Phase transition in positionally disordered Josephson-junction arrays in a transverse magnetic field

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The critical behavior of positionally disordered two-dimensional Josephson-junction arrays in a transverse magnetic field is studied by Monte Carlo simulations. We consider a model in which each superconducting element is randomly displaced from its ideal position on a square lattice by a small amount. The effective strength of the disorder is changed by varying the magnitude of the transverse magnetic field. We consider those values of the magnetic field for which the parameter $\langle f \rangle$ that measures the average flux through an elementary plaquette in units of the flux quantum is integral. For $\langle f \rangle = 1$ and $\langle f \rangle = 3$, we find Kosterlitz-Thouless-type phase transition from a normal to a superconducting phase. For $\langle f \rangle = 5$ and $\langle f \rangle = 7$, the results suggest a spin-glass-like freezing over the time scales of the simulations. We do not find any evidence for a low-temperature reentrant transition for any value of $\langle f \rangle$.

I. INTRODUCTION

Recent experiments on two-dimensional (2D) arrays of Josephson junctions[1] and proximity-coupled grains[2] in an external magnetic field have stimulated considerable interest. An ordered array of Josephson junctions in a transverse magnetic field is described by a uniformly frustrated 2D XY model. The Hamiltonian of the system is given by

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij}) , \tag{1.1}$$

where θ_i is the phase of the superconducting order parameter at the ith site of the array, $\langle ij \rangle$ represents distinct pairs of nearest-neighbor sites, J is the Josephson coupling for a single junction, and A_{ij} is given by

$$A_{ij} = (2\pi/\Phi_0) \int_i^j \mathbf{A} \cdot dl . \tag{1.2}$$

In Eq. (1.2), $\Phi_0 = hc/2e$ is an elementary flux quantum and $\mathbf{A}$ is the vector potential due to the presence of the transverse magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. The amount of uniform frustration, f, is then given by

$$\sum_p A_{ij} = 2\pi f , \tag{1.3}$$

where the sum is over the sides of an elementary plaquette. Consequently, f can be written in terms of the magnetic field B as

$$f = B A_p / \Phi_0 , \tag{1.4}$$

where A_p is the area of an elementary plaquette. The model described by the above equations is periodic in f with period 1 and has reflection symmetry about $f = \frac{1}{2}$ in the interval [0,1]. Hence, it is necessary to consider f only in the range $0 \le f \le \frac{1}{2}$. The case $f = 0$ corresponds to the ordinary XY model which has a Kosterlitz-Thouless[3] (KT) transition and the case $f = \frac{1}{2}$ corresponds to the fully frustrated model investigated by several authors.[4] For commensurate systems with $f = 1/n$, n being an integer, the transition temperature T_c is found[5] to vary with n, while for incommensurate systems (f is irrational) the situation is not clear. Several authors have suggested that there is no finite-temperature phase transition in this case,[6] whereas a recent Monte Carlo simulation[7] suggests that junction arrays in an irrational magnetic field undergo a spin-glass-like transition.

The effects of quenched positional disorder on the critical behavior of Josephson-junction arrays in a transverse magnetic field are also studied by various authors.[8–11] Using a randomly diluted Josephson-junction model, John and Lubensky[8] demonstrated the existence of a spin-glass like phase in mean-field theory for a system near the percolation threshold. This possibility has also been suggested by Shih, Ebner, and Stroud[9] from numerical simulations of a model of disordered superconducting-nonsuperconducting composite. On the other hand, Granato and Kosterlitz[10] have studied the disordered case where the average value of f, denoted by $\langle f \rangle$, is an integer and each site is displaced from its ideal position $\mathbf{r}$ on a square lattice by an amount $\mathbf{u}_r$ with a probability distribution:

$$P(\mathbf{u}_r) \propto \exp\left[-\frac{\mathbf{u}_r^2}{2\Delta^2} \right] . \tag{1.5}$$

They define a critical value of $\langle f \rangle$ given by

$$f_c = (32\pi)^{-1/2} a/\Delta \;, \tag{1.6}$$

where a is the lattice spacing, and suggest that when $\langle f \rangle < f_c$, there are two transitions: a high-temperature transition from the normal to a KT-like phase and a low-temperature reentrant transition back to the normal state. The superconducting phase is predicted to disappear for $\langle f \rangle > f_c$. Their analysis does not suggest the existence of a glassy phase.

