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Similarity between quantum Hall transport coefficients

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A simple scaling model, applied to the observation by Chang and Tsui that $d\rho_{xy}/dn$ and ρ_{xx} are nearly identical in GaAs-$Al_xGa_{1-x}As$ heterostructures, suggests that in the *interplateau regions* scattering is long range and the carrier concentration does not vary with magnetic field.

Recently, Chang and Tsui[1] observed a remarkable similarity in the magnetic field dependence of $d\rho_{xy}/dn$ and ρ_{xx} in the two-dimensional (2D) electron gas in GaAs-$Al_xGa_{1-x}As$ heterostructures. Over a wide range of strong magnetic fields these transport coefficients are nearly identical apart of strong spikes in $d\rho_{xy}/dn$ at the edge of each minima. Chang and Tsui suggested the possibility of a fundamental connection between these quantities.

Pudalov and Semenchinsky,[2] Yoshihiro *et al.*,[3] and Cage *et al.*[4] observed a sample and field-independent linear relationship between $\delta\rho_{xy}$ and ρ_{xx} in the wings of the quantum Hall effect (QHE) plateaus in Si metal-oxide-semiconductor field-effect transistors (MOSFET's). Theoretically, this was studied by Khmelnitskii[5] and by Pudalov and Semenchinsky.[6]

We present here a simple scaling model of the magnetic field dependence of the transport coefficients in the interplateau regions, and demonstrate how the similarity between the transport coefficients may shed some light on the electron scattering mechanisms. We start by assuming a power-law dependence on the magnetic field of the relaxation time and of the electron concentration in the *interplateau region* and derive a relationship between the relevant indices.

In the strong magnetic field limit $\omega_c\tau \gg 1$, the conductivity tensor components are connected in the following way:[7]

$$\sigma_{xx} \propto \frac{\sigma_{xy}}{\omega_c\tau} , \tag{1}$$

where

$$\sigma_{xy} \propto \frac{nec}{B} + \frac{\sigma_{xx}(\text{peak})}{\omega_c\tau} . \tag{2}$$

Let us assume that

$$\tau(B) \propto B^{\beta} \tag{3a}$$

and

$$n(B) \propto B^{\gamma} . \tag{3b}$$

A standard high-field ($\omega_c\tau \gg 1$) expansion with respect to a small parameter $\epsilon = 1/\omega_c\tau$ yields

$$\sigma_{xx} \propto \epsilon B^{\gamma-2-\beta} . \tag{4}$$

Combining now the expressions

$$\rho_{xy} \simeq \sigma_{xy}^{-1} \tag{5a}$$

and

$$\rho_{xx} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2} \tag{5b}$$

with Eqs. (3) and (4), we obtain

$$\frac{d\rho_{xy}}{dB} \propto -B^{-\gamma} + \epsilon^2 B^{-2-2\beta-\gamma} \tag{6a}$$

and

$$\rho_{xx} \propto \epsilon B^{-\gamma-\beta} \equiv \epsilon B^{\alpha} . \tag{6b}$$

We turn now to Eq. (5a) of Chang and Tsui,[1]

$$\frac{d\rho_{xy}}{dB} \propto \left(\frac{n}{B}\right) \rho_{xx} , \tag{7}$$

which in our notations reads

$$\frac{d\rho_{xy}}{dB} \propto B^{\gamma-1} \rho_{xx} . \tag{7'}$$

Substitution of Eqs. (6a) and (6b) into Eq. (7′) yields

$$\beta = -1 , \tag{8a}$$

$$\gamma = 0 . \tag{8b}$$

This corresponds to

$$n \propto B^0 \tag{9}$$

and

$$\tau \propto B^{-1} . \tag{10}$$

From Eqs. (6b) and (8) we get

$$\alpha = 1 . \tag{11}$$

This corresponds to long-range scattering by screened ionized and Gaussian long-range impurities in semiconductors in the extreme quantum limit, as was shown by Bergers and Haidu[8] in the framework of the self-consistent transport equation, derived by Gerhards.[9] In InSb, in the extreme quantum limit, a value of $\alpha \simeq 1$ was observed by Pavlow *et al.*[10] Dornhaus *et al.*[11] reported

$0.5 < \alpha < 1.9$ in $Hg_{1-x}Cd_xTe$. Bruls *et al.*[12,13] have shown that linear magnetoresistance can appear in two-dimensional magnetotransport also when the relaxation time is field independent, as a consequence of spatial variations in ρ_{xy}.

