by an RF model.

In this paper we study the influence of quenched random fields on systems which favor a modulated order at one of a continuum of nonzero wave vectors. Some of these systems are described by Ginzburg-Landau-Wilson (GLW) Hamiltonians with quadratic terms (i.e., inverse susceptibilities) which attain their minimum value not at the point $\mathbf{q}=0$, but on a surface or line in q space.[2] The other systems of this kind, commonly occurring in liquid-crystal physics, are usually described by de Gennes–type Hamiltonians.[3] Nevertheless, for symmetry reasons,[4] they share with the previously mentioned models, at least, the same spin wave description at low temperatures.[5,6] In both cases, at low temperatures an inhomogeneous order of the form $\langle M(x)\rangle \propto \cos(\mathbf{q}_0\cdot\mathbf{x}+\varphi)$ occurs, with $M(x)$ being a smectic density wave, modulated spin moment, *etc.*,[2–7] and with $\mathbf{q}_0$ belonging to the continuous energy minimum. So, the phase φ and the wave vector $\mathbf{q}_0$ are selected, respectively, by spontaneous breaking of translational and orientational symmetries.

The particular model we consider here is the ring model in which the energy minimum is a ring embedded in d-dimensional q space.[2–7] Our most striking prediction is that *orientational* order of the low-temperature phase, is unstable against an arbitrarily weak RF in 3D. The modulated ground state of the system breaks up into domains with random orientations of their local wave vectors so that there is no preferred direction at low temperatures when an arbitrarily weak RF is present in 3D. More precisely, for $d<4.5$, only translational order is unstable. But, for $d<d^*$, plausible arguments can be given (Secs. II and III) that long range orientational order is also unstable due to RF. We argue that $d^*>3$.

An interesting application of these ideas is to magnetic systems such as magnetic superconductors,[7] which can be approximately modeled by GLW models of the type considered here (Sec. IV).[36] Indeed, the disordered state we propose here exhibits some of the phenomenological features, such as translational incoherence of the modulated structure (reflected also in suppression of higher order harmonics) and the existence of orientationally disordered structure composed of domains with different orientations of local wave vectors observed in the coexistence phase of these materials.[8–12] However, in realistic magnetic materials, spatial anisotropy due to the crystal lattice breaks the energy degeneracy of the continuous energy minimum set and the actual state is determined by a competition between the quenched disorder and the anisotropy. We study this competition throughout the paper.

The outline of the paper is as follows: the GLW model, studied at low temperatures throughout the paper is defined in Sec. II A, and the spin-wave (SW) model is constructed. The SW model is studied perturbatively in Secs. II B and II C and possible limitations of a perturbative approach are discussed in Sec. II D. The main outcome of the SW theory is that $d^* > 3$, e.g., $d^* = 3.5$ in the framework of harmonic SW theory (Sec. II B), or that $4 \ge d^* \ge 3.5$, as argued in Sec. II C. Topological defects, e.g., dislocations loops, neglected by the SW approach, are discussed qualitatively in Sec. III and in the Appendix—in these sections we argue that $d^* = 4$. Competition between random fields and spatial anisotropies is discussed throughout Secs. II and III. We argue that sufficiently weak spatial anisotropy, for a given strength of RF, does not restore long-range orientational order. Summary as well as a discussion of application of these ideas to magnetic systems is discussed in Sec. IV.

II. MODEL AND SPIN-WAVE THEORY

A. Spin-wave model

As announced in the Introduction, we shall focus our considerations on low temperature properties of the ring model[2,7] in the presence of random fields (RF). This GLW model has, in d dimensions, the form

$$H(M) = \int \frac{d^d q}{(2\pi)^d} \tfrac{1}{2} K_I(\mathbf{q}) \, |M(\mathbf{q})|^2 + \int d^d x \, f(M(\mathbf{x})) + \int d^d x \, h(x) M(x) \;, \quad (2.1)$$

$$K_I(\mathbf{q}) = a(q_1^2 + q_2^2 - r)^2 + b \sum_{k=3}^{d} q_k^2 \;, \quad (2.2)$$

$h(x)$ is assumed to be Gaussian RF, with $[h(x)] = 0$, $[h(x)h(y)] = \Delta d^d(x-y)$; RF is coupled to one component order parameter $M(x)$ (we shall use angular brackets to denote thermal averages, and square brackets for averages over RF. However, all the results are exhibited in zero-temperature limit.) As usual $f(M) = u_2 M^2 + u_4 M^4 + \cdots$. The most important difference between (2.1) and the standard ferromagnetic GLW model is momentum dependence of the inverse susceptibility $K_I(\mathbf{q})$, Eq. (2.2), which attains its minimum value not at $\mathbf{q} = 0$, but on a ring in q space:

$$q_1^2 + q_2^2 = r = q_0^2, \quad q_k = 0, k = 3, 4, \ldots, d \;. \quad (2.3)$$

At low temperatures and for zero RF, such a model favors a linearly polarized modulated state $\langle M(x) \rangle \propto \cos(\mathbf{q}_0 \cdot \mathbf{x} + \varphi)$. The only restriction on $\mathbf{q}_0$ is that it belong to the ring (2.3). However, it's orientation is not fixed—so the orientational symmetry is broken at low temperatures. Also, the phase φ is not fixed for an incommensurate structure, as the one favored by (2.1). Different choices of φ produce the same state, only translated in real space—so the translational symmetry is also broken. In 3D pure ring model ($\Delta = 0$), both translational, in φ, and orientational orders, in $\mathbf{q}_0$, are stable with respect to thermal fluctuations at low enough temperatures.[7]

In real magnetic systems such as magnetic superconductors[8] an underlying crystal lattice breaks the orientational energy degeneracy of the ring (2.3) so that the wave vector of the condensate $\mathbf{q}_0$ belongs to a discrete set of n points in q space (Fig. 1). For example, $n = 4$, for $ErRh_4B_4$ (Ref. 9), and $n = 6$ for $HoMo_6S_8$ (Refs. 10 and 11), and $HoMo_6Se_8$ (Ref. 12). So, the terms of the form

$$K_A(q) \propto \mathrm{Re}(q_1 + i q_2)^n \;, \quad (2.2')$$

$i = (-1)^{1/2}$, should be added to $K_I(\mathbf{q})$, (2.2), in order to account for this *spatial* anisotropy. Throughout the paper we shall consider also effects of these spatial anisotropies on the physics of the isotropic model (2.1) and (2.2).

In this section we shall construct a spin-wave (SW) model[5] for the random field problem Eq. (2.1). Preliminary, we express the problem in terms of replicas. After a standard development, the effective replica field Hamiltonian H_{eff},

$$\frac{H_{\text{eff}}}{k_B T} = \frac{1}{k_B T} \sum_p H_0(M_p) - \frac{\Delta}{2(k_B T)^2} \sum_{p,p'} \int d^d x M_p(x) M_{p'}(x) \;, \quad (2.4)$$

is obtained from (2.1) [p is replica index, $p = 1, 2, \ldots, m$; $H_0(M)$ in (2.4) is the nonrandom part of (2.1), and, as usual, the limit $m \to 0$ gives all relevant information about the original RF problem (2.1)]. We are primarily interested in the limit of small temperature T in (2.4), i.e., in the ground-state properties of the system, which are nontrivial in the presence of RF.[1] At low T an order of the form $[\langle M(x) \rangle] \propto \cos(\mathbf{q}_0 \mathbf{x} + \varphi)$ is expected. Therefore, small energy fluctuations should be appropriately described by inserting the spin wave ansatz $M_p = M \cos[S_p(x)]$ in (2.4).[5,13] Thus we obtain an effective SW Hamiltonian

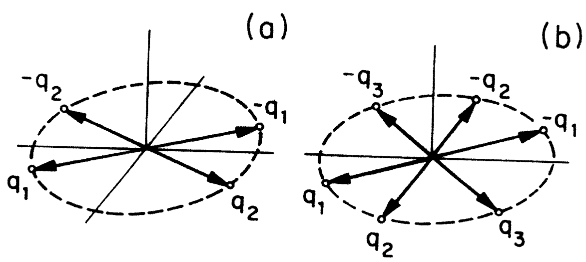

FIG. 1. In magnetic systems, such as magnetic superconductors, spatial anisotropy breaks energy degeneracy of the ring (dashed), so that energy minima occur on a discrete set of n points in q space, (a) $n = 4$, for $ErRh_4B_4$, (b) $n = 6$, for $HoMo_6S_8$ and $HoMo_6Se_8$.

$$\frac{H_{\mathrm{SW}}}{k_B T}=\frac{1}{2t}\sum_p \int d^d x \left\{ a_1 \left[\left[\frac{\partial S_p}{\partial x_1}\right]^2+\left[\frac{\partial S_p}{\partial x_2}\right]^2 - r\right]^2 + a_2 \left[\frac{\partial^2 S_p}{\partial x_1^2}+\frac{\partial^2 S_p}{\partial x_2^2}\right]^2 + b\sum_{k=3}^{d}\left[\frac{\partial S_p}{\partial x_k}\right]^2\right\} - \frac{g}{t^2}\sum_{p,p'}\int d^d x \cos(S_p - S_{p'}) , \tag{2.5a}$$

with $a_1=a_2=a$, $t=k_BT/2M^2$ and $g=\Delta/16M^2$, and for the moment we set the anisotropy (2.2′) zero. Within SW theory, the phases S_p in (2.5a) are treated as continuous real fields $-\infty<S_p(x)<\infty$. Thus, the topological defects, e.g., dislocation loops in 3D,[6] are neglected by SW approach. Dislocations should not qualitatively alter the physics at low temperatures if they are bound in a weak RF. It is frequently and, presumably incorrectly, assumed that dislocations are bound in similar RF problems, e.g., in RF *XY* model even below the lower critical dimension being four.[14,15]

We treat the question of topological defects in Sec. III. However, we note that dislocation loops are almost certainly unbound in the ground state, see Sec. III—so, the SW theory is only a crude preliminary step in understanding the problem. Nevertheless, even within SW theory, some interesting features of the present problem become manifest, e.g., instability of the orientational order in 3D due to RF. Moreover, since dislocations always tend to decrease order, it seems clear on physical grounds that the SW estimate of the lower critical dimension of the orientational order d^* should be a lower bound (see Sec. III). We present the details of SW theory (Sec. II B and II C) also because they are the unique quantitative results which we could construct to estimate whether the physics of the isotropic model (2.1) and (2.2) is relevant for understanding properties of realistic, spatially anisotropic, magnetic systems, such as magnetic superconductors (Sec. IV). SW calculations of this work are performed within SW perturbation theory.[15,16] Such calculations are greatly simplified in the zero temperature ($T=0$) limit, since, to all orders in zero temperature perturbation theory, one can replace RF term $\cos(S_p\text{-}S_{p'})$, appearing in Eq. (2.5a) by a much simpler harmonic RF term[16]

$$1-\tfrac{1}{2}(S_p\text{-}S_{p'})^2 . \tag{2.5b}$$

However, metastable states of the SW action, which are not properly accounted for by a perturbative approach, may significantly affect the details of the physics.[14,15] We discuss this problem in Sec. II D.

