PHYSICAL REVIEW B VOLUME 37, NUMBER 12 15 APRIL 1988-II

Conductivity of Kondo-type systems in the presence of crystal fields

Wei Guo and Peter M. Levy
Department of Physics, New York University, 4 Washington Place, New York, New York 10003

(Received 22 July 1987; revised manuscript received 30 November 1987)

By starting with the expression for the conductivity of Kondo-type systems in the presence of crystal fields as calculated by the Kubo formalism, we derive the expression found from the Boltzmann equation. To rewrite the compact Kubo expression in the more cumbersome expression found in the semiclassical Boltzmann approach, we use a generalization of the optical theorem to finite-temperature propagators. Also, we calculate the conductivity through third order in the coupling constant $J=V^2/E_f$ and obtain agreement with previous results.

The effects of crystal fields on such properties as the susceptibility and conductivity in Kondo systems have been extensively studied by Maekawa, Takahashi, Kashiba, and Tachiki.[1] In their work the Kubo formalism is properly used to obtain these properties. However, in one article the conductivity is found by using the Boltzmann-equation approach.[2] Although the final result is *correct*, a factor is missing in the expression for the relaxation time, which accounts for inelastic-scattering processes due to transitions between crystal-field levels. When one uses the proper expression for the relaxation time, the conventional optical theorem as used in the extant derivation is inapplicable.[2] Here we derive the appropriate extension of this theorem for finite-temperature propagators.

There are at least two ways to calculate the resistivity (conductivity) of alloys containing Kondo-type impurities that are subject to crystal fields. In a semiclassical approach one solves the Boltzmann equation in the relaxation-time approximation and uses the resulting electron relaxation time to evaluate the conductivity

$$\sigma=\frac{e^2}{3m^2}\sum_{\sigma}\int\frac{d\mathbf{k}}{(2\pi)^3}k^2\left[-\frac{\partial f^0}{\partial\varepsilon_{\mathbf{k}}}\right]\tau_{\mathrm{tr}}(\mathbf{k},\sigma)\ . \tag{1}$$

Note we have set $\hbar=1$. The transport relaxation time for inelastic scattering of conduction electrons, due to transitions between crystal field levels of the magnetic impurities, is given as[3]

$$\tau_{\mathrm{tr}}^{-1}(\mathbf{k},\sigma)=\sum_{m,m',\sigma'}w_m R(m,m';\varepsilon_k)\int\frac{d\mathbf{k}'}{(2\pi)^3}\,|\langle\mathbf{k}'\sigma',m'\,|\,T\,|\,\mathbf{k}\sigma,m\rangle|^2\left[1-\frac{k'\cos\theta'}{k\cos\theta}\right]\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}+E_{m'}-E_m)\ , \tag{2}$$

where the inelasticity is characterized by the factor $R(m,m';\varepsilon_k)$ which is defined as

$$R(m,m';\varepsilon_k)=[1-f(\varepsilon_{\mathbf{k}})(1-e^{\beta(E_{m'}-E_m)})]^{-1} \tag{3}$$

and $f(\varepsilon_k)=(1+e^{\beta\varepsilon_k})^{-1}$, $\varepsilon_k=p^2/2m-\mu$, where μ is the chemical potential. The factor R is just 1 when the scattering is elastic.

In the second approach the dc conductivity is expressed in terms of the current-current correlation function by the Kubo formula[4]

$$\sigma=\lim_{\omega\to 0}\mathrm{Im}\left[\frac{1}{3v\omega}\int_0^{\beta}d\tau\, e^{i\omega\tau}\langle T_\tau\mathbf{J}(\tau)\cdot\mathbf{J}(0)\rangle\right]\ , \tag{4}$$

where

$$\mathbf{J}=-\frac{e}{m}\sum_{\mathbf{k},\sigma}\mathbf{k}C_{\mathbf{k}\sigma}^{\dagger}C_{\mathbf{k}\sigma}$$

and v is the volume. For Kondo ions, Fulde and Peschel[5] have shown that when one neglects vertex corrections the dc conductivity is given by Eq. (1), but with a relaxation time given by

$$\tau^{-1}(\mathbf{k},\sigma)=-2\,\mathrm{Im}\Sigma_c(\mathbf{k}\sigma,\omega)\ , \tag{5}$$

where $\Sigma_c(\omega)$ is the self-energy of the conduction electrons. This form of the relaxation time looks considerably simpler than the one arrived at in the semiclassical approach, and it is by no means obvious that the two expressions, Eqs. (2) and (5), are equal. Here we will show their equality, i.e., by starting with Eq. (5) we will derive Eq. (2) by using the Anderson model of local moments in metals. To establish the link between these expressions, we use a generalization of the optical theorem to systems at finite temperatures.

