Limiting response time of double-barrier resonant tunneling structures

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The lifetime of the lowest quasibound state localized within the quantum-well region of a double-barrier resonant tunneling structure is calculated. The results are used to estimate the limiting response time for resonant transport in a $GaAs/Al_xGa_{1-x}As$ structure.

I. INTRODUCTION

The experimental results of Sollner *et al.*[1] have led to a renewal of interest in the phenomenon of resonant tunneling. Epitaxially grown $GaAs/Al_xGa_{1-x}As$ double-barrier resonant tunneling (DBRT) structures, similar to those used by Sollner *et al.*, are now the subject of extensive investigation at many laboratories. Growth methods,[2] barrier widths,[3] quantum-well (QW) composition,[4] spacer-layer width,[5] etc., are all being varied in order to investigate their impact on resulting device *I-V* characteristics. Despite these efforts, the very origin of the negative differential resistance seen in the *I-V* characteristics of these systems remains the subject of controversy.[6–9] Some feel that Fabry-Perot-type resonances are needed to explain the observations, while others believe that kinematic constraints associated with tunneling from a 3D to a 2D system are all that is required (sequential tunneling model). More work is needed to clarify this matter.

In this Brief Report we calculate, within the context of a simple model, the limiting response time τ of a DBRT structure where a substantial fraction of the electrons comprising the current can be associated with states satisfying a Fabry-Perot-type resonance condition. We feel that measured intrinsic response times substantially lower than our estimates would argue in favor of sequential tunneling models where, in principle, limiting response times could be much shorter. In the next section we discuss the conceptual framework of our calculation and describe some of its details. We then use our results to estimate τ for actual $GaAs/Al_xGa_{1-x}As$ DBRT structures.

II. CALCULATION

One of the most attractive features of DBRT structures is their short response times. A significant ac response has been measured at frequencies up to 2.5 THz,[1] suggesting response times shorter than 10^{-13} sec. When the enhanced current in the *I-V* characteristic can be attributed to a Fabry-Perot-type resonance, the structure's intrinsic limiting response time τ can be estimated. Consider such a structure biased just below the lowest resonance voltage V_r. A substantial fraction of the electrons contributing to the steady current in this case are in wave functions which are coherent over the entire structure's width. Electrons in such states benefit from the reinforcing nature of multiple reflections and are able to tunnel through the structure with up to unity transmission. However, as pointed out by Ricco and Azbel,[10] the very electrons which benefit the most from the effects of wave-function coherence are also those primarily affected by the delay times connected with the resonance. This means that if one applies a small-amplitude ac voltage $\delta V(\omega)$ to a DBRT structure biased just below V_r, a significant linear current response $\delta I(\omega)$ will cease to exist for frequencies much greater than $1/\tau$.[11] In fact, numerical studies of the temporal evolution of electron wave packets through DBRT-structure model potentials show that packets satisfying the resonance condition experience significant time delays in their transmission.[12,13] Harada and Kuroda[13] estimated this delay time by studying the time dependence of the wave-function amplitude inside the QW region of a DBRT structure while an incident wave packet propagated through the system. External fields and the different effective masses in the QW and barrier regions of the structure were ignored in their calculation. In the work described below, we approach the problem of calculating the resonance lifetime from a different (although standard[14]) perspective. First, an electron is placed in an initial state localized within the QW region of a DBRT structure. This state is expanded in the complete set of time-dependent eigenstates of the system, states which properly account for its effective-mass profile. After a time t elapses, the evolved state is projected back onto the initial state. From the long-time behavior of this projection coefficient, an analytic expression for the lifetime is obtained. The expression is then evaluated for a number of different $GaAs/Al_xGa_{1-x}As$ DBRT structures. When the effective-mass difference between the QW region and barrier regions is ignored, our results agree with earlier work.[15] For simplicity, we ignore the effects of an externally applied field in this calculation. A calculation including these effects is deferred to future work.