In the next section we present predictions of harmonic approximation to the SW theory, which is obtained by substituting $S_p(\mathbf{x})=q_0x_1+\varphi_p(\mathbf{x})$ in (2.5a) and neglecting nonlinear gradient terms in φ, such as $(\partial\varphi/\partial x_2)^4$ and $(\partial\varphi/\partial x_1)(\partial\varphi/\partial x_2)^2$. The result is

$$\frac{H_{\mathrm{SW}_0}}{k_B T}=\frac{1}{2t}\sum_p \int d^d x \left[4a_1 r\left[\frac{\partial\varphi_p}{\partial x_1}\right]^2 + a_2\left[\frac{\partial^2\varphi_p}{\partial x_1^2}+\frac{\partial^2\varphi_p}{\partial x_2^2}\right]^2 + b\sum_{k=3}^{d}\left[\frac{\partial\varphi_p}{\partial x_k}\right]^2\right] - \frac{g}{t^2}\sum_{p,p'}\int d^d x\cos(\varphi_p-\varphi_{p'}) . \tag{2.6}$$

The most prominent property of SW theory (2.6) is the existence of a single soft direction 2, resulting from the orientational symmetry of the problem, i.e., the fact that the choice of the preferred direction 1 is quite arbitrary.[17] As mentioned in the introduction, because of the same reason, a number of liquid crystal systems has, at low temperatures, the same SW theory as the magnetic model (2.1).[6] For example, the DeGennes' type model for planar nematics of Nelson and Halsey[18] which is, roughly, of the form

$$\frac{H(\psi,\mathbf{N})}{k_BT}=\frac{1}{k_BT}\int d^dx[\,|(\nabla-iq_0\mathbf{N})\psi|^2 + \mathcal{H}(\mathbf{N})+U(|\psi|) + h(x)\,\mathrm{Re}\psi(x)] . \tag{2.7}$$

ψ is a complex translational (i.e., smectic) order parameter, $\mathbf{N}$ is planar orientational (i.e., nematic) order parameter, confined to the plane 1–2, i.e., $\mathbf{N}=(\cos\theta,\sin\theta,0,\ldots,0_d)$, $\mathcal{H}(\mathbf{N})$ is a Frank-type Hamiltonian[18] and $U(|\Psi|)$ contains, as usual, a nonlinear term stabilizing the smectic phase at low temperatures T. We added a random field coupled to the smectic density wave. At low-temperatures smectic order at wave vector $\mathbf{q}_0$ can be expected ($|\mathbf{q}_0|=q_0$ and $\mathbf{q}_0$ belongs to the plane 1-2) and the ansatz

$$\Psi=\Psi_0\exp[iq_0x_1+i\varphi(x)]$$

is appropriate for studying fluctuations. After inserting this ansatz into (2.7) and integrating out of the action nematic fluctuations, a spin wave theory of the form (2.6) is obtained. The form of dislocation-mediated melting theory is the same for all pure systems with a single soft direction,[6] and this fact should be maintained in the presence of weak RF. We shall use this fact in Sec. III to argue about dislocation effects.

B. Harmonic spin-wave theory

Let us consider the harmonic SW theory (2.6), with the replacement (2.5b). A simple calculation gives

$$\langle\varphi_p(\mathbf{q})\varphi_{p'}(-\mathbf{q})\rangle=tG_0(\mathbf{q})\delta_{pp'}+2g[G_0(\mathbf{q})]^2 , \tag{2.8}$$

with

$$G_0^{-1}(\mathbf{q})=4a_1rq_1^2+a_2q_2^4+b\sum_{k=3}^{d}q_k^2\ . \tag{2.9}$$

For $d \le 4.5$ the average $\langle[\varphi_p(x)]^2\rangle$ diverges in the thermodynamic limit, indicating, by a well-known argument of Imry and Ma,[1] the absence of magnetic long-range order, even at zero temperature ($t=0$). So, for $d \le 4.5$, large phase fluctuations split the modulated structure into translationally incoherent domains. As usual,[16,27] the size of the domains can be estimated by calculating spin-spin correlation function $[\langle M(x)M(y)\rangle]$, which should be short ranged for $d \le 4.5$. This calculation is simple within harmonic theory (2.8), and gives the typical size of a domain, for $d<4.5$

$$\xi_2 \approx x^*(a_1/a_2)^{1/(9-2d)}\ , \tag{2.10}$$

along the soft direction 2, or

$$\xi_1 \approx (a_1/a_2)^{1/2}q_0\xi_2^2\ , \tag{2.11}$$

along the wave vector of the modulated state, i.e., direction 1. The length x^*, in (2.10), is

$$x^*=\left[\frac{g a_1^{1/2}a_2^{(d-7)/2}}{r^{1/2}b^{(d-2)/2}}\right]^{-1/(9-2d)} \propto g^{-1/(9-2d)}\ , \tag{2.12}$$

(x^* is related to nonlinear effects, Sec. II C). From (2.10)–(2.12) we see that the domains are highly anisotropic when the disorder g is weak:

$$\frac{\xi_1}{\lambda_0}\approx\left[\frac{a_1}{a_2}\right]^{1/2}\left[\frac{\xi_2}{\lambda_0}\right]^2 \propto g^{-1/(4.5-d)}\ , \tag{2.13}$$

where λ_0 is the wavelength of the modulated structure, $\lambda_0=2\pi/q_0$ [Fig. 2(a)].

For $d<3.5$ another, somewhat unusual divergence occurs. Namely, within the harmonic theory (2.8), the average

$$\left[\left\langle\left[\frac{\partial\varphi}{\partial x_2}\right]^2\right\rangle\right]=\left\langle\left[\frac{\partial\varphi_p}{\partial x_2}\right]^2\right\rangle$$

diverges in the thermodynamic limit. Since the angle $\theta(x)$ defining local orientation of the modulated structure, i.e., the local orientation $\mathbf{N}(x)=[\cos\theta(x),\ \sin\theta(x)]$ of the wave vector $(\partial S/\partial x_1, \partial S/\partial x_2)$ [Fig. 2(a)] is

$$\theta(\mathbf{x})=\arctan\left[\frac{\partial S}{\partial x_2}\Big/\frac{\partial S}{\partial x_1}\right]\approx\frac{\partial\varphi}{\partial x_2}\Big/q_0\ , \tag{2.14}$$

this last divergence indicates instability of the orientational order to an arbitrarily weak random field. One might expect the local orientation $\mathbf{N}(x)$ of the structure to vary randomly over the system so that there is no preferred direction in the presence of RF for $d \le 3.5$, and particularly in 3D [Fig. 2(b)]. So, even in the ground state, the modulated structure is broken into domains with random orientations of local wave vectors as depicted in Fig. 2(b). The size of a domain inside of which the system is locally orientationally ordered can be estimated by calculating orientational correlation function:

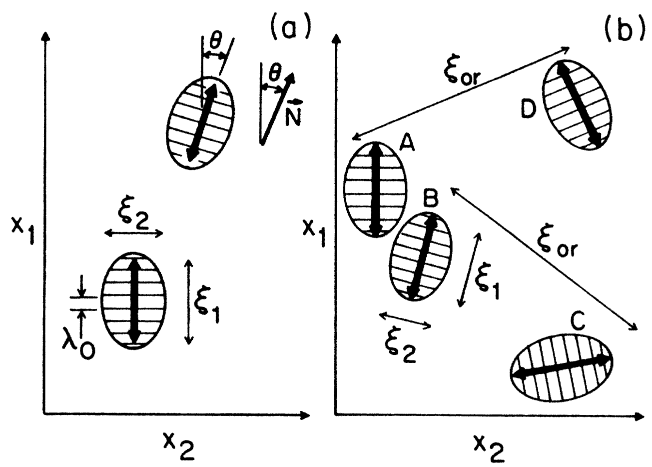

FIG. 2. (a) For $d<4.5$ long-range magnetic, i.e., translational order is destroyed by SW fluctuations. The system is still translationally coherent inside of anisotropic domains of size ξ_2, along the soft, and ξ_1, along the hard direction. For $d^*<d\le 4.5$ ($d^*=3.5$ in harmonic SW theory), the directions of local wave vectors (specified by $\mathbf{N}$) are correlated throughout the sample, and there is long-range orientational order. (b) For $d \le d^*$ there is no long-range orientational order. The directions of local wave vectors are correlated only up to distance of order ξ_{or}, the orientational correlation length.

$$K(\mathbf{x})=[\langle\mathbf{N}(\mathbf{x})\cdot\mathbf{N}(0)\rangle]\ . \tag{2.15}$$

