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Resistance fluctuations in multiprobe microstructures: Length dependence and nonlocality

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We study numerically the resistance fluctuations, δR, in a disordered metallic microstructure with several probes within a single quantum-coherent region. The strong influence of the probes causes δR to depend on the geometry of the coherent region. In a particular four-probe structure, δR is only weakly dependent on the voltage probe separation, in agreement with experiment, while the strong dependence of δR on separation in other structures should be experimentally observable. Nonlocal fluctuations decay slowly as the distance between the current path and voltage probes increases.

Quantum-mechanical coherence in disordered metals and the associated interference phenomena are at the heart of many topics which have generated intense interest in the last decade, including localization, resonant tunneling, and universal conductance fluctuations. In all of these phenomena, the spatial extent of the quantum coherence plays a crucial role; in disordered metallic systems (nonmagnetic), this phase-coherence length, L_ϕ, is simply the distance a particle diffuses elastically before scattering inelastically, $L_\phi = (D\tau_{in})^{1/2}$. Quantum coherent phenomena in metals have usually been treated in systems which are considerably larger than a single coherent region. Recently, however, the voltage fluctuations of a single quantum-coherent region were probed in devices with several voltage leads attached within L_ϕ.[1,2] The strikingly different behavior seen in these experiments raised new questions about our understanding of quantum coherence in disordered metals. Further, the interpretation of these experiments required including the effects of the probes on the "sample," thus bringing transport measurements closer towards confronting the traditional problems of quantum measurement theory.

Fluctuations in the conductance as a function of magnetic field or Fermi energy have been observed in a variety of systems.[1-3] These fluctuations result from the interference of diffusing particles such that scattering from particular impurity configurations rather than ensemble-averaged behavior is seen.[3-6] For a disordered region with two ordered probes attached, the conductance fluctuation measured at length L_ϕ is of order e^2/h and is independent of the degree of disorder and the value of L_ϕ,[4-6] implying that the resistance fluctuation δR behaves as $\delta R \propto R^2$. In contrast, the most recent experiments have shown that for voltage probes spaced closer than L_ϕ, δR (not δG) is nearly independent of the spacing between the probes and hence of the average resistance R.[1,2] Because the two-probe theories, such as those used to understand conductance fluctuations at length scale L_ϕ, necessarily fix $L \geq L_\phi$, any explanation of the length independence of the resistance fluctuations must include the four-probe nature of the experiment.

In this paper, we present the results of microscopic, numerical calculations of the resistance in multiprobe structures where the probes have finite width and are strongly coupled to the sample. As noted above, the multiprobe nature of our calculation is essential for any meaningful comparison with experiment for $L < L_\phi$. Previous theoretical treatments of this problem have been based on phenomenological arguments and the Onsager relations,[1] or qualitative arguments using a generalized Landauer formula.[7] Very recently, a model similar to the one we study numerically has been solved in perturbation theory for certain quasi-one-dimensional geometries.[8-10]

The structures that we consider, shown as insets in the figures, have a finite disordered region connected to several ordered leads and are described by a nearest-neighbor tight-binding Hamiltonian[4,11] with diagonal disorder in the disordered regions. The current in lead k, I_k, is the sum of the pairwise currents between that lead and each of the others. The pairwise current is simply the probability for transmitting intensity between two leads k and l at the Fermi energy, T_{kl}, times the chemical potential difference, $V_l - V_k$:[12]

$$I_k = -(e^2/h)\sum_{l\neq k}^{n} T_{kl}(V_l - V_k) \ . \tag{1}$$

A derivation of Eq. (1) from the Kubo formula[13,14] shows that the transmission coefficients are simply related to the retarded Green's function between the two ordered leads.[4,14] We find the Green's functions needed by using the recursive Green's-function technique[4,11] which must be generalized to calculate the Green's functions between all interfaces in our multiprobe structure.[14] Such calculations have not been done previously and, surprisingly, require the same order of steps as the two-probe calculations.[14]

We solve Eq. (1)[7,12] with the constraints that the current is fed in lead 1 and taken out lead 3 with the current in all other leads equal to zero.[14] By ensemble averaging the resistance $R_{lk}=(V_l-V_k)/I$ over disorder configurations for fixed geometries, we find the rms fluctuations δR_{lk}.[15] A magnetic field is not included since in that case the current depends on Green's functions between points within the disordered region,[9,14] a computationally more difficult problem.

Our basic results are first, that the transmission fluctuations do not depend on either the absolute size scale or the value of disorder, a generalization of the universality found in the diffusive regime.[5,6] Second, we do find that the fluctuations are sensitive to the overall geometry, that is, the shape and lead configuration.