In the limit of infinite intra-atomic Coulomb energy for the Anderson model, the restriction of one local electron per site can be accounted for by introducing a slave boson.[6] By introducing this auxiliary particle, the local f-electron operators entering the Anderson Hamiltonian obey fermion commutation relations, instead of the more complicated ones for the original problem. In terms of slave bosons the Anderson Hamiltonian in the presence of a crystalline electric field is written as[6]

$$H=\sum_{\mathbf{k},\sigma}\varepsilon(\mathbf{k})C^{\dagger}_{\mathbf{k}\sigma}C_{\mathbf{k}\sigma}+\sum_{i,m}E_m f^{i\dagger}_m f^i_m + \frac{1}{\sqrt{N_s}}\sum_{\mathbf{k},\sigma,i,m}(V_{\mathbf{k}\sigma,m}e^{i\mathbf{k}\cdot\mathbf{R}_i}f^{i\dagger}_m b^i C_{\mathbf{k}\sigma}+\text{H.c.})+\lambda Q^i\ , \tag{6}$$

where

$$Q^i=\sum_m f^{i\dagger}_m f^i_m + b^{i\dagger}b^i\ ,$$

N_s is the number of sites, and the restriction of one local electron per site is given by $Q^i=1$. When we limit ourselves to spherical mixing, only the $l=3$ partial-wave component of the conduction electron is scattered by the mixing interaction $V_{\mathbf{k}\sigma,m}$. After averaging over the positions i of the Kondo impurities one finds that when all the scattering is limited to one angular-momentum channel, there are no vertex corrections to the current-current correlation function.[7] For the Anderson model the self-energy of the conduction electrons is

$$\Sigma_c(\mathbf{k}\sigma,\omega)=c_i\sum_m |V_{\mathbf{k}\sigma,m}|^2 G^{4f}_m(\omega)\ , \tag{7}$$

where c_i is the concentration of magnetic ions and the relaxation time (5) is given as

$$\tau^{-1}(\mathbf{k},\sigma)=2c_i\sum_m |V_{\mathbf{k}\sigma,m}|^2\rho^{4f}_m(\omega)\ , \tag{8}$$

where $\rho^{4f}_m(\omega)$ is the local $4f$-electron spectral density function.

In the limit of large degeneracy of the $4f$ state, vertex corrections to self-energy loops can be neglected, and the $4f$ spectral density function is[6]

$$\rho^{4f}_m(\omega)=\frac{1}{Z_{4f}}(1+e^{\beta\omega})\int\frac{d\omega'}{\pi}e^{-\beta\omega'}[A_m(\omega')B(\omega'-\omega)]\ , \tag{9}$$

where Z_{4f} is the partition function, $A_m(\omega)$ the spectral density function for the pseudo-f-electron (f_m), and $B(\omega)$ the spectral function for the slave boson. These spectral functions are determined by self-consistently solving the equations[8]

$$A_m(\omega)=-\text{Im}G_m(\omega)=-\text{Im}\frac{1}{\omega-E_m-\Sigma_{fm}(\omega)}\ , \tag{10a}$$

$$\Sigma_{fm}(\omega)=\int\frac{d\omega'}{\pi}B(\omega')K_m(\omega-\omega')\ , \tag{10b}$$

$$B(\omega)=-\text{Im}D(\omega)=-\text{Im}\frac{1}{\omega-\Sigma_b(\omega)}\ , \tag{10c}$$

$$\Sigma_b(\omega)=\int\frac{d\omega'}{\pi}\sum_m A_m(\omega')K_m(\omega-\omega')\ , \tag{10d}$$

where

$$\begin{aligned}K_m(\omega-\omega')&=\frac{1}{\pi}\sum_{\mathbf{k},\sigma}\frac{|V_{\mathbf{k}\sigma,m}|^2 f(\varepsilon_k)}{\omega-\omega'+\varepsilon_k}\\&=\frac{1}{\pi}\sum_{\sigma}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{|V_{\mathbf{k}\sigma,m}|^2 f(\varepsilon_k)}{\omega-\omega'+\varepsilon_k}\\&=\frac{1}{\pi}\int d\xi\frac{\langle|V_{\mathbf{k},m}|^2\rangle\rho(\xi)f(\xi)}{\omega-\omega'+\xi}\end{aligned} \tag{10e}$$

and

$$\langle|V_{\mathbf{k},m}|^2\rangle\equiv\frac{1}{4\pi}\sum_\sigma\int d\Omega_k|V_{\mathbf{k}\sigma,m}|^2\ . \tag{10f}$$