This calculation is easy within the harmonic theory (2.8) and the approximation (2.14).[19] For $d=3.5$ one obtains nonuniversal behavior,

$$K(\mathbf{x})\propto|\mathbf{x}|^{-u},\quad u\propto g\ , \tag{2.16}$$

while for $d<3.5$

$$K(\mathbf{x})=\exp(-|\mathbf{x}/\xi_{or}|^{2(3.5-d)})\ , \tag{2.17}$$

for $\mathbf{x}$ lying in x_1-x_2 plane,[19] with

$$\xi_{or}\approx\xi_2(\xi_2/\lambda_0)^{1/(3.5-d)}\propto g^{-1/2(3.5-d)}\ , \tag{2.18}$$

being the orientational correlation length. So, the following picture depicted in Fig. 2(b) emerges: short-range translational (e.g., modulated magnetic) order exists inside of domains characterized by ξ_1 and ξ_2, Eqs. (2.10)–(2.13). These domains are very anisotropic for weak RF—by Eq. (2.13)

$$\frac{\xi_1}{\lambda_0}\approx\left[\frac{\xi_2}{\lambda_0}\right]^2 \tag{2.13$'$}$$

since for magnetic systems $a_1=a_2$ [see the line below Eq. (2.5)]. For example, for a structure coherent some hundred wavelengths along the local wave-vector ($\xi_1/\lambda_0 \approx 100$), we have by (2.13′), $\xi_2/\lambda_0\approx 10$, for the coherence length in the direction perpendicular to the wavevector (in x_1-x_2 plane). At longer scales, spin waves destroy translational order, so that two neighboring domains, e.g., A and B in Fig. 2(b), are completely out of phase φ. Nevertheless, these two neighboring domains have almost the same orientation. However, pairs of domains, e.g., A and C or A and D in Fig. 2(b), which are separated by a distance bigger than $\xi_{\rm or}$, have uncorrelated orientations. In 3D, we have from (2.18),

$$\frac{\xi_{\rm or}}{\xi_2} \approx \left[\frac{\xi_2}{\lambda_0}\right]^2, \quad d=3\ , \qquad (2.18')$$

so that, for the previous example with $\xi_2/\lambda \approx 10$, we have $\xi_{\rm or} \approx 100\xi_2 \approx 1000\lambda_0$. So, for this example: $\xi_1 \approx 100\lambda_0, \xi_2 \approx 10\lambda_0$ and $\xi_{\rm or} \approx 1000\lambda_0$. This example has been chosen deliberately, since $\xi_1/\lambda_0 \approx 100$ could be appropriate for magnetic superconductors (see Sec. IV). With a typical wavelength for these materials $\lambda_0 \approx 100$ Å we have

$$\xi_1 \approx 10^4\ \text{Å}, \xi_2 \approx 10^3\ \text{Å}, \xi_{\rm or} \approx 10^5\ \text{Å}\ , \qquad (2.19)$$

as typical figures for magnetic superconductors, however, in the absence of spatial anisotropies to be considered later. In spin systems, such as (2.1), it could be very difficult to measure orientational correlations and to check the harmonic theory prediction (2.17).[20] Nevertheless, destruction of the orientational order can easily be observed in the simplest scattering experiment probing two-spin correlation function in q space (Fig. 3): Below and sufficiently close to the lower critical dimension of translational, say modulated, magnetic order, peaks corresponding to the modulated state are somewhat smeared [Fig. 3(a)]; however, the orientation of the pair of peaks is well defined and fixed by spontaneous breaking of orientational symmetry. Below the lower critical dimension d^* of the *orientational* order (being 3.5, within the harmonic SW theory), orientational order is destroyed and the peak is smeared all over the ring, Eq. (2.3), in q space as shown in Fig. 3(b). The translational coherence length along the local wave vector ξ_1 [see Fig. 2(b)] can be estimated experimentally, see Fig. 3(b). Then, $\xi_{\rm or}$ can be estimated by (2.13′) and (2.18′):

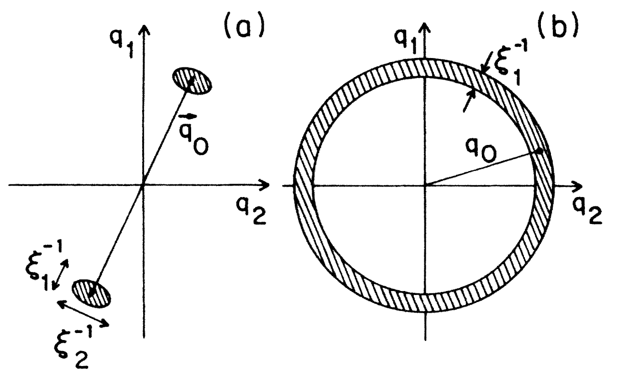

FIG. 3. Typical scattering patterns in q space (in the absence of spatial anisotropies). (a) For $4.5 \geq d > d^*$ only translational order is broken by a weak RF—the direction of the pair of smeared peaks at $\pm \mathbf{q}_0$ (corresponding to the modulated structure) is chosen by spontaneous breaking of the orientational symmetry. (b) For $d \leq d^*$ orientational order is unstable. The peak of (a) is smeared all over the ring.

$$\frac{\xi_{\rm or}}{\lambda_0} \approx \left[\frac{\xi_1}{\lambda_0}\right]^{3/2}. \qquad (2.18'')$$

As noted above, in magnetic systems it could be very difficult to measure $\xi_{\rm or}$ directly.[20]

Thus far we have neglected spatial anisotropies of the form of (2.2′), which break the degeneracy of the continuous energy minimum (2.3), so that condensation occurs on *one* of $(n/2)$ pairs of energy minima in q space (Fig. 1). After a development similar to that used to generate nonlinear SW theory (2.5a), terms of the form (2.2′) produce SW terms of the form:

$$\mathrm{Re}\left[\frac{\partial S_p}{\partial x_1} + i\frac{\partial S_p}{\partial x_2}\right]^n, \qquad (2.20)$$

and also some terms containing higher order derivatives of S_p. Interactions (2.20) can, in principle, be rather efficient in stabilizing the orientational order. In the presence of terms (2.20), the harmonic SW propagator (2.9) is modified to

$$G_0^{-1}(q) = 4a_1 r q_1^2 + a_2 q_2^4 + A q_2^2 + b\sum_{k=3}^{d} q_k^2\ . \qquad (2.21)$$

The term Aq_2^2 in Eq. (2.21) arises from the term $(\partial\varphi/\partial x_2)^2$ which is generated from (2.20) after the shift $S_p = q_0 x_1 + \varphi_p$ and neglecting SW nonlinear (in φ) terms (which are discussed later in this section). A is proportional to the strength of the anisotropic terms (2.20) or (2.2′) and direction 1 is chosen along one of $(n/2)$ preferred directions. So, there are no soft directions in (2.21). The anisotropy A provides an effective cutoff q_A to long wavelength fluctuations,

$$a_2 q_A^4 = A q_A^2, \text{i.e., } q_A = (A/a_2)^{1/2}\ , \qquad (2.22)$$

which stabilizes the orientational order within the harmonic approximation (2.8) since, with $G_0(q)$ as in (2.21), one obtains from (2.14) for $3.5 > d > 2$

$$\theta_0^2 \equiv \left[\left\langle \theta^2 \right\rangle\right] \approx \left[\frac{q_A^{-1}}{\xi_{\rm or}}\right]^{2(3.5-d)}. \qquad (2.23)$$

$\xi_{\rm or}$ is the previously introduced orientational correlation length in the *absence* of the anisotropy—in (2.23) it serves as a measure of the strength of RF [see Eq. (2.18)]. The anisotropy length q_A^{-1} measures the strength of spatial anisotropy [see Eq. (2.22)]. So, the local orientational fluctuations of θ, Eq. (2.14), are finite for a nonzero q_A and the orientational order is preserved at least within the harmonic SW theory. Then, the scattering pattern in Fig. 4(a) can be expected: a strong pair of peaks having angular spread θ_0, Eq. (2.23), along the ring (2.3). Fluctuations which are not well described by the harmonic SW theory can produce $(n-2)$ weaker peaks at other minima—they are also exhibited in Fig. 4(a). These other peaks correspond to domains in the sample having different directions from the one taken by the strongest peak. Nevertheless, the system is orientationally ordered.

Can this picture be maintained for arbitrarily small spatial anisotropy A? In the absence of the anisotropy

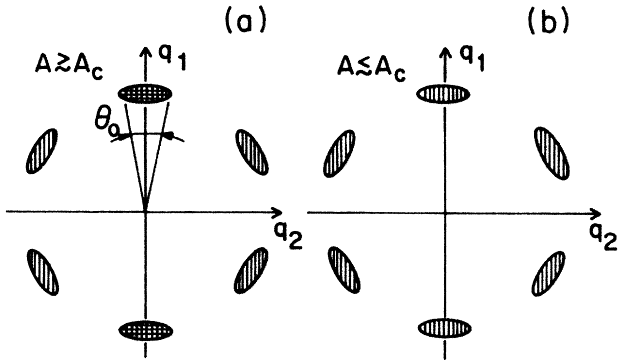

FIG. 4. Scattering patterns in q-space of a 3D magnetic system with sixfold spatial anisotropy (the q_1-q_2 plane is exhibited). (a) When the anisotropy A is stronger than the critical value A_c, the scattering is dominated by two strong peaks (double hatched) having angular spread θ_0, see Eq. (2.23). Weaker peaks can be expected to be observed at other minima if $A > A_c$. (b) When A is somewhat smaller than A_c, all peaks are equally strong and the orientational order is destroyed. The transition at $A = A_c$ could be the first order.

and for $d < 3.5$, we predicted instability of the orientational order. It is unlikely that arbitrarily weak spatial anisotropy would restore the orientational order for $n > 2$: switching on a small anisotropy would somewhat reorientate domains in Fig. 2(b) so as to have their directions more aligned to *all* of $(n/2) > 1$ preferred directions. For example, let a domain Γ of size ξ_{or}, have orientation $\mathbf{N}$ in the absence of spatial anisotropy. After switching on a weak anisotropy, Γ will take an orientation $\mathbf{N}'$ somewhat closer to the nearest of the $n/2 > 1$ preferred directions. Obviously, switching on a weak anisotropy would turn the initial collection of randomly orientated domains (orientated uniformly in all directions) into another collection of randomly orientated domains (orientated in all directions, but slightly nonuniformly). So, sufficiently weak n-fold spatial anisotropy would not restore the orientational order for $n > 2$, and a scattering pattern with n peaks of the same intensity should be observed [Fig. 4(b)]. However, strong anisotropy restores the orientational order and a phase transition at some critical value of the anisotropy A_c should exist. A detailed study of this transition is beyond the scope of this paper.[21] However, an estimate of the critical value of the anisotropy can be obtained from (2.23) by imposing $\theta_0 \approx 2\pi/n$: if θ_0, Eq. (2.23), is larger than $2\pi/n$, SW fluctuations would simply "melt" the orientationally ordered state. θ_0, being the angular spread of orientational fluctuations is unlikely to be larger than the angle between two neighboring preferred directions being $2\pi/n$. For such large orientational fluctuations, with $\theta_0 \approx (2\pi/n)$, nonlinearities of the anisotropic part of SW action (2.20) become important. These nonlinearities had been neglected in deriving the harmonic SW propagator (2.21), which was used to estimate orientational fluctuations in the presence of the anisotropy (2.23). However, it is rather obvious from the form of full nonlinear anisotropic SW terms (2.20) that these nonlinearities enhance orientational fluctuations and, actually, allow for the transition to the orientationally disordered phase if the orientational fluctuations predicted by the harmonic theory, Eq. (2.23), are large enough. In the absence of these nonlinearities the harmonic propagator (2.21) stabilizes some orientational order by fixing the orientational correlation function (2.15) at large distances to some finite value of order

$$K(\infty) = \exp(-\theta_0^2) = \exp(-|q_A^{-1}/\xi_{or}|^{2(3.5-d)}) . \quad (2.24)$$