In this paper, we investigate the effects of quenched positional disorder on Josephson-junction arrays by doing a detailed Monte Carlo (MC) simulation. We concentrate on the case $\langle f \rangle$ an integer and change the effective strength of the disorder by varying the magnitude of the transverse magnetic field B. Our main conclusions are the following. For small values of $\langle f \rangle$, the system exhibits a transition from the normal to the KT-like phase. The critical behavior at this transition is qualitatively similar to that at the KT transition in a finite system. The fact that weak disorder does not produce a qualitative change in the critical behavior has also been seen in a recent MC simulation[11] for $\langle f \rangle = \frac{1}{2}$. On the other hand, for large value of $\langle f \rangle$, the critical behavior of the system is modified considerably and calculations of various quantities such as the specific heat, the helicity modulus and the Edwards-Anderson[12] order parameter suggest that the system behaves very much like a spin glass over the time scales of the simulations. The nature of the various metastable states of the system supports the spin-glass analogy. We do not see any signature of a reentrant transition for any value of $\langle f \rangle$.

The rest of the paper is organized as follows. In Sec. II we introduce the quantities calculated in the simulation and describe the calculational procedure. In Sec. III we present and discuss our results for various values of $\langle f \rangle$. Possible connections between our results and experiments are discussed in Sec. IV.

II. CALCULATIONAL PROCEDURE

To simulate the behavior of model (1.1), we consider a magnetic field in the $\hat{\mathbf{z}}$ direction, $\mathbf{B} = B\hat{\mathbf{z}}$, and use the gauge $\mathbf{A} = Bx\hat{\mathbf{y}}$. The phase factor A_{ij} given by Eq. (1.2) then takes the form

$$A_{ij} = (2\pi/\Phi_0) B X_{ij} (y_j - y_i) \;, \tag{2.1}$$

where $X_{ij} = (x_i + x_j)/2$ is the average x coordinate along the bond joining lattice sites i and j. To introduce the positional disorder, we allow each lattice site to randomly move from its ideal position $(ma\hat{\mathbf{x}}, na\hat{\mathbf{y}})$ by a small amount such that the new position is

$$((m + \delta r_1) a\hat{\mathbf{x}}, (n + \delta r_2) a\hat{\mathbf{y}}) \;,$$

where r_1 and r_2 are two random numbers distributed uniformly between -1 and $+1$ and $\delta = 0.05$. We keep δ constant throughout the simulation and change the effective strength of the disorder by considering $B = 0,1,3,5,7$ in units of Φ_0/a^2. These values of B correspond to $\langle f \rangle = 0,1,3,5,7$ respectively. With this construction the critical value f_c given by Eq. (1.6) is approximately equal to 3.45.

In these simulations we mostly considered a 32×32 lattice with periodic boundary conditions. However, we also studied 16×16 and 22×22 lattices to understand the finite-size effects. Using a standard Metropolis algorithm, we used 50 000 MC steps/spin for temperatures above J/k_B and 100 000–150 000 MC steps/spin for temperatures below J/k_B for 32×32 samples. At each temperature thermodynamic averages were performed over the last half of the time steps. We also considered averages over 5–10 independent runs with different realizations of the disorder.