In each semiconductor layer comprising the structure, a one-band effective-mass Hamiltonian can be written in the form

$$H=-\frac{\hbar^2}{2m_c}\nabla^2+V(z)\ , \tag{1}$$

where m_c is the conduction-band effective mass characterizing the layer (say, m_1 in the QW region and beyond the barriers, and m_2 in the barrier regions), and the potential $V(z)$, is given by

$$V(z)=\begin{cases} V_0, & \text{if } a<|z|<a(1+\xi)\ , \\ 0, & \text{otherwise}\ . \end{cases}$$

In this expression, ξ is the width of the barriers (in units of a) and V_0 is the energy difference between the conduction-band edges in the two different semiconductors. Since H is translationally invariant in the x and y directions, its eigenstates can be written in the form

$$\psi(\mathbf{r})=\frac{1}{L}\exp(ik_x x+ik_y y)\phi(z)\ , \tag{2}$$

where periodic boundary conditions [$\psi(x+L,y,z)=\psi(x,y,z)$, etc.] along the x and y directions lead to the identification $k_{x,y}=2\pi n_{x,y}/L$, with $n_{x,y}=0,\pm1,\pm2,\ldots$, etc. Substituting (2) into (1) leads to the reduced equation

$$\frac{d^2\phi(z)}{dz^2}+\frac{2m_c}{\hbar^2}\left[E-\frac{\hbar^2\mathbf{k}^2}{2m_c}-V(z)\right]\phi(z)=0\ , \tag{3}$$

where E is the energy eigenvalue and $\mathbf{k}^2\equiv k_x^2+k_y^2$. We choose vanishing boundary conditions for ϕ at $z=\pm L/2$, force ϕ to be continuous at $z=\pm a$ and $z=\pm a(1+\xi)$, and require that $m_c^{-1}d\phi(z)/dz$ be continuous at the interface coordinates $z=\pm a$ and $z=\pm a(1+\xi)$.[16] The normalized solutions to (3) satisfying the specified boundary conditions are easily found; the even functions inside the QW region are given by

$$\phi(z)=C(q)\cos(qz)\ , \tag{4}$$

where the coefficient $C(q)$ satisfies the relation

$$\lim_{a/L\to0}\frac{L}{2}\,|C(q)|^2=F(qa) \tag{5}$$

with

$$F(u)=\frac{u^2}{D(u)}\ . \tag{6}$$

The function $D(u)$ is given by

$$D(u)=u^2+\tfrac{1}{2}[\mu\tilde{\gamma}+(1-\mu)u^2]\left[\cos^2(u)+\frac{u^2}{\mu(\tilde{\gamma}-u^2)}\sin^2(u)\right]\{\cosh[2\xi w(u)]-1\}$$
$$-\frac{1}{2\mu w(u)}[\mu\tilde{\gamma}+(1-\mu)u^2]u\sin(2u)\sinh[2\xi w(u)]\ , \tag{7}$$

where $\mu\equiv m_1/m_2$, the function $w(u)$ is given by

$$w(u)\equiv\left[\frac{\tilde{\gamma}-u^2}{\mu}\right]^{1/2}\ , \tag{8}$$

and $\tilde{\gamma}$ is defined by $\tilde{\gamma}\equiv\gamma-(1-\mu)v^2$. Here γ is the dimensionless potential strength given by $\gamma\equiv2m_1V_0a^2/\hbar^2$, and $v\equiv ka$ is the magnitude of the transverse wave vector characterizing the state. The allowed values for q in (4) are given by the roots of a transcendental equation; however, in the limit when $a/L\to0$ the root density approaches $2\pi/L$ and the equation's solution becomes unnecessary.[17]

We are now in a position to investigate the decay of a state initially localized within the QW region. Since sharp resonances are best viewed from the infinite γ (or infinite ξ) limit, it is reasonable to let ψ_0 (ψ at $t=0$) be the ground state corresponding to $\gamma=\infty$, normalized to unity within the QW region.[18] Setting the transverse wave vector to $(\bar{k}_x,\bar{k}_y)$, we let

$$\psi_0(\mathbf{r})=\frac{1}{L}\exp(i\bar{k}_x x+i\bar{k}_y y)\frac{1}{\sqrt{a}}\cos(\pi z/2a)\quad\text{for } |z|\le a\ , \tag{9}$$

and $\psi_0=0$ for $|z|>a$. In order to determine how long the electron stays inside the QW region, we calculate the probability amplitude $\mathcal{A}(t)$ for finding the electron in the initial state ψ_0 after a time t has elapsed. This is given by