We outline that the field independence of the carrier concentration in the interplateau region, following from Eq. (9), is in agreement with recent de Haas-van Alphen measurements of Eisenstein *et al.*[14] on GaAs-$Al_xGa_{1-x}As$ heterojunctions. They observed a positive slope of the magnetization curve (in the interplateau region) which corresponds to a fixed number of particles (Shoenberg[15]), to be contrasted with the negative slope in the plateau region, where the chemical potential is fixed (Vagner and Maniv[16]).

In a general case

$$\frac{d\rho_{xy}}{dB} \propto \rho_{xx}^{\eta} , \tag{12}$$

the scaling relationships are

$$\beta + \gamma + 1 = 0 , \tag{13a}$$

$$\gamma + \eta = 0 . \tag{13b}$$

Thus, provided η is known experimentally, the indices β and γ may be defined from Eq. (13).

To summarize, using the high-field expansion for the conductivity tensor we have found, via a simple scaling approach, that the similarity between the quantum Hall transport coefficients reported by Chang and Tsui[1] suggests a long-range impurity scattering in GaAs-$Al_xGa_{1-x}As$ and a field-independent carrier concentration in the interplateau regions.

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[1]A. M. Chang and D. C. Tsui, Solid State Commun. **56**, 153 (1985).

[2]V. M. Pudalov and S. G. Semenchinsky, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 173 (1983) [JETP Lett. **38**, 202 (1983)]; Solid State Commun. **51**, 19 (1984).

[3]K. Yoshihiro, J. Kinoshita, K. Inagoki, and G. Yamananouchi, Physica **117-118B**, 706 (1983).

[4]M. E. Cage, B. F. Field, R. F. Dziuba, S. M. Girvin, A. C. Gossard, and D. C. Tsui, Phys. Rev. B **30**, 2286 (1984).

[5]D. E. Khmelnitskii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1983)].

[6]V. M. Pudalov and S. G. Semenchinsky, Zh. Eksp. Teor. Fiz. **86**, 1431 (1984) [Sov. Phys. JETP **59**, 838 (1984)].

[7]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[8]D. Bergers and J. Haidu, Solid State Commun. **20**, 683 (1976).

[9]R. Gerhardts, Z. Phys. B **22**, 327 (1975).

[10]S. T. Pavlow, R. V. Parfeniev, Yu. A. Firsov, and S. S. Shalyt, Zh. Eksp. Teor. Fiz. **48**, 1565 (1965) [Sov. Phys. JETP **21**, 1049 (1965)].

[11]R. Dornhaus, G. Nimtz, W. Schlabitz, and P. Zaplinski, Solid State Commun. **15**, 495 (1974).

[12]G. J. C. L. Bruls, J. Bass, A. P. van Gelder, H. van Kempen, and P. Wyder, Phys. Rev. Lett. **46**, 553 (1981).

[13]G. J. C. L. Bruls, J. Bass, A. P. van Gelder, H. van Kempen, and P. Wyder, Phys. Rev. B **32**, 1927 (1985).

[14]J. P. Eisenstein, H. L. Stormer, V. Narayanamurti, A. Y. Cho, A. C. Gossard, and C. W. Tu, Phys. Rev. Lett. **55**, 875 (1985).

[15]D. Shoenberg, *Magnetic Oscillations in Metals* (Cambridge Univ. Press, Cambridge, 1984), Chap. 2; in *Condensed Matter Physics,* edited by R. L. Orbach (Springer-Verlag, New York, 1987), p. 129.

[16]I. D. Vagner and T. Maniv, Phys. Rev. B **32**, 8398 (1985).