The specific heat per site (C) was calculated from fluctuations of the internal energy and the helicity modulus (γ) which is a measure of the stiffness of spins, was computed using the relation[9]

$$\gamma_{xx} = JN^{-1}\left[\sum_{\langle ij \rangle} x_{ij}^2 \langle \cos(\theta_i - \theta_j - A_{ij})\rangle - (1/k_B T)\left\langle \left[\sum_{\langle ij \rangle} x_{ij} \sin(\theta_i - \theta_j - A_{ij})\right]^2\right\rangle + (1/k_B T)\left[\left\langle \sum_{\langle ij \rangle} x_{ij}\sin(\theta_i - \theta_j - A_{ij})\right\rangle^2\right]\right] , \tag{2.2}$$

where $x_{ij} = x_j - x_i$ and $\langle\ \rangle$ denotes a thermodynamic (Monte Carlo) average. An analogous expression holds for γ_{yy}. The helicity modulus γ was calculated by taking an average of γ_{xx} and γ_{yy} and then averaging over different realizations of the disorder.

To examine the extent to which the system is frozen into one state, we calculate the Edwards-Anderson[12] (EA) order parameter q defined by

$$q = \langle (1/N) \sum_i |\langle \mathbf{S}_i \rangle|^2 \rangle_c \;, \tag{2.3}$$

where $\langle\ \rangle_c$ denotes an average over different realizations of the disorder and $\mathbf{S}_i = (\cos\theta_i, \sin\theta_i)$. Since the Hamiltonian (1.1) is invariant under an overall rotation of the spin system, the MC updating procedure generates, in general, uniform rotations of the spins for finite samples. Unless care is taken to correct for the effects of uniform rotations, the EA order parameter defined as

$$q = \lim_{t\to\infty} \left\langle\left\langle \left[(1/N) \sum_{i=1}^{N} \mathbf{S}_i(0) \cdot \underline{R}\, \mathbf{S}_i(t) \right] \right\rangle\right\rangle_c \tag{2.4}$$

vanishes[13] at long times. If there is no overall rotation of the spin system then from (2.4) one finds that

$$(1/N)\sum_{i=1}^{N} S_i^{\alpha}(0)S_i'^{\beta}(t)=(\tfrac{1}{2})q\delta^{\alpha\beta}\ , \qquad (2.5)$$

where $t>\tau$, the maximum relaxation time of the system, α,β are the cartesian components and we have omitted the angular brackets for simplicity. Now for the finite size of the samples, there is always some rotation associated with the updating of the spins, and $S_i^{\beta}(t)$ is given by

$$S_i^{\beta}(t)=R_1^{\beta\gamma}S_i'^{\gamma}(t)\ , \qquad (2.6)$$

where $\underline{R}_1$ is the corresponding rotation matrix. Then the quantity

$$(1/N)\sum_i S_i^{\alpha}(0)R^{\alpha\beta}S_i^{\beta}(t)$$

becomes equal to $(1/2)\mathrm{Tr}(\underline{R}\underline{R}_1)q$ with the help of (2.6). Now $\mathrm{Tr}(\underline{R}\underline{R}_1)$ is maximum $(=2)$ if $\underline{R}=\underline{R}_1^{-1}$ and then

$$\max(1/N)\sum_i \mathbf{S}_i(0)\cdot\underline{R}\mathbf{S}_i(t)=q\ . \qquad (2.7)$$

Thus, in order to take account of this unphysical rotation we have considered the quantity

$$(1/N)\sum_{i=1}^{N}\mathbf{S}_i(0)\cdot\underline{R}\mathbf{S}_i(t) \qquad (2.8)$$

and determined the general SO(2) matrix $\underline{R}$ which makes this quantity a maximum. The way it is done is the following. If we write $\mathbf{S}_i(0)=(\cos\theta_i(0),\sin\theta_i(0))$, $\mathbf{S}_i(t)=(\cos\theta_i(t),\sin\theta_i(t))$ and $\underline{R}$ as $R(\phi)$ then the angle ϕ which maximizes (2.8) is given by

$$\tan\phi=\sum_i \sin(\theta_i(0)-\theta_i(t))/\sum_i \cos(\theta_i(0)-\theta_i(t))\ . \qquad (2.9)$$