However, because of the nonlinearities, a transition to an orientationally disordered phase is expected when $\theta_0 = [\langle \theta^2 \rangle]^{1/2} \approx 2\pi/n$, so that by Eq. (2.23),

$$2\pi/n \approx (q_A^{-1}/\xi_{or})^{3.5-d}, \quad n \geq 4 \quad (2.25)$$

at the transition. The transition could be of the second order [pattern in Fig. 4(a) may continuously turn into the pattern in Fig. 4(b) at the transition point]; however, the line of arguments presented here may suggest that the transition is first order.[21] From Eqs. (2.18), (2.22), (2.23), and (2.25) we obtain the relation between critical value of the anisotropy A_c and the strength of the RF g,

$$A_c \propto g^{1/(3.5-d)} , \quad (2.26)$$

which should hold for weak RF. For applications to realistic system (see Sec. IV), relation (2.25) is more useful. After omitting the numerical factor, this relation reads

$$1 \approx q_A^{-1}/\xi_{or} . \quad (2.27)$$

Equation (2.27) is appropriate for the weak RF (large ξ_{or}) and small anisotropy (large q_A^{-1}) case. For systems with $\xi_{or} \ll q_A^{-1}$ ($A \ll A_c$) spatial anisotropy is quite uninfluential—it becomes important at distances $x \approx q_A^{-1}$ where the orientational correlation function (2.17) is already small and, by the previous arguments, no orientational order will be restored if $n > 2$. So, for $q_A^{-1} \gg \xi_{or}$, the system is in the orientationally disordered phase of the isotropic model; however, local orientations of wave vectors of the domains (of size ξ_{or}) are somewhat rearranged due to the anisotropy, as explained before. For $q_A^{-1} \ll \xi_{or}$ ($A \gg A_c$) the anisotropy dominates and the orientational order is restored. Both situations could be observed in scattering patterns, Fig. 4.

The reasoning leading to the condition Eq. (2.25), to be satisfied at the transition, is similar to the classical picture of crystal melting, according to which the melting transition is driven when atomic displacements due to thermal phonons [corresponding to orientational fluctuations due to RF, Eq. (2.23)] become comparable to the crystal lattice constant (corresponding to the angle between neighboring preferred directions $2\pi/n$). In the case of crystal melting such estimates (known as Lindeman criterion) usually underestimate thermal fluctuations—atomic thermal fluctuations, as estimated from harmonic phonon theory, may be much smaller than the lattice constant at the actual first order melting transition temperature. As explained before, in our case harmonic SW theory also underestimates the disordering effect of RF—so it may happen that θ_0, Eq. (2.23), is much smaller than $2\pi/n$ at the transition. However, in Sec. IV we shall use Eq. (2.27) obtained within a harmonic SW theory. In that way we shall not *overestimate* the disordering effect of RF. A second point: the picture of crystal melting solely in terms of phonons is poor, since topological defects are necessary to describe disordered phases involved in melting (liquid and cubic liquid crystal phase).[22] However, in our case all phases involved in the transition (orientationally ordered and orientationally disordered one) are characterized by SW fluctuations which disorder both of them (in the framework of this section). So, the spin wave scenario for the transition proposed here is rather appropriate. However, the physics of the transition may be altered if topological defects are relevant for a complete description of the phases already disordered by spinwaves (Sec. III). Nevertheless the crude reasoning presented here to argue that sufficiently weak (i.e., $A < A_c$) n-fold spatial anisotropy A, with $n > 2$, does not restore the orientational order below d^* could hold quite generally—such an anisotropy somewhat reorientates domains without producing long range orientational order.

In conclusion, in this section we conjectured the existence of the lower critical dimension of the orientational order d^* of the model in absence of the spatial anisotropy (within harmonic SW theory $d^*=3.5$). For a given strength of RF, sufficiently weak spatial anisotropy does not restore the orientational order. We used crude arguments to reach these conclusions—in the remaining part of this section and in Sec. III we shall try to give some more solid argument in favor of the same conclusions.

C. Anharmonic effects

What is the nature of the instability of the orientational order conjectured by the simple SW theory of Sec. II B? Are the orientational correlations indeed given by (2.17) at large distances? The results of Sec. II B were derived by means of harmonic SW theory based on the neglect of nonlinear gradient terms. For example, in deriving (2.9), we had neglected gradient interactions such as $(\partial\varphi/\partial x_2)^4$ and $(\partial\varphi/\partial x_1)(\partial\varphi/\partial x_2)^2$. Even at the lowest order (beyond the harmonic approximation) of perturbation theory these terms produce infrared divergent behavior in the isotropic model for $d \leq 4.5$.[23] So the results of the harmonic SW theory are suspect. Note, however, that there were no problems in evaluating various quantities within the harmonic theory, at least for $d \lesssim 4.5$. As usual, this indicates that the physics is not drastically modified for $d < 4.5$, and that the divergences may be well controlled by an expansion in $\epsilon = 4.5 - d$. The harmonic fixed point, which has been used in the previous section, is unstable with respect to previously mentioned nonlinear terms, and the true behavior at large distances [e.g., along the soft direction, for $x_2 \gg x^*$, with x^* given by Eq. (2.12)] is controlled by a non-Gaussian fixed point. Behavior of this kind has been studied both in pure[5] and in random systems[24] with soft directions and a similar study is appropriate for the present problem. Here we shall omit technical details and present results relevant for understanding the orientational order behavior. The most interesting prediction of this section is that the decay of orientational correlations is given by an algebraic law [Eq. (2.38), below]. Also, we give a somewhat different interpretation of the competition between disorder and the spatial anisotropy discussed in Sec. II B.

So, the problem at hand is the full nonlinear isotropic SW theory (for the moment we set the anisotropy zero) Eqs. (2.5a) and (2.5b) [replacement (2.5b) is assumed, see Secs. II B and II D]. For $t = g = 0$ there are no fluctuations and the behavior is determined simply by minimizing (2.5a)—there are two phases (a) orientationally ordered phase for $r > 0$; $\nabla S = \mathbf{q}_0$, with $\mathbf{q}_0$ belonging to the ring (2.3), $|\mathbf{q}_0| = r^{1/2}$; (b) orientationally disordered one for $r < 0$; $\nabla S = \mathbf{0}$. For $r = r_c = 0$, a Lifshitz point with *two* soft directions for SW fluctuations occurs.[25] For $d < 4.5$, the orientationally ordered phase is in *presence* of RF, the domain of attraction of the previously mentioned non-Gaussian fixed point, which is well controlled for small $\epsilon = 4.5 - d$. We call it μ_1^* [see Fig. 5(a) which exhibits schematic RG flow and ground state phase diagram]. One can pass from this phase to the orientationally disordered one by crossing the Lifshitz boundary being the attraction domain of another fixed point exhibited in Fig. 5(a)—we call it μ_2^*. This Lifshitz-type transition occurs, in the presence of fluctuations, at positive values of r, i.e., $r_c > 0$, instead of being zero as in the mean field theory (by the way, note that Fig. 5 exhibits interesting region, with $r > 0$, favoring within *mean-field* theory, some orientational order). For $d > 5$, the mean-field theory of the transition is qualitatively correct, e.g., $[\langle \nabla S \rangle] \propto (r - r_c)^B, B = \frac{1}{2}$. For $d < 5$, one obtains from a simple RG calculation[26] $B = \frac{1}{2} - 5(5-d)/18 + O((5-d)^2)$ and for the single relevant eigenvalue at μ_2^*

$$y(\mu_2^*) = 2 - (8/9)(5-d) + O((5-d)^2) , \tag{2.28}$$

a quantity which will be of some interest later. Correlations in the orientationally disordered phase (i.e., for $r < r_c$) are easy to calculate at *large* distances, simply by setting $r < 0$ in (2.5a) (since, for all $r < r_c$ the long distance behavior is unique) and omitting nonlinear gradient terms, which are, in fact, irrelevant [so for *all* $r < r_c$ the

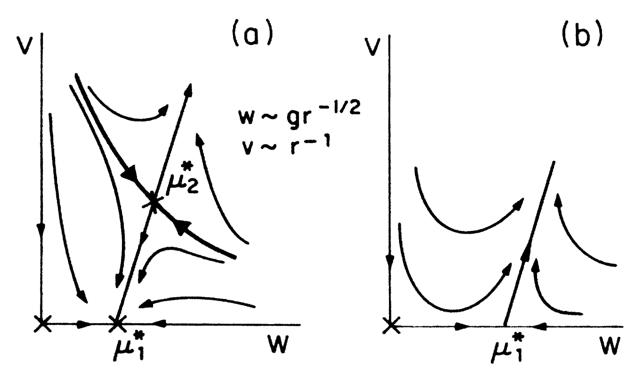

FIG. 5. Schematic RG flow and ground state phase diagrams. The parameters exhibited in the figure are $w \propto g r^{-1/2}$ and $v \propto r^{-1}$. (a) $4.5 > d > d^*$; the orientationally ordered state is the domain attraction of the fixed point μ_1^*. (b) $d < d^*$ the system is orientationally disordered and RG flow is away from the fixed point μ_1^*.