The density of states for conduction electrons is $\rho(\xi)$, and $f(\xi)$ is the Fermi function. These equations are equally valid for strong coupling, $T<T_k$, as for weak coupling $T>T_k$, where T_k is the Kondo temperature.

To leading order in $1/N_f$ the $4f$ spectral density function is found by using the bare spectral density function for the pseudo-f-electron, i.e.,[6,8]

$$A^0_m(\omega)=\pi\delta(\omega-E_m) \tag{11}$$

so that the $4f$ spectral function, Eq. (9), is

$$\rho^{4f}_m(\omega)=f^{-1}(\omega)\int\frac{d\omega'}{\pi}\frac{e^{-\beta\omega'}}{Z_{4f}}\pi\delta(\omega'-E_m)B(\omega'-\omega) \tag{12}$$

$$=f^{-1}(\omega)w_m B(E_m-\omega)\ , \tag{13}$$

where $w_m\equiv e^{-\beta E_m}/Z_{4f}$. So to leading order in $1/N_f$ we find the relaxation time, Eq. (8), is given as

$$\tau^{-1}(\mathbf{k},\sigma)=2c_i f^{-1}(\omega)\sum_m|V_{\mathbf{k}\sigma,m}|^2 w_m B(E_m-\omega)\ . \tag{14}$$

Also, to leading order in $1/N_f$ the renormalized boson spectral function corresponds to the propagator shown in Fig. 1. This renormalized propagator satisfies the Dyson equation

$$D(\omega)=D^0(\omega)+D^0(\omega)\Sigma_b(\omega)D(\omega)\ , \tag{15}$$

where the superscripts refer to the order of the hybridization coupling constant used to derive the quantity, i.e., zero denotes a bare propagator. The zeroth-order boson propagator is[9]

$$D^0(\omega)=\frac{1}{\omega}\ , \tag{16}$$

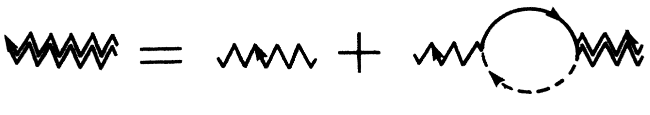

FIG. 1. The renormalized boson propagator (bold wiggly line) in terms of the bare propagator (thin line) and the lowest-order boson self-energy represented by the bubble made up from conduction- (solid line) and pseudo-f- (dashed) electron propagators.

and the self-energy of the boson $\Sigma_b(\omega)$ represented by the bubble in Fig. 1 is given by Eqs. (10d) and (11):

$$\begin{aligned}\Sigma_b(\omega)=\Sigma_b^{(2)}(\omega)&=\int d\omega' \sum_{m'} \delta(\omega'-E_{m'})K_{m'}(\omega-\omega')\\&=\sum_{m'} K_{m'}(\omega-E_{m'}) \ .\end{aligned} \tag{17}$$

As we will presently show by placing Eqs. (16) and (17) in Eq. (15) and taking the imaginary part, we find the boson spectral density correction to second order in the mixing parameter V. When used in Eqs. (9) and (8) this yields the relaxation time correct to fourth order in V. This procedure is then repeated as in Fig. 1 to produce higher-order corrections to the relaxation time. However, first we will show how our expression, Eq. (8), can be written as Eq. (2). By taking the Hermitian adjoint of Eq. (15) and recognizing that $D^0(\omega)$ has been taken to be real,[9] we find

$$D^\dagger(\omega)=D^0(\omega)+D^\dagger(\omega)\Sigma_b^\dagger(\omega)D^0(\omega) \ . \tag{18}$$

Therefore we can write the bare propagator in terms of the renormalized one as

$$D^0(\omega)=D^\dagger(\omega)[1-\Sigma_b^\dagger(\omega)D^0(\omega)] \ . \tag{19}$$