Accordingly, we defined our single-spin autocorrelation function as[13]

$$q(t)=\left\langle\left\langle \max\left[(1/N)\sum_{i=1}^{N}\mathbf{S}_i(0)\cdot\underline{R}\mathbf{S}_i(t)\right]\right\rangle\right\rangle_c\ . \qquad (2.10)$$

The EA order parameter q is then obtained from $q(t)$ as

$$q=\lim_{t\to\infty}q(t)\ . \qquad (2.11)$$

We also looked at the various metastable equilibrium configurations (EC) of the system. To generate the different EC's we followed the prescription given by Walker and Walstedt[14] in a different context. In this method, one starts from a random configuration of the spins and then rotates the spins sequentially into coincidence with their instantaneous local fields. A sequence of N such rotations constitute a single iteration. One is guaranteed a reduction in the energy of the system at every step with this algorithm, and one stops when the required convergence in energy (in our case 10^{-6}) is achieved. EC's were generated with the above procedure starting from 20 different random configurations of the spins. Since the Hamiltonian has rotational symmetry, one has to be very careful to distinguish genuinely independent EC's from those which differ only by a uniform rotation. For this reason, we calculated the quantity $O^{\alpha\beta}$, the "true" overlap between EC's α and β, as

$$O^{\alpha\beta}=\max\left[(1/N)\sum_i \mathbf{S}_i^{\alpha}\cdot\underline{R}\mathbf{S}_i^{\beta}\right] \qquad (2.12)$$

where $\underline{R}$ is a general SO(2) matrix. If for any newly generated EC the overlap with any of the existing EC's is greater than 0.95, we considered that new EC to be not independent. We also calculated the vortex-pair density in the metastable states in the following way. If $\{\theta_i^0\}$ are the angles of the lowest-energy EC thus found, for any other EC, $\{\theta_i\}$, we calculated the quantity $\theta_i'=\theta_i-\theta_i^0$ for every lattice site i. We then computed the changes $\Delta\theta_i'$ in the angle θ_i' from one spin to the next, defining the angle difference between neighboring spins to lie within the interval $(-\pi,\pi)$. We then calculated the vorticity around each elementary square in the lattice as $1/(2\pi)$ times the sum of the $\Delta\theta_i'$'s along the four sides of the square. The vortex pair density was then calculated as the sum of positive (or negative) vortices divided by the area of the lattice. The results of the calculations described above for various values of $\langle f\rangle$ are discussed in the next section. In the following, we set J,k_B, and a equal to unity, so that the internal energy is measured in units of J, the temperature in units of J/k_B and the helicity modulus is measured in units of Ja^2.

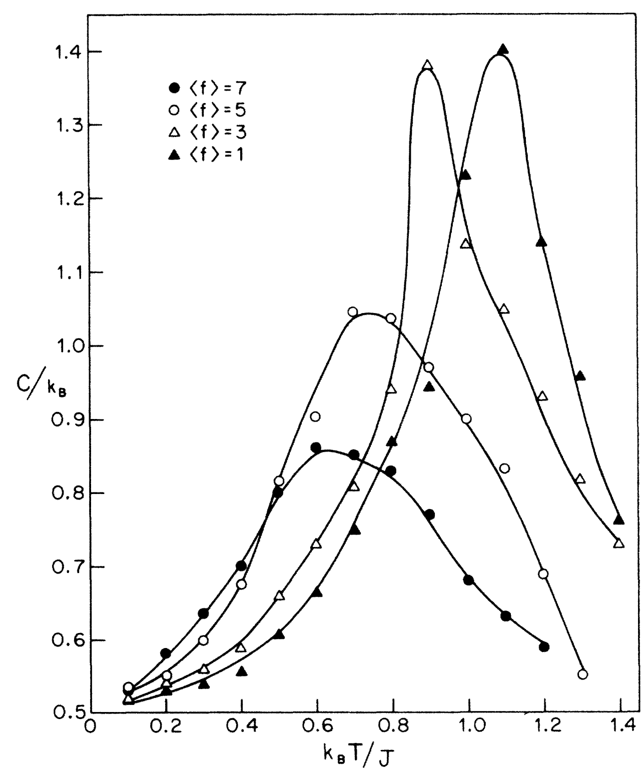

FIG. 1. The temperature (in units of J/k_B dependence of the specific heat per spin, C/k_B, for different values of $\langle f\rangle$. The solid lines are guides to the eye.