model (2.5a) has, at large scales, the same asymptotic behavior as the RF *XY*-model spin-wave theory]. In this way one obtains, for example:

$$[\langle \nabla S(\mathbf{x})\cdot\nabla S(\mathbf{0})\rangle]\propto 1/|\mathbf{x}|^{d-2}\ , \qquad (2.29)$$

for $r<r_c$, i.e., in the orientationally disordered phase. On the other side, in the orientationally ordered phase we have, for weak RF:

$$[\langle \nabla S(\mathbf{x})\cdot\nabla S(\mathbf{0})\rangle]\cong q_0^2 \qquad (2.30)$$

for large $\mathbf{x}$, at least for $d\lesssim 4.5$. In this phase scaling of SW correlations is anisotropic, e.g., within the harmonic approximation the SW propagator has a single soft direction, see (2.9). However, as alluded before, gradient nonlinearities modify the large distance behavior for $\epsilon=4.5-d>0$, so that at small momenta SW correlations are given by (2.8), with G_0 being replaced by G_r, such that

$$\begin{aligned}&G_r^{-1}(q_1=0,q_2,\mathbf{q}_\perp=0)\propto q_2^{\beta},\quad q_2<<(x^*)^{-1}\ ,\\&G_r^{-1}(q_1,q_2=0,\mathbf{q}_\perp=0)\propto q_1^{\alpha},\quad q_1<<(x^*)^{-2}/q_0\ , \qquad (2.31)\\&G_r^{-1}(q_1=0,q_2=0,\mathbf{q}_\perp)\propto |\mathbf{q}_\perp|^2,\quad \text{any } \mathbf{q}_\perp\ ,\end{aligned}$$

with $\mathbf{q}_\perp=(q_3,q_4,\ldots,q_d)$ and

$$\alpha=2+\epsilon/3+O(\epsilon^2),\ \beta=4-4\epsilon/3+O(\epsilon^2)\ , \qquad (2.32)$$

as can be obtained from calculations carried around μ_1^*. These calculations, leading to (2.32), are similar to those of Feigel'man and Ioffe[24] for a random system with soft directions. The exponents α and β are related by a scaling relation

$$5=\beta/\alpha+(6-d)\beta/2\ , \qquad (2.33)$$

being a consequence of the orientational symmetry of the theory (2.5a) and (2.5b). For $d<4.5$ the fixed point μ_1^* is stable—one of the irrelevant eigenvalues is

$$y(\mu_1^*)=-2(\beta/\alpha-1)=-[2-2\epsilon+O(\epsilon^2)]\ , \qquad (2.34a)$$

$$y(\mu_1^*)=-2[d-3.5+O(\epsilon^2)]\ , \qquad (2.34b)$$

[it corresponds to irrelevant parameter v, see Fig. 5(a)].

We are now in a position to discuss in more detail the instability of the orientational order which occurs, within harmonic SW theory, for $d\le d^*$, $d^*=3.5$. The relations (2.28) and (2.34b) were obtained respectively by expansions in (5-d) and (4.5-d). Of course, it is extremely risky to extrapolate them to, say $d=3.5$. However, their tentative extrapolation is rather suggestive. So, from (2.34a) and (2.34b) we see that nonlinearities tend to render the SW spectrum more isotropic, so that at some $d=d^*$ ($d^*\approx 3.5$)$y(\mu_1^*)$ goes to zero and α becomes equal to β. If so, we obtain by (2.33), $\alpha=\beta=8/(6-d^*)$ to be satisfied for $d=d^*$. With this value of $\alpha=\beta$, the average $[\langle(\partial\varphi/\partial x_2)^2\rangle]$, evaluated with the renormalized propagator, outlined in (2.31), diverges logarithmically with the size of the system, indicating that the situation with $\alpha=\beta$ occurs at the same dimension d^* as the onset of the orientational order instability. Note that the situation with $\alpha=\beta$ is reminiscent of a Lifshitz point behavior with two soft directions. The single relevant eigenvalue $y(\mu_2^*)$ at the Lifshitz point μ_2^* is given by (2.28). From this figure it seems that, at $d\approx 3$,$y(\mu_2^*)$ goes to zero, indicating that μ_1^* and μ_2^* presumably exchange their stabilities at some dimension. If so, this dimension should be identified with previously introduced d^*, in which $y(\mu_1^*)$ goes to zero. The fixed points μ_1^* and μ_2^* could exchange their stabilities via merging or, in an exotic manner, via a fixed line joining μ_1^* and μ_2^* at $d=d^*$ [as suggested by (2.16)]. Below d^* we expect the situation in Fig. 5(b), an unstable fixed point μ_1^*, and the ground state being orientationally disordered, i.e., with $[\langle\nabla S\rangle]=0$. Of course, the precise determination of d^* is difficult due to standard uncertainties of an ϵ expansion. However, it seems clear on physical grounds that fluctuations causing divergent behavior of the perturbation theory, controlled by the ϵ expansion, can only increase d^*; i.e., the harmonic SW estimate $d^*=3.5$ should be a lower bound. So, presumably, $d^*\ge 3.5$. Other arguments suggest that $d^*\le 4$. For example, to lowest order in perturbation theory, the critical value r_c of r at the Lifshitz transition is given by

$$r_c\propto g\int d^dq(q_1^2+q_2^2)[G_{\rm LP}(q)]^2\ , \qquad (2.35)$$

with $G_{\rm LP}(q)$ being the harmonic Lifshitz point propagator, given roughly by

$$G_{\rm LP}^{-1}(q)\propto(q_1^2+q_2^2)^2+\sum_{k=3}^{d}qk^2\ . \qquad (2.36)$$

Then, by a simple power counting one can see that r_c diverges for $d\le 4$, indicating that an infinite value of r_c, i.e., of the bare wave vector q_0, is necessary to produce long range orientational order. So, for any finite q_0 the system is in an orientationally disordered phase [Fig. 5(b)]. This may suggest that $d^*=4$. This value of d^* is probably also inaccurate, since as mentioned before, the perturbation theory at the Lifshitz transition is divergent below $d=5$. However, as explained before, a value of d^* bigger than 3.5 can be expected. Finally, it is rather clear that at d^* the quantity $[\langle(S(x))^2\rangle]$ should already be divergent in both phases in Fig. 5(a)—so the conditions $d^*\le 4.5$ and $d^*\le 4$ should be satisfied. Thus, an estimate of the form

$$3.5\le d^*\le 4 \qquad (2.37)$$

is plausible.

What is the nature of orientational correlations below d^*—harmonic SW estimate (2.17) suggests an exponential decay. For $d>d^*$ the two phases are present in the system, Fig. 5(a): one with long range orientational order, Eq. (2.30), and the other with algebraic decay at the orientational correlations, Eq. (2.29). For $d<d^*$, the entire phase diagram [Fig. 5(b)], is occupied by the orientationally disordered phase—orientational correlations in this phase are, at large distances, still given by Eq. (2.29) (we remind that [Eq. (2.29)] may be obtained from the theory (2.5a) and (2.5b) by setting $r<0$). So, at large enough distances, orientational correlations of the form

$$K(\mathbf{x})=[\langle\mathbf{N}(\mathbf{x})\mathbf{N}(\mathbf{0})\rangle]\propto 1/|\mathbf{x}|^{d-2}\ , \qquad (2.38)$$

say, for $\mathbf{x}$ in $x_1 - x_2$ plane, should replace the harmonic result (2.17). Crossover from (2.17) to (2.38) presumably occurs at $|\mathbf{x}| \approx \xi_{\rm or}$. Note that at $|\mathbf{x}| = \xi_{\rm or}$, $K(x)$ in Eq. (2.17) is still of order unity. So for $|\mathbf{x}| > \xi_{\rm or}$, the equation

$$K(\mathbf{x}) \approx 1/|\mathbf{x}/\xi_{\rm or}|^{d-2} \quad (2.39)$$

should hold, while for $|\mathbf{x}| < \xi_{\rm or}$, Eq. (2.17) should be correct. So, for $|\mathbf{x}| > \xi_{\rm or}$, a system with positive bare value of $r = q_0^2 > 0$ behaves as a system with $r < 0$, Eq. (2.39), which does not favor any long range orientational order at all (nevertheless, short-range orientational order exists inside of domains of size $\xi_{\rm or}$). Random fields produce such prominent renormalization of r at distances larger than $\xi_{\rm or}$. If a spatial anisotropy of the form (2.20) is applied to the isotropic system, the orientational order would not be restored if the characteristic anisotropy length q_A^{-1} (defined in Sec. II B as the length scale where the anisotropy starts to influence spin wave correlations) is larger than $\xi_{\rm or}$—this was asserted in Sec. II B in framework of heuristic arguments. How does this statement cope with the picture proposed in this section? If $q_A^{-1} \gg \xi_{\rm or}$ then, on scales where the anisotropy becomes important, orientational correlations are of the form (2.39) and the renormalized value of r is already negative—so, no long range orientational order can be restored by such an anisotropy. On the other hand, if $q_A^{-1} \ll \xi_{\rm or}$ the anisotropy starts to dominate on scales on which no significant orientational disorder had been produced by RF. Such an anisotropy stabilizes the orientational order. So, once again, we arrive at the condition (2.27)

$$q_A^{-1}/\xi_{\rm or} \approx 1 ,$$

indicating the borderline between the two phases.

D. Discussion

Two important steps involved in deriving the results of Secs. II B and II C may be troublesome. They are (1) neglect of topological excitations (dislocation loops in 3D and their analogs in other dimensions), and (2) in the treatment of the SW theory we employed the replacement (2.5b)—this is valid to all orders of zero temperature perturbation theory.[16] It has already been pointed out that the first step is presumably incorrect, as explained in more detail in Sec. III and the Appendix. The second step also may be troublesome, since a formal perturbation theory almost certainly breaks down at lengthscales larger than the translational correlation length. This has been argued for other RF spin systems with continuous symmetries, e.g., for RF XY spin wave model.[14] However, qualitative arguments in favor of the breakdown of the perturbative SW theory could be the same for the SW theory of our problem: at large lengthscales multiple extrema (metastable states and local maxima) of the SW action are, almost certainly, improperly taken into account by the perturbation theory.[14,15] So, in principle, the physics of nonlinear SW theory (2.5a) [without the replacement (2.5b)] may be different at large scales from that predicted in Secs. II B and II C.