By placing this in Eq. (15) we find

$$\begin{aligned}D(\omega)=D^0(\omega)&+D^\dagger(\omega)\Sigma_b(\omega)D(\omega)\\&-D^\dagger(\omega)\Sigma_b^\dagger(\omega)D^0(\omega)\Sigma_b(\omega)D(\omega) \ .\end{aligned} \tag{20}$$

When we take the Hermitian adjoint of this expression and subtract the propagator $D(\omega)$ from its adjoint, we find

$$D(\omega)-D^\dagger(\omega)=D^\dagger(\omega)[\Sigma_b(\omega)-\Sigma_b^\dagger(\omega)]D(\omega) \ . \tag{21}$$

This is the relation we need to relate Eq. (8) to Eq. (2); it is an extension of the conventional optical theorem to temperature-dependent propagators.

From our central result of Eq. (21) the boson spectral density can be written as

$$\begin{aligned}B(\omega)&=-\mathrm{Im}D(\omega)\\&=-D^\dagger(\omega)\mathrm{Im}\Sigma_b(\omega)D(\omega) \ .\end{aligned} \tag{22}$$

By taking the imaginary part of the boson self-energy Eq. (17), and from the definition of the kernel, Eq. (10e), we find

$$\begin{aligned}\mathrm{Im}\Sigma_b(\omega)=-c_i \sum_{m',\sigma'} \int \frac{d\mathbf{k}'}{(2\pi)^3} \mid V_{\mathbf{k}'\sigma',m'} \mid^2 f(\varepsilon_{k'})\\ \times\delta(\varepsilon_{k'}-E_{m'}+\omega) \ .\end{aligned} \tag{23}$$

When we place Eqs. (22) and (23) in Eq. (14), we find the relaxation time is

$$\tau^{-1}(\mathbf{k},\sigma)=2c_i f^{-1}(\omega) \sum_{m,m',\sigma'} \mid V_{\mathbf{k}\sigma,m} \mid^2 w_m \int \frac{d\mathbf{k}'}{(2\pi)^3} \mid V_{\mathbf{k}'\sigma',m'} \mid^2 f(\varepsilon_{k'})\delta(\varepsilon_{k'}-E_{m'}+E_m-\omega)D^\dagger(E_m-\omega)D(E_m-\omega) \ . \tag{24}$$

By defining the elements of a "t matrix" as

$$\langle \mathbf{k}'\sigma',m' \mid t \mid \mathbf{k}\sigma,m \rangle \equiv V_{\mathbf{k}'\sigma',m'} V^*_{\mathbf{k}\sigma,m} D(E_m-\omega) \mid_{\omega=\varepsilon_k} \tag{25}$$

and concomitantly setting $\omega=\varepsilon_k$ we find

$$\begin{aligned}\tau^{-1}(\mathbf{k},\sigma)&=2c_i f^{-1}(\varepsilon_k) \sum_{m,m',\sigma'} w_m \int \frac{d\mathbf{k}'}{(2\pi)^3} f(\varepsilon_{k'})\delta(\varepsilon_{k'}-E_{m'}+E_m-\varepsilon_k) \mid \langle \mathbf{k}'\sigma',m' \mid t \mid \mathbf{k}\sigma,m \rangle \mid^2\\&=2c_i f^{-1}(\varepsilon_k) \sum_{m,m',\sigma'} w_m f(\varepsilon_k+E_{m'}-E_m) \int \frac{d\mathbf{k}'}{(2\pi)^3} \delta(\varepsilon_{k'}-E_{m'}+E_m-\varepsilon_k) \mid \langle \mathbf{k}'\sigma',m' \mid t \mid \mathbf{k}\sigma,m \rangle \mid^2 \ .\end{aligned} \tag{26}$$

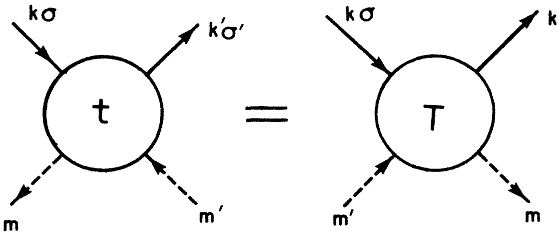

FIG. 2. The relation of the electron-hole t matrix defined by Eq. (25) to the electron-electron T matrix used for the transition probability in Eq. (2).