III. RESULTS

Our results for the specific heat per spin (C) versus temperature T for 32×32 lattice and for various values of $\langle f\rangle$ are displayed in Fig. 1. We note that for $\langle f\rangle=1$, the effect of disorder is small and the graph looks very similar to that found[15] in a pure 2D XY model. For $\langle f\rangle=3$, the shape of the curve remains unchanged, although the temperature at which C shows a peak is shifted to a smaller value. On the other hand, substantial modification in the shape of the C versus T curve is noted for higher values of $\langle f\rangle$. The specific heat curve gets more and more rounded as the disorder is increased and the maximum value of C is continuously reduced. Most spin-glass models exhibit a broad maximum in the specific heat at a temperature close to the freezing temperature. However, the progressive roundedness of the specific heat peak observed here is not conclusive evidence for glassy behavior—it might also indicate that the phase transition disappears at larger values of the disorder. We also note that any signature of a re-entrant transition at a lower temperature is absent in the specific heat curves.

The γ versus T and q versus T curves are shown in Figs. 2 and 3 respectively. In Fig. 2 we find that for $\langle f\rangle=1$ and $\langle f\rangle=3$, the shape of the graphs is very similar to that without disorder ($f=0$). A line of slope $2/\pi$ indicates the universal jump in $\gamma(T_c)/T_c$ of a KT transition in the absence of disorder. From the intersection of this straight line with the γ versus T curve, we estimate $T_c\approx 0.94$ for $f=0$, which agrees well with previous results.[15] According to the analysis of Ref. 10, the value of $\gamma(T_c)/T_c$ at the normal to superconducting transition for $\langle f\rangle<f_c$ is not universal. This value is predicted to increase with $\langle f\rangle$. The behavior of the γ versus T curves for $\langle f\rangle=1$ and $\langle f\rangle=3$ is consistent with this prediction. However, the picture completely changes for higher values of $\langle f\rangle$. For $\langle f\rangle=5$ and $\langle f\rangle=7$, we do not see any sign of a jump in the helicity modulus and the graphs closely resemble those found[9] in simulations of dilute granular superconductors where the authors have suggested the existence of a glassy phase. Again, no evidence for a re-entrant transition is found in the γ versus T data.

The picture is very similar when we consider the q versus T graphs in Fig. 3. Here also $\langle f\rangle=1$ and $\langle f\rangle=3$ fall in the same class and $\langle f\rangle=5$ and $\langle f\rangle=7$ show qualitatively different behavior. The shape of the q versus T graphs for $\langle f\rangle=5$ and $\langle f\rangle=7$ is similar to that found in a simulation[7] of a model of a Josephson-junction array in an irrational magnetic field. This model is predicted[7] to exhibit a glass transition for finite cooling rates. For all values of $\langle f\rangle$, the EA order parameter increases continuously as T is decreased, thus indicating