$$\mathcal{A}(t)=\sum_{\mathbf{k},q,\mu}\exp(-i\omega_{\mathbf{k},q}t)\,|\langle\mathbf{k},q,\mu\,|\,\psi_0\rangle|^2\ , \tag{10}$$

where μ labels the parity of the state and $\omega_{\mathbf{k},q}\equiv(\hbar/2m_1)(\mathbf{k}^2+q^2)$. Since ψ_0 is even, it will have a projection onto only the even states given in (4). Furthermore, only states with $\mathbf{k}=(\bar{k}_x,\bar{k}_y)$ will contribute to the sum in (10) since transverse momentum is a good quantum number. A simple integration leads to the result

$$|\langle\bar{k}_x,\bar{k}_y,q\,|\,\psi_0\rangle|^2=\pi^2a\,|C(q)|^2\times\frac{\cos^2(qa)}{(qa-\pi/2)^2(qa+\pi/2)^2}\ , \tag{11}$$

which, when used in (10), gives

$$\mathcal{A}(t)=\frac{\pi}{2}\int_{-\infty}^{+\infty}du\,F(u)\frac{\cos^2(u)}{(u-\pi/2)^2(u+\pi/2)^2}\times\exp(-i\lambda^2u^2)\ . \tag{12}$$

To obtain (12), we made use of the relation $\sum_q\to(L/2\pi)\int dq$ and for convenience let $\bar{\mathbf{k}}=0$ (i.e., we set $v=0$).[19] The variable λ is related to t through the expression $\lambda^2\equiv8t/\pi\tau_0$, where $\tau_0\equiv16m_1a^2/\pi\hbar$ is the period of the lowest bound state when $\gamma=\infty$. To compute the integral in (12), we first replace $\exp(-i\lambda^2u^2)$ with its Fourier transform. This leads to the expression

$$\mathcal{A}(t)=\frac{\sqrt{\pi}}{4\lambda}\exp(-i\pi/4)\int_{-\infty}^{+\infty}dk\,\exp(ik^2/4\lambda^2)\times\int_{-\infty}^{+\infty}du\,f(u)\exp(iku), \quad (13)$$

where $f(u)$ is defined by

$$f(u)\equiv F(u)\frac{\cos^2(u)}{(u-\pi/2)^2(u+\pi/2)^2}.$$

The integral over u in (13) can be done formally by noting that $f(u)$ has an infinite sequence of simple poles in the complex plane and is bounded at infinity. This fact allows us to use the Mittag-Leffler theorem[20] to replace $f(u)$ by the expansion

$$f(u)=f(0)+\sum_{s,\sigma}\left[\frac{b_{s,\sigma}}{u-u_{s,\sigma}}+\frac{b_{s,\sigma}}{u_{s,\sigma}}\right],$$

where $\{u_{s,\sigma}\}$ are the simple poles of f and $\{b_{s,\sigma}\}$ are the corresponding residues. In labeling these poles (and residues) we make use of the fact that for each pole z of f in the first quadrant of the u plane one can find associated poles at $-z^*$, $-z$, and z^*. Hence, positive increasing s labels the poles in the first quadrant according to the ordering $|u_{1,1}|<|u_{2,1}|<|u_{3,1}|<\cdots$, etc., while σ is $+1$ or -1, depending on whether the pole is in the upper or lower half plane, respectively. The three poles associated with $u_{s,1}$, located in the second, third, and fourth quadrants, are then denoted by $u_{-s,1}$, $u_{-s,-1}$, and $u_{s,-1}$, respectively. For purposes of illustration, the first few poles of $f(u)$ are plotted in Fig. 1 for the case $\gamma=10$, $\xi=0.15$, and $\mu=0.73$. We also superimposed on the same plot the value of $F(u)$ as a function of real u for this parameter set. As can be seen in the figure, each complex-conjugate pair of poles in F (for values of real u plotted) leads to a Lorentzian-type feature in F. These Lorentzians correspond to the (even) quasi-bound-states between the barriers; they approach δ functions as γ goes to ∞. After inserting this expansion for f into (13), using the fact that $f(0)=0$ for positive γ, and performing the required integrations, we obtain

$$\mathcal{A}(t)=\frac{i\pi^2}{2}\sum_{s,\sigma}\sigma b_{s,\sigma}\exp(-i\lambda^2u_{s,\sigma}^2)\times\text{erfc}[\sigma\lambda u_{s,\sigma}\exp(-i\pi/4)]+\frac{\pi^{3/2}}{2\lambda}\exp(-i\pi/4)\sum_{s,\sigma}\frac{b_{s,\sigma}}{u_{s,\sigma}}. \quad (14)$$