An effort to go beyond the perturbative approach was done by Villain and Fernandez, for RF XY-model spin-wave theory, which is similar to Eq. (2.6), however, with all directions being hard. An infinite susceptibility phase was predicted by them to occur in the ground state of the SW model for $4 > d > 2$. The perturbative treatment of the same SW problem gives a finite susceptibility ground state.[16,27] Aharony and Pytte[28] considered RF continuous spin systems by a scaling analysis, which, *a priori*, does not exclude possible presence of topological defects neglected by a SW approach. Their results are in favor of a finite susceptibility ground state. Moreover, this susceptibility scales with spin-spin coherence length in the same way as it does in the result obtained by the perturbative SW approach.[16,27] Also, the experimental data on RF and random anisotropy systems with continuous symmetries are frequently interpreted to indicate the presence of a finite susceptibility phase at low temperatures.[29] This may suggest that perturbative theory is at least qualitatively correct. However, this is, we believe, not necessarily so, since a finite density of unbound topological defects, say dislocation loops in 3D RF XY model, might be responsible for the finite susceptibility ground state. With[16,27] or without[14] the use of the perturbative theory, the SW approach, excluding the possible presence of topological defects, is almost certainly a poor description of RF problems of the kind considered here. In Sec. III and the Appendix we shall argue that inclusion of topological defects may greatly increase the disordering effect of random fields, e.g., by shifting the lower critical dimension of the orientational order from the SW result 3.5 to 4. Thus, presumably, perturbative harmonic SW theory of Secs. II B and II C significantly underestimates the actual disorder induced by RF.

III. TOPOLOGICAL DEFECTS

Thus far we discussed the physics in the framework of SW theory, i.e., in the absence of topological defects. As discussed in Sec. II, they may be relevant for a complete understanding of the problem. Dislocation-mediated melting theories of *pure* systems described at low temperatures by a SW theory with a single soft direction, are constructed in $d = 2$,[30] and $d = 3$.[6] The most striking effect of thermally activated dislocations on systems favoring an inhomogeneous order on a ring in q space is the possible appearance of a nematiclike phase characterized by short range translational and long-range orientational order—such a phase, which intervenes between a low temperature both translationally and orientationally ordered (i.e., smecticlike) and a high temperature fully disordered phase is quite unexpected in framework of mean-field theories of the majority of systems with a single soft direction, e.g., of magnetic or hydrodynamic systems.[6,30] This nematiclike phase is generally characterized by a finite density of unbound dislocations (dislocation loops) in 2D (3D), which destroy translational order above some temperature T_N, less than the transition temperature to the fully disordered phase. In $d = 2$, T_N is zero, i.e., the fully ordered, smecticlike, phase exists only at zero temperature.[30] It turns out that $d = 2$ is the lower critical dimension of the orientational order of the nematiclike phase—at $d=2$ orientational correlations exhibit nonuniversal algebraic decay.[30] The existence of a finite density of unbound dislocations is crucial for the instability of the orientational order, since spin waves alone are insufficient to produce that instability in $d=2$.[30] So $d=2$ plays a special role for the pure systems favoring a modulated state on a ring in q space. In the following we shall argue that $d=4$ may play a similar role for these systems in the presence of RF. In fact, we shall construct some nonrigorous arguments for RF systems with a single soft direction, which suggest that for $d<4$, dislocations are unbound, presumably even in the ground state (see the Appendix). We suspect that the orientational order of such a phase is unstable, also for $d<4$.

For a system having a spin wave theory with no soft directions, i.e., for the RF XY model, Villain and Fernandez[14] have constructed a simple argument showing that dislocations of arbitrarily large size are favorable in the ground state, for $d<4$ (see Appendix). The argument presumably indicates that a finite density of free dislocations is created in the ground state in addition to dislocation free Imry-Ma domains (which are promoted by spin waves). The length scale on which dislocations become important scales with the strength of RF in the same way [up to a logarithmic factor, see Eq. (A1)] as the size of the SW promoted Imry-Ma domains, indicating that the SW description, in the absence of topological defects, is rather incomplete. Similar arguments can be constructed for systems with a single soft direction, to argue that dislocation loops are unbound, in particular, in $d=3$, and, more generally, for $d<4$ (not 4.5, as one might expect, so that dislocation free phase, considered in Sec. II, may exist for $4 \leq d \leq 4.5$), see the Appendix.

So, both for XY and single soft direction systems, a finite density of free dislocation loops can be expected in the presence of RF, even at zero temperature for $d<4$. For the nonrandom XY model, a finite density of *thermally* excited dislocations creates the ordinary paramagnetic state in any d; systematic Debye-Hückel theory[31] can be used to verify this assertion. So, free dislocations produce a state with paramagnetic correlations, at least when dislocations are thermally unbound. It is difficult to construct a similar systematic theory for the XY model at zero temperature but nonzero RF. Nevertheless, the simplest and the most probable possibility for $d<4$ is that the system is a disordered paramagnet even in the ground state.[32] Similarly, ground state of RF systems with a single soft direction behaves, presumably, as a nematiclike phase for $d<4$, because of a finite density of unbound dislocations present even in a weak random field. Both dislocations and spin waves are active in the destruction of the translational order of this nematiclike phase. Is the orientational order of such a phase stable against random fields? For pure systems with a single soft direction, orientational correlations can be worked out easily in the phase with unbound dislocations, by means of a Debye-Hückel approximation applied to the dislocation mediated melting theory.[6,30] As noted above, it is difficult to implement similar calculations in the presence of RF at zero temperature. However, as explained in Sec. II A, planar-nematic liquid crystal systems,[18] Eq. (2.7), have a SW theory with a single soft direction, even in presence of RF. The form of dislocation-mediated melting theory depends only on the form of the corresponding spin wave theory and this fact should be maintained for weak RF.[33] This suggests that, even without studying dislocation effects in detail, one may draw qualitative conclusions about the nematiclike phase in, say, magnetic systems by considering the nematic phase of the planar nematic liquid crystal system[18] in the presence of RF coupled to the smectic density wave, Eq. (2.7). Assuming zero translational order, $[\langle\Psi\rangle]=0$, as appropriate for a nematic phase, it is straightforward to generalize the calculation of orientational correlations of Halsey and Nelson,[18,34] to the case when RF coupled to Ψ is present, as in Eq. (2.7). The advantage of doing calculations with (2.7), with respect to GLW model (2.1), is that the orientational order parameter $\mathbf{N}(x)=[\cos\theta(x), \sin\theta(x), 0, \ldots, 0_d]$ is explicitly present in (2.7). To see qualitatively how the randomness influences the orientational order parameter in the nematic phase, it is sufficient to calculate $[\langle(\theta(x))^2\rangle]$ in a manner of Halsey and Nelson,[34] to a low (one loop) order of the perturbation theory around the state with $[\langle\Psi(x)\rangle]=0$. The result is simple:

$$[\langle(\theta(x))^2\rangle] \propto \int d^dq (G(q))^2 \; . \tag{3.1}$$

The propagator $G(q)$ is anisotropic—however, in all directions it behaves as q^{-2} for small q. So, the feedback of RF coupled to $\Psi(x)$ is another effective RF or random anisotropy type interaction coupled to the orientational order parameter $\mathbf{N}(x)$. Another way to describe this is to state that after integrating out (or minimizing, at zero temperature) translational degrees of freedom Ψ from (2.7) (this elimination is a priori unambiguous in the nematic phase) a RF-like interaction coupled to the orientational degrees of freedom is generated.

The most striking effect of this random interaction is, as one can see from (3.1), to destroy the orientational order of a nematic phase for $d<4$. If properties of nematic like phases are common to all systems with a single soft direction, as we suggested before, then this instability also occurs in magnetic systems with a continuous set of energy minima (2.3). Moreover, as suggested in the Appendix, even the ground state of the system may be nematiclike for $d<4$ due to unbound dislocations being present even for arbitrarily weak RF. So, the orientational order is unstable for $d<4$ probably even in the ground state. Thus, the lower critical dimension of this order is $d^*=4$, to be compared, e.g., with the SW estimate $d^*=3.5$, Sec. II B. Long range order of the continuous orientational order parameter $\mathbf{N}(x)$ is unstable for $d<4$ because there is an effective random anisotropy coupled to $\mathbf{N}(x)$. For realistic magnetic systems favoring an inhomogeneous order, continuous orientational symmetry is broken by the spatial anisotropy due to the crystal lattice, Sec. II A. In the context of the present discussion, these lattice effects act as a *nonrandom* anisotropy coupled to spin-like field $\mathbf{N}(x)$. The effects of nonrandom anisotropy in RF and random anisotropy continuous spin systems were recently considered by Goldschmidt and Aharony.[35] They show that, for $d<4$ and for a given quenched disorder, sufficiently weak nonrandom anisotropy is insufficient to restore long range order of spins. A critical value of nonrandom anisotropy is necessary to produce, via a phase transition, an ordered state (this critical value diverges at $d=2$, i.e., at lower critical dimension of RF systems with discrete symmetries). Similar physics should occur in the present problem—for a given disorder sufficiently weak spatial anisotropy would not restore the orientational order—to do this, a critical strength (diverging for $d \to 2$) of the spatial anisotropy is necessary.

Finally, let us summarize the main points of this section: (a) Dislocations are favorable in the ground state in RF systems with a single soft direction for $d<4$, as in the RF XY model.[14] In the XY case this presumably indicates the onset of a ground state with paramagnetic correlations,[15] while for the system with a soft direction, unbound dislocations probably produce a nematiclike ground state. (b) The orientational order of such a nematiclike state is unstable for $d<4$, i.e., $d^*=4$. So, as expected, topological defects increase the lower critical dimension of the orientational order with respect to the SW result. (c) Sufficiently weak spatial anisotropy applied to a magnetic system with a continuous set of energy minima does not restore the orientational order below $d^*=4$.