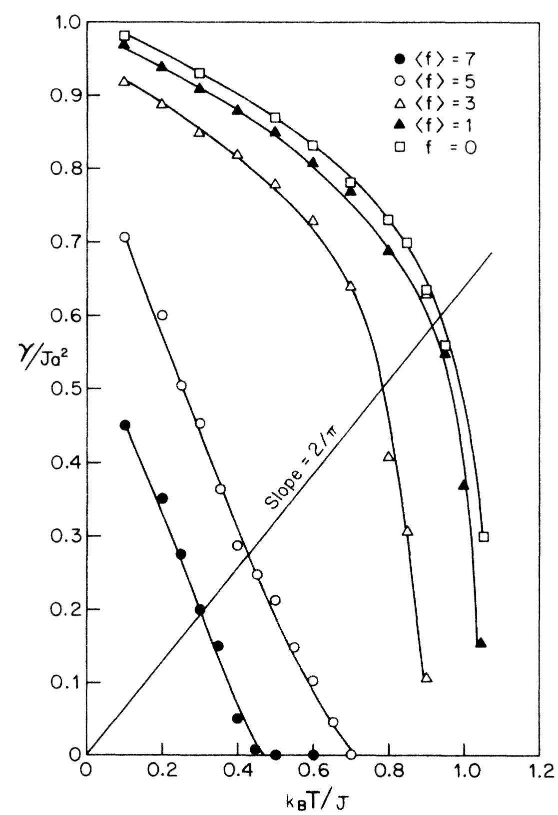

FIG. 2. The helicity modulus γ/Ja^2 vs the temperature k_BT/J for different values of $\langle f\rangle$. The solid curves are guides to the eye. The straight line of slope $2/\pi$ represents the universal jump in γ/T at the Kosterlitz-Thouless transition for $f=0$.

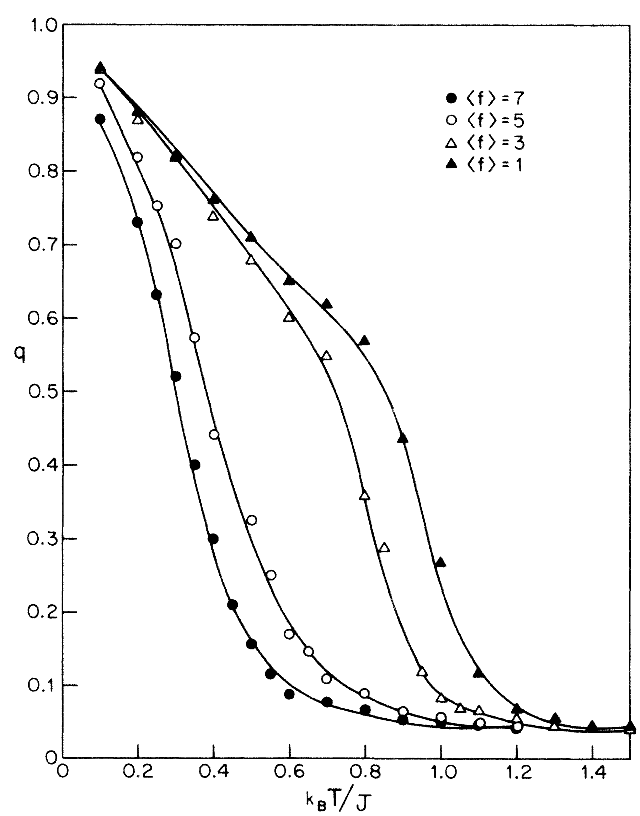

FIG. 3. The Edwards-Anderson order parameter q as a function of temperature (k_BT/J) for different values of $\langle f\rangle$. The solid lines are guides to the eye.

the absence of any reentrant transition. We looked for finite-size effects by carrying out the simulation for 16×16 and 22×22 lattices. We did not find any significant sample-size dependence in the low-temperature behavior of C,γ, and q.