The product

$$\begin{aligned}w_m f^{-1}(\varepsilon_k) f(\varepsilon_k+E_{m'}-E_m)&=\frac{w_m e^{\beta\Delta_{mm'}}}{1-f(\varepsilon_k)(1-e^{\beta\Delta_{mm'}})}\\&=w_{m'}R(m',m;\varepsilon_k) \ ,\end{aligned} \tag{27}$$

where $\Delta_{mm'}\equiv E_m-E_{m'}$.

The t matrix we defined in Eq. (25) describes electron-hole scattering processes, while the T matrix defined in Eq. (2) describes electron-electron scattering. The relation between these two as shown in Fig. 2 is given by

$$\langle \mathbf{k}'\sigma',m' | t | \mathbf{k}\sigma,m \rangle = \langle \mathbf{k}'\sigma',m | T | \mathbf{k}\sigma,m' \rangle \ . \tag{28}$$

By substituting Eqs. (27) and (28) into Eq. (26) and interchanging the variables m and m', we find

$$\tau^{-1}(\mathbf{k},\sigma) = 2c_i \sum_{m,m',\sigma'} w_m R(m,m';E_k) \times \int \frac{d\mathbf{k}'}{(2\pi)^3} |\langle \mathbf{k}'\sigma',m' | T | \mathbf{k}\sigma,m \rangle|^2 \times \delta(\varepsilon_{k'} + E_{m'} - E_m - \varepsilon_k) \ . \tag{29}$$

This is the semiclassical expression Eq. (2) for scattering processes for which the factor $(k'\cos\theta')/(k\cos\theta)$ does not enter because there are no vertex corrections when all the scattering takes place in one angular-momentum channel.[7] This completes the derivation of the semiclassical expression for the Kubo formula.

To compare the results we obtain for the conductivity of Kondo ions in the presence of crystal fields to previous ones,[10] we now calculate the conductivity for $T \gg T_k$, i.e., the weak-coupling regime, by directly evaluating the boson spectral density Eq. (22). To lowest order we place the bare boson propagator (16) in Eq. (22) and after integration over intermediate states we find

$$B^{(2)}(\omega) = \frac{\rho(\Delta_{m'm} + \omega)}{\omega^2} \sum_{m'} \langle |V_{\mathbf{k}',m'}|^2 \rangle f(E_{m'} - \omega) \tag{30}$$

and

$$\rho^{(2)}_{4fm} = f^{-1}(\omega) w_m \frac{\rho(\Delta_{m'm} + \omega)}{(E_m - \omega)^2} \times \sum_{m'} \langle |V_{\mathbf{k}',m'}|^2 \rangle f(\Delta_{m'm} + \omega) \ . \tag{31}$$

As we will be interested in the density about $\omega = 0$, the Fermi surface, and as the crystal field splittings are small compared to the distance of the levels from the Fermi level, we can set $E_m - \omega \approx E_f - \omega \approx E_f$, in the denominator of Eq. (31). Furthermore, we can set $\rho(\Delta_{m'm} + \omega) \approx \rho(0)$, the conduction electron density of states at the Fermi level. By setting the hybridization constant $\langle |V_{\mathbf{k}',m'}|^2 \rangle \equiv V^2$ and defining V^2/E_f as the effective exchange coupling J, we find

$$\rho^{(2)}_{4fm}(\omega) = w_m (1 + e^{\beta\omega}) \frac{\rho J}{E_f} \sum_{m'} f(E_{m'} - E_m + \omega) \ . \tag{32}$$

By placing this density in Eq. (8) and noticing that

$$\frac{1}{4\pi} \int d\Omega_k \, |V_{\mathbf{k}\sigma,m}|^2 = \tfrac{1}{2} V^2 \ ,$$

we find the first contribution to the relaxation time is

$$\begin{aligned} \tau_1^{-1} &= c_i \rho \sum_m \frac{V^2 J}{E_f} w_m (1 + e^{\beta\omega}) \sum_{m'} f(E_{m'} - E_m + \omega) \\ &= c_i \rho J^2 (1 + e^{\beta\omega}) \sum_{m,m'} w_m f(E_{m'} - E_m + \omega) \end{aligned}$$

or

$$\tau_1^{-1} = c_i J^2 \rho \sum_{m,m'} w_m \frac{1 + e^{\beta w}}{1 + e^{\beta(\Delta_{m'm} + \omega)}} \ . \tag{33}$$

This is just the first term R_k found by Cornut and Coqblin[10] when one recognizes that we have excluded impurity scattering from mechanisms other than mixing, i.e., we set V_{mm} of Ref. 10 to zero.