Expression (14) gives the amplitude for finding the electron in the state ψ_0 after a time t has elapsed. For large times, $t\gg\tau_0$, we expect $\mathcal{A}(t)$ to decay exponentially with time. In order to extract the lifetime associated with the decay, we replace the complementary error functions in (14) by their asymptotic expansions.[21] This replacement produces a number of cancellations and leads to the form

$$\mathcal{A}(t)\underset{\lambda\gg1}{\sim}2i\pi^2\sum_{s=1}^{\infty}b_{s,1}\exp(-i\lambda^2u_{s,1}^{*2}), \quad (15)$$

where the sum in (15) is restricted to the poles of f which lie in the first quadrant of the complex plane; symmetry relations between the poles and their residues in the other quadrants have been used to obtain this result.[22] Examination of this expression shows that the largest contribution to $\mathcal{A}$ for $\lambda\gg1$ comes from the pole with the smallest imaginary part, the pole corresponding to the lowest quasi-bound-state in the QW region. If we approximate the sum by this term and define $u_{1,1}\equiv\alpha+i\beta$, we obtain

$$\mathcal{A}(t)\underset{\lambda\gg1}{\sim}2i\pi^2b_{1,1}\exp[-i\lambda^2(\alpha^2-\beta^2-2i\alpha\beta)]. \quad (16)$$

The probability $\mathcal{P}(t)=|\mathcal{A}(t)|^2$ then becomes

$$\mathcal{P}(t)\underset{t\gg\tau_0}{\sim}4\pi^4|b_{1,1}|^2\exp(-t/\tau), \quad (17)$$

where the lifetime of the quasi-bound-state is given by

$$\tau=\pi\tau_0/32\alpha\beta. \quad (18)$$

We have computed α and β from expression (7) and have plotted in Fig. 2 the logarithm of τ/τ_0 as a function

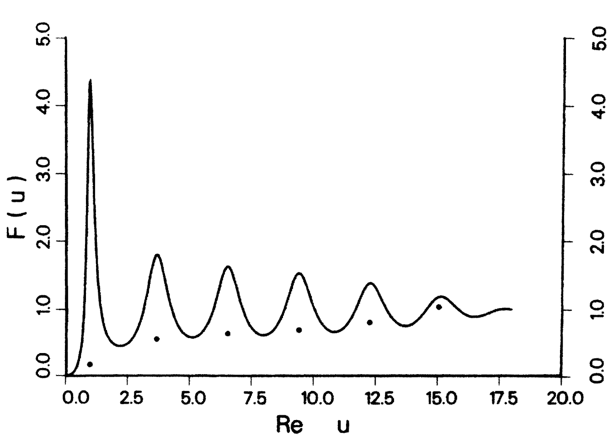

FIG. 1. The simple poles of $f(u)$ in the first quadrant of the u plane (for the case $\gamma=10$, $\xi=0.15$, and $\mu=0.73$) are marked by darkened circles. (The imaginary part of the poles is given by the vertical axis on the right.) The function $F(u)$ [see Eq. (6)] is also plotted vs real u for the same parameter set.