The geometry dependence comes from the presence of "inelastic scattering" which is included in our calculation through the reservoirs at the end of each perfect lead which are implicit in both the Kubo formalism for a finite region and Landauer-type formulas such as Eq. (1). Because the leads introduce the inelastic scatters,[16] the amount or character of inelastic scattering is determined by the geometry. To see that a perfect lead reduces coherence, consider the resistance of the main channel in a three-probe situation (Fig. 1). If the perfect lead attached to probe 2 were disconnected, then $R_{13}^{-1}\propto T_{13}$ where, in the Feynmann path language, T_{13} is the modulus squared of the sum of the amplitudes over all paths from probe 1 to probe 3. With probe 2 present, however, Eq. (1) yields $R_{13}^{-1}=(e^2/h)\ [T_{13}+T_{12}T_{23}/(T_{21}+T_{23})]$.[7] Here, T_{13} is the intensity derived from paths which do not touch the perfect lead while the second term corresponds to paths which do touch probe 2. Thus, the amplitudes for paths which touch probe 2 are added incoherently to those which do not touch probe 2 in contrast to the two-probe case. This "inelastic scattering" introduced by a perfect lead is particularly clearly demonstrated by adding evenly spaced perfect leads to a long main channel. The side probes divide the system into separate quantum-coherent regions so that the distance between them is approximately the phase-coherence length. The good agreement between the resistance fluctuations of the main channel (R_{13}) for y coherent regions (inset of Fig. 2) and the expected (Ref. 17) $\delta R_{13}\propto y^{1/2}$ from adding uncorrelated random variable supports the view that perfect leads act as inelastic scattering centers.[18]

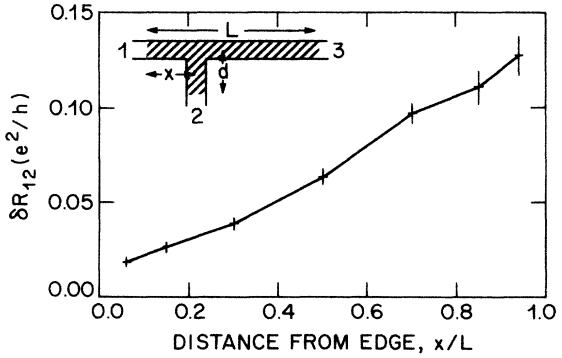

FIG. 1. The rms fluctuation of R_{12} as a function of x/L for the structure shown (shaded regions are disordered). δR_{12} depends strongly on the placement of probe 2 ($d=L/2=100$, solid line).

In contrast to inelastic processes in real homogeneous wires, our inelastic processes occur at fixed, definite places in space. The correspondence between our calculation and the resistance-fluctuation experiments which have been done[1,2] can only be made for particular geometries in which our abrupt inelastic processes mimic those in the real system; we will discuss these geometries below. However, our calculation does correspond to an experiment that could be done: by selectively adding magnetic impurities to some of the wires as has been done for studying localization,[19] regions with very small inelastic lengths could be defined which would behave as the perfect leads in our samples. The rich variety of behavior that we find suggests that such an experiment would be interesting.

We first consider a three-probe conductor and ask whether the fluctuation of the voltage on the third probe depends on where the third probe is attached. In the terminology of Fig. 1, δR_{12} would be independent of x either if every voltage probe attached to this quantum area defines a local chemical potential which fluctuates independently,[2] or if previously suggested general arguments about transmission coefficients are valid.[7] In fact, we did not find length-independent resistance fluctuations in any three-probe geometry, presumably because of the

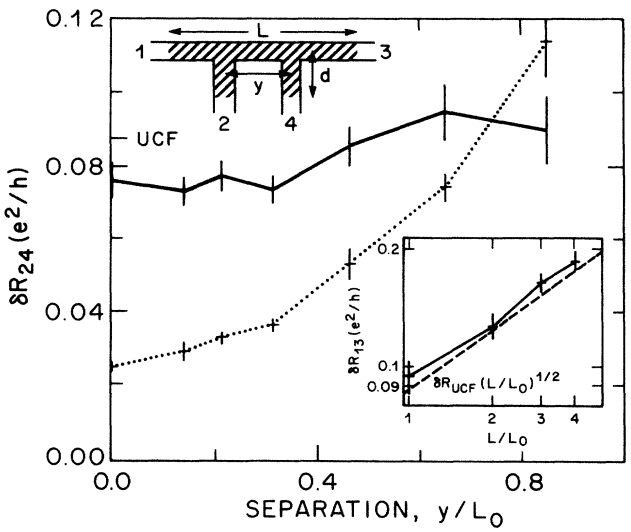

FIG. 2. The rms fluctuation of R_{24} as a function of the separation between the voltage probes, y/L_0, for the structure in the inset (shaded regions are disordered, $L=800$). $L_0=2d+y$ is approximately the phase-breaking length. When L_0 is fixed (solid line, $L_0=160$) the fluctuation is independent of the separation of the voltage probes while when d is fixed (dotted line, $d=10$), the fluctuation depends on y. In the length-independent case, δR_{24} is approximately the universal two-probe value (indicated by UCF). The inset shows δR_{13} on a log-log scale when there are perfect probes attached to the channel between 1 and 3 (in these cases, y is the distance between probes 1 and 3 and $L_0=165\approx L_\phi$ is the distance between adjacent probes). For $y>L_0$, $\delta R_{13}\approx y^{1/2}\delta R_{\rm UCF}$.

length-dependent "inelastic scattering" introduced by the third lead. For the case in Fig. 1, the approximately linear dependence of δR_{12} on x implies a decrease in the fluctuations of the naive conductance, $G=1/R$.[20] (R_{12} itself varies linearly with x.) The first major result, then, is that the resistance fluctuations in multiprobe phase-coherent structures are not, in general, independent of geometry.