To calculate τ^{-1} to next order in J we consider the first correction to the boson propagator in Eq. (22), which, by using the same simplifications as in Eqs. (30)–(32), can be written as

$$B^{(4)}(\omega) = \rho \sum_{m'} \langle |V_{\mathbf{k}',m'}|^2 \rangle f(E_{m'} - \omega) [D^{(2)}(\omega)^\dagger D^{(0)}(\omega) + D^{(0)}(\omega) D^{(2)}(\omega)] \ . \tag{34}$$

By using Eq. (16) for $D^0(\omega)$ and Eq. (17) for $\Sigma_b^{(2)}(\omega)$ from Eq. (15), we find

$$D^{(2)}(\omega) = \frac{1}{\omega^2} \sum_{m'} K_{m'}(\omega - E_{m'}) \tag{35}$$

and

$$[D^{(2)\dagger}(\omega) D^{(0)}(\omega) + D^{(0)}(\omega) D^{(2)}(\omega)] = \frac{2}{\omega^3} \sum_{m''} \mathrm{Re} K_{m''}(\omega - E_{m''}) \tag{36}$$

$$= \frac{2\rho}{\omega^3 \pi} \sum_{m''} \langle |V_{\mathbf{k}',m''}|^2 \rangle \mathrm{P} \int d\xi \frac{f(\xi)}{\omega - E_{m''} + \xi} \ . \tag{37}$$

The resultant boson spectral function is

$$B^{(4)}(\omega) = \frac{2\rho^2 V^4}{\pi \omega^3} \sum_{m',m''} f(E_{m'} - \omega) \mathrm{P} \int d\xi \frac{f(\xi)}{\omega - E_{m''} + \xi} \ . \tag{38}$$

The $4f$ spectral density function is

$$\rho_m^{(4)}(\omega)=2\frac{\rho^2V^4}{(E_m-\omega)^3}w_m f^{-1}(\omega)\sum_{m'} f(E_{m'}-E_m+\omega)\sum_{m''}\mathrm{P}\int d\xi\,\frac{f(\xi)}{E_m-E_{m''}-\omega+\xi}\ . \tag{39}$$

And the relaxation time, Eq. (8), to third order in the coupling J is

$$\tau_2^{-1}=2c_i\rho^2J^3\sum_{m,m',m''} w_m f^{-1}(\omega)f(E_{m'}-E_m+\omega)\times\mathrm{P}\int d\xi\,\frac{f(\xi)}{E_m-E_{m''}-\omega+\xi}\ . \tag{40}$$

By comparing this to Eq. (40) of Ref. 10, we find it agrees with the second term found by Cornut and Coqblin when one sets their impurity potential V_{mm} equal to zero and when one recognizes that to lowest order in $1/N_f$ the term with $m=m'=m''$ does not contribute to τ^{-1}.

We have shown for the infinite-U Anderson model that the relaxation time and thus the conductivity, at least through third order in the coupling J, calculated on the basis of Eq. (5) is identical to that found by using Eq. (2).[10] Whereas the first is derived on the basis of the Kubo formalism, the latter comes from a Boltzmann-equation approach. The relaxation time, Eqs. (5) and (29), for the Anderson model can be written as in Ref. 2 as

$$\tau^{-1}=-2c_i\sum_m w_m\,\mathrm{Im}\langle \mathbf{k}\sigma,m\,|\,t\,|\,\mathbf{k}\sigma,m\rangle\ . \tag{41}$$

On comparing this expression with Eq. (2), used by Cornut and Coqblin, it is remarkable how such a simple form, which does not explicitly take note of inelastic processes, reproduces the results of the more cumbersome looking expression. The key observation is that the "t matrix" in Eq. (41) represents a temperature ensemble average over the states of the system; it does not have the conventional meaning as the expectation value of the transition operator

$$T=V+VGV \tag{42}$$

in one state $|\,\mathbf{k}\sigma,m\rangle$.