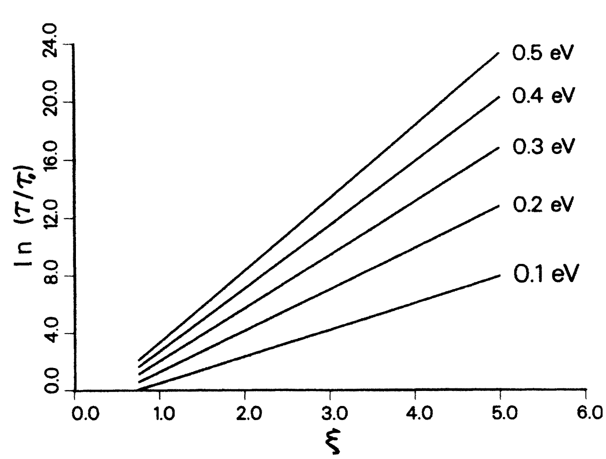

FIG. 2. The logarithm of τ/τ_0 is plotted as a function of ξ for various conduction-band discontinuities V_0.

of ξ for a number of different barrier heights V_0. To compare these results with experiment, we choose parameters consistent with the work of Sollner *et al.*;[1] $m_1=0.065m_0$ and $a=25.4$ Å leads to $\tau_0=1.89\times10^{-14}$ sec. If we furthermore let $V_0=0.23$ eV, $m_2=0.0914m_0$ ($\mu=0.73$), and $\xi=2.0$, we obtain a lifetime $\tau=2.05\times10^{-12}$ sec. This lifetime corresponds to an intrinsic cutoff frequency of 0.49 THz, a value which is substantially smaller than the 2.5 THz frequency quoted earlier.[1] We are not in a position to infer anything from this difference since we ignored the effects of an external field in our work.

III. CONCLUDING REMARKS

We have calculated the lifetime of the lowest quasi-bound-state localized between the barriers of a DBRT structure. A simplified view of the barrier structure was employed, ignoring the effects of other bands, band nonparabolicity, and most importantly, an applied electric field. Since an applied field will modify the shape of the potential acting upon an electron, it may have a significant effect on the lifetime.[23] (We suspect this to be the case for the Sollner structure.) In circumstances where the effect of the field cannot be ignored, one should regard our estimates as upper limits on the lifetime since an external field would tend to decrease the structure's ability to confine charge.

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[14]The technique of estimating a quasi-bound-state's lifetime by propagating forward in time an initially localized wave function is commonly employed. For an explicit description of the method, see, for instance, E. N. Economou, in *Green's Functions in Quantum Mechanics*, Vol. 7 of Springer Series in Solid-State Sciences, edited by P. Fulde (Springer-Verlag, Berlin, 1983), pp. 97–104.

[15]T. B. Bahder, Clyde A. Morrison, and John D. Bruno, Appl. Phys. Lett. 51, 1089 (1987).

[16]The boundary condition on the slope of ϕ, although conventional, is somewhat arbitrary. Some researchers prefer to require that the slope of ϕ be continuous. We have calculated our main results using this latter boundary condition, and they are only slightly modified.

[17]Using the correct root density leads to corrections of order a/L.

[18]A more appealing choice for ψ_0 might be the ground-state wave function in the infinite ξ limit; however, the main results of this work are not modified by this choice.

[19]In principle, the lifetime we calculate depends upon the transverse kinetic energy (this can be seen from the dependence of $\tilde{\gamma}$ on v). Therefore, a wave packet made up of many transverse wave-vector components and initially confined to the QW region of the structure would not "leak out" with a single characteristic time. Despite this fact, setting v to zero can be justified as follows. When a DBRT structure is carrying a current, the transverse wave vectors of the occupied electronic states roughly satisfy the condition $\mathbf{k}^2<k_f^2$, where k_f is the magnitude of the Fermi wave vector characterizing the n-doped regions beyond the barriers in the absence of a field. (The "turning on" of an external field along the z direction does not affect the transverse wave vector of a state). For the structures considered in our lifetime calculation, we estimate that the largest value for v relevant in an experimental circumstance leads to a difference between $\tilde{\gamma}$ and γ of about 5%. The dependence of τ on v is therefore small enough to neglect in this work.

[20]See, for instance, E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis,* (Cambridge University Press, Cambridge, England, 1978), pp. 134–136.

[21]See, for instance, *Handbook of Mathematical Functions,* edited by M. Abramowitz and I. E. Stegun (U.S. Department of Commerce, National Bureau of Standards, Washington, D.C., 1972), p. 298.

[22]We have used the fact that $b_{s,l}=-b_{-s,-l}$ and $b_{s,l}=b^*_{s,-l}$.

[23]This is true at least for the range of barrier thicknesses generally used in DBRT structures.