Turning now to a four-probe structure (Fig. 2), we wish to consider a structure in which the inelastic scattering is fixed in order to compare to experiment. With a long main channel and short side probes near the center, only the distance between the ordered region of the side probes (2 and 4) is important for a diffusing particle near the center of the structure: the space-filling property of two-dimensional diffusion implies that the particles are likely to "see" the side probes well before they approach the ends of the main channel. Thus, if one fixes the length between the ordered regions of the side probes, there is a single fixed length controlling the diffusion near the center of the main channel and one would expect to find behavior similar to that in the experimental system. The resistance fluctuations that we calculate for this case ($L_0 \equiv y+2d$ fixed), shown as the solid line in Fig. 2, are indeed only weakly dependent on the separation between the probes which is consistent with experiment[1,2] but shows somewhat less length dependence than the perturbative calculations.[8-10] In contrast, for a fixed length of disorder in each side probe (dotted line), the changing distance between the ordered part of the voltage probes leads to length dependence of the fluctuations, as in the three-probe case. Similarly, use of a shorter main channel leads to greater length dependence: a main channel half as long produces $\delta R_{24}(y=0)/\delta R_{24}(y=L_0) \approx \frac{1}{2}$.

In addition to the length independence of the resistance fluctuation, the magnitude of the fluctuation agrees with experiment.[1,2] By using the resistance (R_0) of a length L_0 of material as an estimate for the resistance at length scale L_ϕ, we convert the universal two-probe result for the conductance fluctuation[5,6] (denoted UCF) to a resistance fluctuation, $\delta R_{\rm UCF} = \delta G_{\rm UCF} R_0^2$, and obtain very good agreement with our four-probe resistance fluctuation. The magnitude of the fluctuation is not sensitive to the length L because the main channel is localized, in contrast to the $L^{1/2}$ increase in the fluctuation for a metallic main channel as derived diagrammatically.[9]

We also studied the nonlocal aspects of the quantum fluctuations[1,2,21] in the structure shown in Fig. 3 where

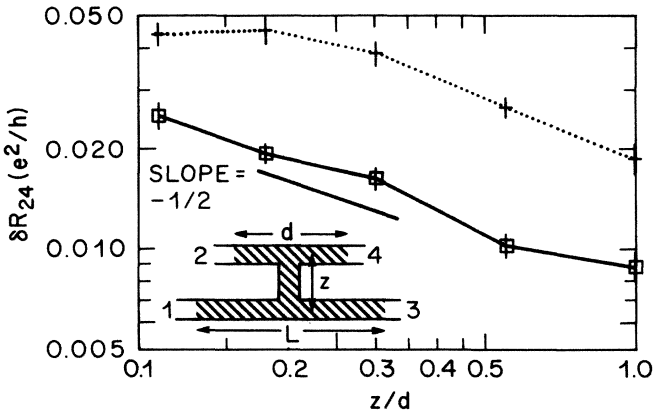

FIG. 3. The rms fluctuation of R_{24} as a function of the distance between the current and voltage paths, z, for the structure shown (shaded regions are disordered). The decay of these nonlocal fluctuations with z is exceedingly weak ($z^{-1/2}$)[($d=180$, $L=180$) solid line, (180,420) dotted line].

the current path (probe 1 to probe 3) and the path between the voltage probes (probe 2 to probe 4) do not intersect. As the separation, z, between the current and voltage paths increases, the fluctuations decay very slowly ($z^{-1/2}$), in contrast to the exponential decay expected on length scales larger than L_ϕ.

In summary, we find that the addition of a lead causes a substantial perturbation to the system, as indicated by experiments. However, we find a far greater variety of length dependences, $\delta R(L)$, than the experiments because of geometrical effects present in our calculation which while not present in the experiments done[1,2] correspond to experiments which could be done. We do obtain nearly length-independent resistance fluctuations for $0 < L < L_\phi$ in a *particular* geometry while in most geometries δR depends on length. The experiments suggested a simple interpretation of length independence in terms of independent chemical potential fluctuations; the variety of behavior that we see implies that such an explanation is not generally correct. Finally, we explicitly verify the nonlocal nature of the resistance fluctuations and find they decay very slowly with the distance away from the current path.

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