The facts that for larger values of the disorder ($\langle f \rangle=5,7$), the specific heat peak gets rounded and the γ versus T and q versus T show a different behavior from those for small disorder ($\langle f \rangle=1,3$), suggest that the low-temperature state for $\langle f \rangle=5$ and $\langle f \rangle=7$ may be the glassy state predicted by various authors,[7–9] although in slightly different contexts. To investigate this possibility, we calculated C,γ,q and the average energy per spin E both on heating and cooling the samples. For small values of disorder, these quantities are indistinguishable in these two different runs. For $\langle f \rangle=5$ or 7, the values of these quantities are different in the two cases for each individual samples, whereas when averaged over different realizations, no meaningful conclusion about the presence or absence of freezing can be drawn, considering the error bars. We also studied the difference between field-cooling (FC) and zero-field-cooling (ZFC) behavior. In the ZFC case we start from the configuration where all the spins are pointing in the same direction (this is the ground state of the system in the absence of the magnetic field) and warm up the samples in the presence of the transverse magnetic field B. At various temperatures we calculate the difference in energy per spin (ΔE) between the FC and ZFC states for various values of $\langle f \rangle$. The ΔE versus T graph for $\langle f \rangle=7$ is shown in Fig. 4. The energy-differences for $T>0.45$ are statistically negligible whereas for $T<0.45$ this difference is significant. The temperature ($T\approx0.45$) at which ΔE goes to zero is very close to that where γ goes to zero and q shows a sharp change for the same value of $\langle f \rangle$. This temperature is lower than but close to the specific heat maximum for the same value of $\langle f \rangle$. These features strongly suggest a frozen-in glassy state below $T\approx0.45$ for $\langle f \rangle=7$.

The structure of the metastable EC's also supports our conclusion of the existence of a glassy state for large values of $\langle f \rangle$. For small disorder (e.g., $\langle f \rangle=1$) we always find a lowest energy EC which is separated from other EC's by considerable energy differences. However as $\langle f \rangle$ is increased, this spacing decreases continuously and for $\langle f \rangle=7$ all the independent EC's have energies very close to one another. This feature is similar to that found[14] in simulations of spin glasses. At sufficiently strong fields, nearly all the phase factors A_{ij} are large compared to 2π. Thus the couplings will tend to orient the phases at essentially random angles and the system is in effect a physical realization of the "gauge glass" mentioned by several authors.[15,16] Since this happens for any chosen gauge of the vector potential **A** we believe that our main conclusions are independent of any particular choice of gauge. The other important feature is that the EC's contain a large number of vortices when calculated[17] with respect to the lowest energy EC. For this reason the overlap between the independent EC's are very small (varying from ≈0.01 to a maximum of ≈0.5) even when they have almost the same energy. In the simulations at very low temperatures, if true thermo-

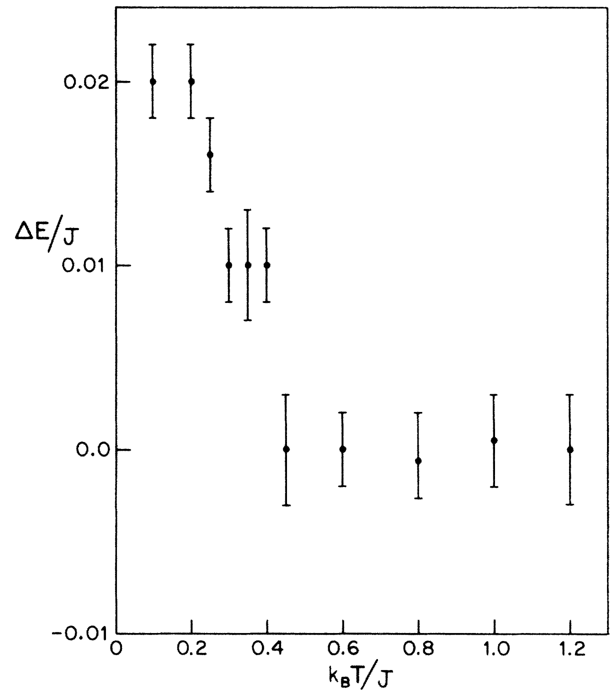

FIG. 4. Temperature dependence of $\Delta E/J$, the difference in energy per spin between zero-field-cooled and field-cooled runs for $\langle f \rangle=7$.

dynamic equilibrium were established, then the system would have explored all of these metastable configurations and the resultant q would have been much smaller than what we have found. This observation, therefore, shows that equilibrium was not established at low temperatures due to the presence of long relaxation times associated with a low-temperature glassy state.