The optical theorem as conventionally used in quantum mechanics[11] refers to expectation values of the transition operator and not to thermal averages. Therefore it is *not* possible to write, at finite temperatures,[12]

$$\mathrm{Im}\langle \mathbf{k}\sigma,m\,|\,t\,|\,\mathbf{k}\sigma,m\rangle=-\pi\sum_{\sigma',m'}\int\frac{d\mathbf{k}'}{(2\pi)^3}\,|\langle \mathbf{k}'\sigma',m'\,|\,t\,|\,\mathbf{k}\sigma,m\rangle|^2\delta(\varepsilon_{\mathbf{k}}-E_m-\varepsilon_{\mathbf{k}'}+E_{m'})\ . \tag{43}$$

As we have shown, when one properly interprets these t matrices as temperature-dependent propagators or effective interactions, see Eqs. (15) and (25), the right-hand side of Eq. (43) contains the inelastic-scattering factor $R(mm',\varepsilon_k)$, Eq. (3). Thus the sum over the intermediate states on the right-hand side of Eq. (43) contains this factor which does not appear in the optical theorem as used in quantum-mechanical scattering theory.[11]

In summary, we have derived the cumbersome expression for the relaxation time found by using the Boltzmann-equation approach from the compact expression found from the Kubo formula. The transcription made use of a generalization of the optical theorem to finite-temperature propagators, Eq. (21). Also we have calculated the effects of crystal fields on the conductivity of the infinite-U Anderson model in the limit of large degeneracy N_f by using the Kubo formalism. To compare our results with the weak-coupling calculation of Cornut and Coqblin we have expanded our results to sixth order in the mixing parameter $V_{\mathbf{k}\sigma,m}$, i.e., to third order in the effective exchange parameter J.

We would like to thank Professor Albert Fert, Professor Peter Riseborough, and Professor Leonard Rosenberg for very useful discussions and Professor S. Maekawa for his correspondence. This project was supported by a New York University Research Challenge Fund Grant.

---

[1] S. Maekawa, S. Takahashi, S. Kashiba, and M. Tachiki, J. Appl. Phys. **57**, 3169 (1985); J. Phys. Soc. Jpn. **54**, 1955 (1985); S. Maekawa, S. Kashiba, S. Takahashi, and M. Tachiki, J. Magn. Magn. Mater. **52**, 149 (1985); **54-57**, 355 (1986); *Solid State Science Series* (Springer, Berlin, 1985).

[2] S. Kashiba, S. Maekawa, S. Takahashi, and M. Tachiki, J. Phys. Soc. Jpn. **55**, 1379 (1986), see Eq. (2.3).

[3] T. Van Peski-Tinbergen and A. J. Dekker, Physica (Utrecht) **29**, 917 (1963).

[4] See, for example, G. D. Mahan, *Many-Particle Physics* (Plenum, New York, 1981), pp. 185–203 and 598.

[5] P. Fulde and I. Peschel, Adv. Phys. **21**, 1 (1972).

[6] P. Coleman, Phys. Rev. B **29**, 3035 (1984). Note that the states $|\,m\rangle$ in Eq. (6) are eigenstates of the crystal field. Therefore they are linear combinations of states $|\,jm_j\rangle$, which represent components of the angular momentum. For a very recent review, see N. E. Bickers, Rev. Mod. Phys. **59**, 845 (1987).

[7] See Ref. 2, and N. E. Bickers, D. L. Cox, and J. W. Wilkins, Phys. Rev. B **36**, 2036 (1987), Appendix D.

[8] D. L. Cox, Ph.D. thesis, Cornell University, 1985; Bickers, Cox, and Wilkins, Ref. 7.

[9] We are interested in the boson spectrum $B(\omega)$ about $\omega=E_m$. As $E_m \ll 0$ we are not concerned with the pole about $\omega=0$; therefore it is not necessary to introduce an infinitesimal

imaginary term in $D^0(\omega)$. However, in other contexts if one is interested in the pole about $\omega=0$, one has to introduce an infinitesimal imaginary term. Then Eq. (21) contains an additional term, see J. Carew and L. Rosenberg, Phys. Rev. D **7**, 1113 (1973).

[10] B. Cornut and B. Coqblin, Phys. Rev. B **5**, 4541 (1972).

[11] See, for example, L. Schiff, *Quantum Mechanics,* 3rd ed. (McGraw-Hill, New York, 1968) pp. 135–137.

[12] See Kashiba, Maekawa, Takahashi, and Tachiki, Ref. 2. We hasten to add that if the scattering is elastic, Eq. (43) does follow from the optical theorem.