IV. SUMMARY AND DISCUSSION

We presented a detailed study of random field effects in models with a continuous set of energy minima in q-space, having the form of a ring [Eq. (2.3)]. The line of arguments given in Secs. II and III, leads to the conclusion that, in presence of RF, the long range orientational order is unstable when the spatial dimensionality d is below some critical dimensionality d^*. Thus, in these RF models, two lower critical dimensionalities exist: one d_c associated with instability of long-range translational order, $d_c=4.5$, and another one, being the lower critical dimensionality of the orientational order d^*. We argue that $d^*>3$. Harmonic SW results (Sec. II B) indicate that $d^*=3.5$, while the analysis of anharmonic SW effects (Sec. II C) indicates that $4 \geq d^* \geq 3.5$. Most likely, $d^*=4$, as argued in Sec. III, in which the interplay between random fields and topological defects (being neglected in framework of SW theory) has been considered.

Models with a continuous set of energy minima can approximately describe physics in some magnetic systems favoring, at low temperatures, an incommensurate modulated order.[7] However, the presence of underlying crystal lattice produces spatial anisotropy breaking the energy degeneracy of the continuous energy minimum, so that condensation is favored to a discrete set of points in q space (see Sec. II A and Fig. 1). Spatial anisotropy tends to stabilize the orientational order. However, in presence of RF's and for $d<d^*$, for any given strength of RF's g, there exists critical strength of the spatial anisotropy $A_c(g)$, such that a system characterized by $A<A_c(g)$ is orientationally disordered. In framework of harmonic SW theory, Sec. II B, $A_c(g) \propto g^{1/(3.5-d)}$, Eq. (2.26). In a scattering experiment, a system having $A \ll A_c$, should be characterized by patterns almost like Fig. 3(b). For $A=A_c$ a phase transition occurs, and for $A \approx A_c$ patterns like those in Fig. 4 should be observed—they indicate either relatively large orientational disorder [for $A \gtrsim A_c$, Fig. 4(a)], or absence of orientational order [for $A \lesssim A$, Fig. 4(b)]. So, an estimate of the ratio A/A_c is important in applying ideas of this paper to realistic systems: the physics of systems having $A \ll A_c$ or even $A \sim A_c$ is dominated by random field effects described here. A ratio equivalent to A/A_c was introduced in Sec. II B:

$$R = q_A^{-1}/\xi_{\rm or} \,, \tag{4.1}$$

[see Eq. (2.27)]. $R \gg 1$ ($R \ll 1$) corresponds to $A \ll A_c$ ($A \gg A_c$). $\xi_{\rm or}$ is the orientational correlation length, introduced in Sec. II B. This length is a quantitative measure of the strength of RF [see Eq. (2.18a)]. For a realistic system, this length can be estimated if one has estimates of translational correlation lengths [see Eqs. (2.18b) and (2.18c)]. On the other side, the anisotropy length q_A^{-1} in Eq. (4.1) is a quantitative measure of the strength of the spatial anisotropy [see Eq. (2.22)]. For a realistic system, q_A^{-1} can be estimated by knowing the details of the inverse susceptibility of the Ginzburg-Landau-Wilson Hamiltonian (see Sec. II B).

Finally, let us give some comments on applicability of these ideas to magnetic superconductors (MS).[9–12] Quenched disorder seems to be responsible for some of the phenomenological features which are common to all of these materials.[8] For example, no higher order harmonics of the modulated structure have been found in neutron scattering experiments[9–12] in all presently known MS—this may signal the absence of true magnetic order, likely due to a quenched disorder.[8] A modulated structure exhibiting a large but finite translational coherence length, due to imperfections of the system, can be appropriately interpreted by a random field model.[1] This motivated study of RF effects in the model for magnetic superconductors Eqs. (2.1), (2.2), and (2.2′) (see also Refs. 7 and 36). We demonstrated that, in limit of weak spatial anisotropy (2.2′), this model exhibits somewhat unusual behavior due to RF—orientational order is unstable and the ground state is expected to consist of domains having different orientations of their local wave vectors, Sec. II B. Experimental data[9] on $ErRh_4B_4$, are interpreted to indicate that the ordered magnetic state, in the coexistence phase of this material, is composed of domains having different orientations of their local wave vectors. This orientationally disordered structure can be produced by nonequilibrium effects, or by inhomogeneity of the sample. This paper suggests another explanation for this orientational disorder—it could be a random field effect (see also Ref. 36).

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APPENDIX

As mentioned in Sec. III, dislocations are unbound both in the XY model and in models with a single soft direction for $d<4$ and when a weak RF is present. Here we shall review the domain argument supporting this statement for RF XY model[14] and present a corresponding argument for models with soft directions.

When excited from the ferromagnetic state of the XY model, a Thouless-Kosterlitz dislocation pair in 2D [Fig. 6(a)], or dislocation loop in 3D [Fig. 6(b)], create domains which are dephased (i.e., with reversed spins) with respect to the rest of the system. Random fields may favor creation both of such domains belted by dislocations and of dislocation free domains (as in the original arguments of Imry and Ma[1]). When a dislocation of size L (creating a domain of volume L^d, see Fig. 6; L is, in 2D, the distance between opposite dislocations in a pair, or the length of a dislocation loop, in 3D, etc.) is excited, then the energy of order:

$$E(L)\approx L^{d-2}\ln L-g^{1/2}L^{d/2},\quad d>2\ , \tag{A1}$$

is spent.[14] The first term in (A1) is the energy necessary to create the dislocation in a pure XY system—this is a well known logarithm multiplied by the "length" of the topological defect [being a $(d-2)$ dimensional object in a d-dimensional system]. The second term in (A1) is the standard energy decrease due to RF (RF is supposed to be uncorrelated, with mean square deviation g).[1] As usual, this term is proportional to the square root of the domain volume.[1] In (A1) and hereafter we suppress all proportionality factors, except g. For $d<4$ and L large enough, $E(L)$ in Eq. (A1), is negative even for arbitrarily weak RF. So, dislocations of arbitrarily large size are expected to be present in the true ground state of the system. That state is most likely a paramagnetic (spin glass) state[15]—a conclusion drawn by analogy to the paramagnetic state of the pure XY system, which is also characterized by the presence of arbitrarily large dislocations. Function $E(L)$, Eq. (A1), has, for $d<4$, a single maximum at $L=L_0$ given by:

$$L_0\approx g^{-1/(4-d)}(\ln g^{-1})^{2/(4-d)}\ , \tag{A2}$$

for small g. A loop *smaller* than L_0 would be infavorable

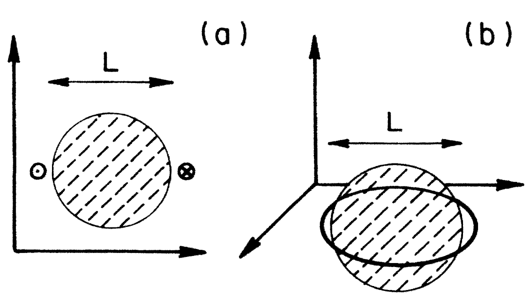

FIG. 6. Topological defects in XY model (a system with no soft directions). Pair of opposite dislocations, in 2D, (a), or a dislocation loop, in 3D, (b), create domains (dashed) which are dephased (i.e., with reversed spins) with respect to the rest of the systems.

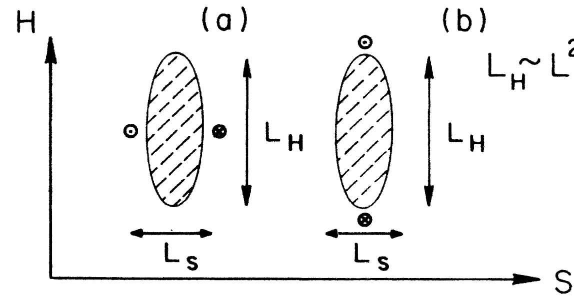

FIG. 7. Domains (dashed) created by dislocation pairs in a 2D system with a soft (S) and a hard (H) direction.

in the ground state—such a loop would shrink to a point in order to decrease the energy. However, at distances larger than L_0 the system is disordered both by spin waves and by unbound topological defects (note that L_0 is, up to the logarithmic factor, the same as the size of dislocation free Imry-Ma domains, being of the order $g^{-1/(4-d)}$).

Now we are going to present similar arguments for systems with soft directions. For example, in a 2D system with a soft and a hard direction,[30] a pair of opposite dislocations, separated by the distance L_S along the soft direction [see Fig. 7(a)], creates a domain of area L_SL_H, with $L_H\approx L_S^2$.[30] So the energy gain due to RF is $-(gL_SL_H)^{1/2}\approx -g^{1/2}L_S^{3/2}$. Nonrandom part of the dislocation pair energy [corresponding to the first term of (A1)] is a constant independent of L_S (for large L_S; see Ref. 30). So

$$E(L_S)\approx \text{const}-g^{1/2}L_S^{3/2}\ ; \tag{A3}$$

obviously, RF favors unbound dislocations [similar argument applies for dislocations in Fig. 7(b)].