In connection with the gauge-glass regime it is interesting to note that renormalization group calculations by Hertz[15] indicate that for $d\leq4$, the frustration parameter characterizing his model exhibits a runaway. Hertz interprets these results in terms of a mean-field theory in which the transition possibly takes place into an ordered phase with many local minima. We do not believe that the freezing observed in our simulations represents a thermodynamic phase transition to a "superconducting glass" phase. Since short-range Ising[18] and XY models[19] of spin glass do not exhibit any thermodynamic phase transition at a finite temperature in two dimensions, there is no reason to expect the model considered here to do so. Our results, however, do suggest that behavior resembling a spin-glass like phase transition may be observed in disordered Josephson-junction arrays in a transverse magnetic field over short or intermediate time scales.

As discussed earlier, the results of our simulations do not show any evidence for a low-temperature reentrant transition at any value of $\langle f \rangle$. This observation, however, does not necessarily imply the absence of a reentrant transition, in experimental situations. Due to the following reasons, it is very difficult to see the low-temperature reentrant transition predicted in Ref. 10 in a finite-size and finite-time MC simulation. Even when the interaction between vortices is screened out by the disorder, the creation of a vortex pair requires a finite amount of energy. In a small sample at a low temperature, the entropy factor may not balance this energy cost. Secondly, since the vortices can be pinned at favorable sites, the low-energy states with different configurations of vortex pairs are separated from one another by energy barriers which are difficult to overcome at low temperatures. Because of these reasons, we can not be sure about whether our simulation results showing the absence of reentrant behavior reflect the real physics in experimental situations.

IV. DISCUSSIONS

To summarize, our simulations of the thermodynamic properties of a model of a positionally disordered Josephson-junction arrays in a transverse magnetic field show that for small integral values of the parameter $\langle f \rangle$ ($\langle f \rangle = 1,3$) the system exhibits a Kosterlitz-Thouless type transition from the normal to the superconducting phase. We do not find any evidence for a low-temperature reentrant transition back to the normal phase, although we can not rule out the occurrence of such a transition in experiments from our study of finite samples. For larger integral values of $\langle f \rangle$ ($\langle f \rangle = 5,7$), the system exhibits a spin-glass like freezing at low temperatures over the time scales of our simulations. This freezing is not expected to represent a true phase transition. However, one may still be able to experimentally observe a "superconducting glass" transition over short or intermediate time scales. One may possibly look for a continuous increase of the helicity modulus from zero as the temperature is lowered below the freezing temperature. In contrast, the helicity modulus is expected to show a discontinuity at a KT like transition. For pure XY case the helicity modulus is closely related to the exponent α of the nonlinear $I-V$ characteristics. One might look for a continuous increase of the exponent α in the disordered case as a sign of "glass" transition as the temperature is lowered below the freezing temperature. However, the relation between the helicity modulus and the exponent α is not clear for the disordered case. Since the kinetic inductance of the array is proportional to the helicity modulus it could be used as an experimental probe to observe the suggested freezing in the disordered case. It may also be possible to see signatures of the presence of long relaxation times in measurements of the low-frequency *ac* conductivity. One may hope to see remanence—if the applied magnetic field is switched off at a low temperature, the tunneling current may decay slowly with time. Other phenomena associated with slow relaxation such as differences between *ac* and *dc* susceptibilities[20] and differences between field-cooled and zero-field-cooled measurements may also be observable. The type of positional disorder considered in this paper is of direct relevance to experiments because some amount of disorder of this type, however small, is always present in real arrays. It would be very interesting to look for some of the effects discussed above in experiments on such arrays.

Note added. After this work was completed we came to know of an experimental paper[21] which reaches very similar conclusion to our work.

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