The situation is more subtle for 3D systems with a single soft direction—but the conclusion will be the same. To see this, let us consider a dislocation loop lying in a plane perpendicular to a hard direction (Fig. 8). For simplicity, we take the loop to have the form of a rectangle

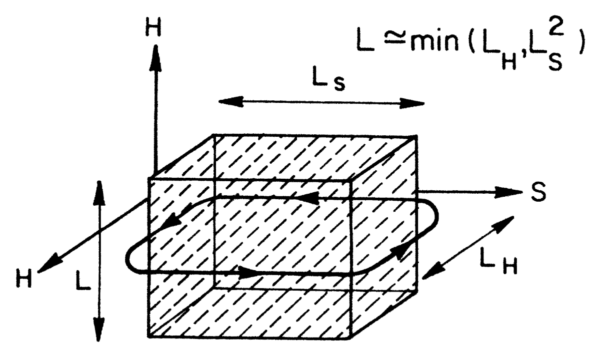

FIG. 8. Dislocation loop (thick line with arrows) in a 3D system with a single soft direction (S) and two hard directions (H). The loop produces a domain (dashed) of height L.

with one pair of its edges (of length L_H) parallel to the remaining hard direction and the other pair (of length L_S) being parallel to the soft direction (Fig. 8). The loop produces a domain of height L (Fig. 8). It is easy to estimate the dependence of L on L_S and L_H. When L_H is infinite (or very large) for fixed L_S, then $L \approx L_S^2$, as for the pair of point dislocations in 2D [Fig. 7(a)]. When L_S is infinite (or large enough) for fixed L_H, then $L \approx L_H$, as in a 2D system with no soft directions [Fig. 6(a)]. Crossover between the two regimes occurs obviously when $L_H \approx L_S^2$. So, it follows:

$$L = L(L_H, L_S) \approx \min\{L_H, L_S^2\} \; . \qquad \text{(A4)}$$

Then the energy gain due to RF is given by

$$E_2 \approx -(g L_S L_H L)^{1/2} \; . \qquad \text{(A5)}$$

By a similar reasoning it is easy to estimate the energy necessary to create the loop in the absence of RF. It is of the form

$$E_1 \approx L_H + L_S \ln L(L_H, L_S) \; . \qquad \text{(A6)}$$

The first term in (A6) is the contribution from the edges of the loop being parallel to the hard direction—there is no Kosterlitz-Thouless logarithmic factor multiplying this term—since there is only one hard direction being perpendicular to these edges; see Fig. 8.[30] However, such a factor is present in the second term of (A6), which gives the contribution to the energy from edges which are parallel to the soft direction (so that there are two hard directions perpendicular to these edges). Finally, from (A5) and (A6) we have total energy gain:

$$\begin{aligned} E(L_H, L_S) &= E_1 + E_2 \\ &\approx L_H + L_S \ln L(L_H, L_S) \\ &\quad - [g L_H L_S L(L_H, L_S)]^{1/2} \; , \end{aligned} \qquad \text{(A7)}$$

with $L(L_H, L_S)$ given by (A4).

Now, it is easy to see that dislocations of arbitrarily large size are favorable: e.g., take $L_S \approx L_H \to \infty$ [then $L(L_H, L_S) = L_H$], or $L_H \approx k L_S^2 \to \infty$, $k > 1$ [then $L(L_H, L_S) = L_S^2$]—in both cases $E(L_S, L_H)$ tends to $-\infty$. So, the ground state of the system should be populated by loops of arbitrarily large size. Most likely, this state is nematiclike by analogy to the nematic state of the pure system with a single soft direction, which is also characterized by the presence of arbitrarily large loops, see Refs. 6, 22, and 30. If so, one of the consequences is the instability of the orientational order, as explained in Sec. III.

In the case of RF XY model a single extremum of (A1) occurs at $L = L_0$, Eq. (A2): L_0 serves as the estimate of the minimal size of dislocation loops, as explained before. Function $E(L_H, L_S)$, Eq. (A7), also has a single extremum S at (L_{H0}, L_{S0}) being a saddle point (Fig. 9), with:

$$L_{S0} \approx g^{-1}, L_{H0} \approx g^{-1} \ln g^{-1} \; , \qquad \text{(A8)}$$

for small g. In Fig. 9 we indicated the lines of the flow

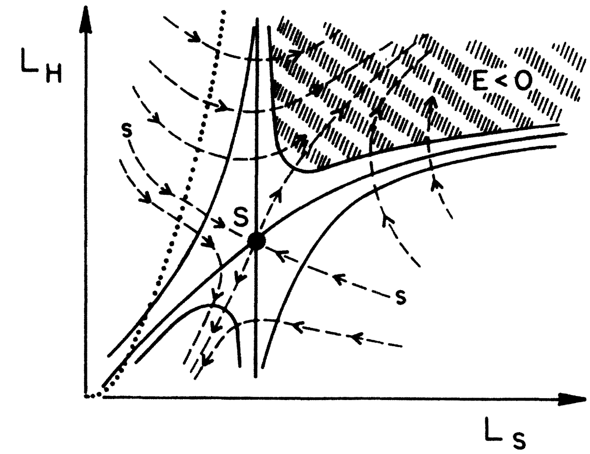

FIG. 9. Schematic topography of the function $E(L_S, L_H)$. Solid lines are levels of constant E. The dashed arrowed lines are the flow, Eq. (A9); E decreases along the flow. The region with $E < 0$ is dashed. The line $L_H = L_S^2$ is dotted.

$$\left[-\frac{\partial E}{\partial L_S}, -\frac{\partial E}{\partial L_H} \right] , \qquad \text{(A9)}$$

so that the energy decreases along the flow. If starting in the region below the separatrix s-s, flow lines terminate at $L_S = L_H = 0$. This corresponds to dislocations for which it is favorable to shrink in order to decrease the energy. Flow lines, starting above s-s, terminate in the region of negative energy and arbitrarily large dislocation loop size. The flow starting at the separatrix s-s terminates at (L_{S0}, L_{H0})—that point determines in framework of a domain argument, the smallest size of dislocation loops present in the system. At distances shorter than L_{S0}, along the soft direction, and L_{H0}, along hard directions, the system is disordered only by spin waves. Note that, up to a logarithmic correction in (A8), $L_{H0} \approx L_{S0} \approx g^{-1}$, while the corresponding SW lengths are given by Eqs. (2.10)–(2.12): $\xi_H \approx \xi_S^2 \approx g^{-2/3}$—so, the disordering effect of spin waves on translational correlations should dominate for weak RF.

The most interesting aspect of these considerations is the isotropic scaling (up to the logarithmic correction) of the dislocation coherence lengths L_{S0} and L_{H0}, see Eq. (A8). One might expect that, say $L_{H0} \approx L_{S0}^2$, rather than $L_{H0} \approx L_{S0}$. However, the present domain argument is entirely inconsistent with such anisotropic scaling. In particular, all flow trajectories leading to the region with $E < 0$ and starting in the region with $L_H > L_S^2$ eventually terminate in region with $L_H \approx L_S \ll L_S^2$, see Fig. 9. So the state with the isotropic scaling is energetically favored. In this respect, the situation in 3D systems with a single soft direction is similar to 3D RF XY model, having all directions hard. Moreover, straightforward extension of these considerations to $d > 3$ shows that this isotropic scaling persists below $d = 4$, which turns out to be the marginal dimension for dislocation unbinding for models with a single soft direction. Thus the dislocation free disordered phase, considered in Sec. II, may exist for $4 \leq d \leq 4.5$. Similar domain arguments applied to RF d-dimensional systems with m soft directions, show that if $d \geq m+2$, then dislocations are unbound below $d=4$ and the scaling of dislocation coherence lengths is isotropic (up to logarithmic corrections). For systems with $d=m+1$ (i.e., with a single *hard* direction) scaling is anisotropic, with $L_H \approx L_S^2$ and dislocations are unbound below $d=5$ due to RF (the simplest of these systems is $m+1=d=3$; for this system it is easy to check that dislocation loops are favorable by considering a loop lying in the plane defined by the two soft directions). Unbound dislocations produce, presumably a nematiclike ground state in all of these systems and the orientational order should be unstable for $d<4$, for reasons similar to those presented in Sec. III.

Note that we have drawn conclusions on dislocations in a 3D system with a single soft direction by considering a dislocation loop perpendicular to a hard direction. Similar considerations show that loops perpendicular to the soft direction are unbound in 3D. However, their minimal size is, for weak RF, much larger than that given by (A8)—it scales as g^{-2}. So, these dislocations are much less efficient in disordering the system. In reality, orientations of loops are not restricted as in the examples considered here, and, in principle, characteristic coherence length due to dislocations should be determined by loops which are the most strongly unbound, i.e., by (A8).

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[11]J. Rossat-Mignod *et al.*, J. Phys. (Paris) Lett. **46**, L373 (1985).

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[13]Spin waves in the context of modulated states, are spatial fluctuations of the phase of the modulated structure. The applicability of SW theory to the *one* component GLW models, favoring linearly polarized states, stems from the fact that continuous translational symmetry is expected to be broken at least at low temperatures. S_p in Eq. (2.5a) satisfies roughly $|\nabla S_p| \approx q_0$ as forced by the nonlinear term in (2.5a), i.e., at zero temperature and in absence of the disorder we have $[\langle \nabla S_p \rangle] = \mathbf{q}_0$ with $\mathbf{q}_0$ belonging to the ring. Spin wave approach typically neglects fast terms obtained by inserting the spin wave ansatz in the original GLW model: for example, in addition to the random term $\cos(S_p - S_{p'})$ exhibited in (2.5a), there is also a fast term of the form $\cos(S_p + S_{p'}) = \cos(2\mathbf{q}_0\mathbf{x} + \varphi_p + \varphi_{p'})$ which can be neglected in the weak disorder limit, when the coherence length is much larger than the wavelength of the structure. Note that the random term in Eq. (2.5a) is the same as in RF XY model, see e.g., Ref. 15. The two component, i.e., XY, GLW model with the continuous set of minima such as (2.3), favoring helical spiral order, has the same spin-wave description, (2.5a), as the one component model (2.1), favoring linearly polarized states (as appropriate for magnetic superconductors, Refs. 7 and 8). Both models should have the same physics at low temperatures and at long length scales for weak random field, as explained above. (Of course, this statement applies also to one and two component GLW models favoring condensate on a discrete set of points in q space.)

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[19]See, e.g., Ref. 30. The approximation in Eq. (2.14) is consistent with the use of the harmonic theory (2.8), which, strictly speaking, gives anisotropic correlations in x_1-x_2 plane even for $d<3.5$. So, within the harmonic theory, Eq. (2.17) holds for $\mathbf{x}$ along the soft direction 2. Inclusion of SW nonlinearities, Sec. II C, removes that artificial anisotropy.

[20]Operators bilinear in spin variables are necessary to define an orientational order parameter. So the determination of orientational correlations requires measurement of, at least, four spin correlation functions.

[21]One might hope to study the nature of that transition in $2+\epsilon$ dimensions, since for $d=2$ spin waves destroy orientational order for arbitrarily weak RF, even in presence of the spatial anisotropy [at least in the framework of perturbation theory, Eq. (2.5b)].

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[31]See, e.g., R. A. Pelcovits, J. Phys. A **14**, 1693 (1981).

[32]See, e.g., Ref. 15. However, it is difficult to construct some solid argument in favor of this assertion.

[33]Note that SW theory is a good common description for all systems with a single soft direction only for weak RF, see comments in Ref. 13.

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