Theory of hot-electron energy loss in polar semiconductors: Role of plasmon-phonon coupling

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The theory for hot-electron relaxation in bulk III-V semiconductors is developed within a many-body formalism by considering energy loss to optical and acoustic phonons, with special emphasis on the influence of the plasmon-phonon coupling phenomenon. The effects of quantum statistics, screening, hot phonons, and acoustic phonons are quantified in order to assess their relative physical importance. We find that for low-electron-density samples there is a temperature range in which it is essential to include plasmon-phonon coupling in the theory, and predict how this effect will manifest itself experimentally. Our theoretical results are in good agreement with the available experimental data in GaAs and $Ga_xIn_{1-x}As$.

I. INTRODUCTION

Recently a great deal of experimental[1-4] and theoretical[5-7] attention has been focused on the problem of hot-electron relaxation in semiconductor systems. In spite of this substantial interest in the subject no overall clear quantitative picture of the various mechanisms affecting carrier power loss in a semiconductor has really emerged. In GaAs quantum well experiments, using steady-state field-induced techniques, different authors[1,2] show qualitative agreement only in a limited temperature region, and there the power loss disagrees by almost an order of magnitude. In bulk GaAs,[3,4] steady-state and picosecond time-resolved measurements give relaxation times which are factors of 2–20 slower than the simple theoretical result. No quantitative resolution of these disagreements has been provided, either in three or in two dimensions, even though various mechanisms such as degeneracy, screening, and the hot-phonon effect have often been invoked to assert that the relaxation rate should indeed go down, consistent with the experimental findings.[5-7] Understanding of hot-electron experiments is very important since they constitute a direct probe of a very fundamental interaction in solid-state physics, namely the electron-phonon interaction. The newly developed steady-state electric-field-induced techniques provide a very adequate tool since the electronic measurement of the power input P to the electron system, together with the optical determination of the electron temperature T, give a direct determination of the power loss of hot electrons as a function of temperature for a fixed electron density n. These kinds of experiments have so far been performed only in quasi-two-dimensional systems.[1,2]

In order to understand the behavior of hot electrons in semiconductor systems we studied the steady-state power loss to optical and acoustic phonons in bulk GaAs, $Ga_xIn_{1-x}As$, and GaAs quantum wells. In this paper we develop the bulk three-dimensional (3D) analysis while the two-dimensional (2D) results will be published elsewhere.[8]

Throughout this work we will assume that the hot-electron gas is in equilibrium with itself so that it may be described by a single temperature T (which we assume to be large compared to the lattice temperature T_L). This assumption is supported by the profile of the luminescence spectrum and is a common practice in this field. The lattice is assumed to be in equilibrium with a heat reservoir so that the acoustic phonons, produced by the relaxation of LO phonons or of hot electrons, do not raise the lattice temperature. We will also assume that the LO-phonon dispersion and the nonparabolicity of the electronic bands can be neglected.

The simplest approach to the problem is to consider a nondegenerate electron gas at a temperature T interacting with a lattice at zero temperature. We find that this simple model gives an intuitive insight into the physics of this problem and forms a basis for understanding the more complicated models that we will come to later. Although the expression of power loss within this model is well known,[4,9] we include here a brief derivation for the sake of completeness. Ignoring phonon-reabsorption processes, the power loss of electrons with energy E to LO phonons is given by the golden rule:

$$\frac{dE}{dt}=\frac{2\pi}{\hbar}\sum_q \hbar\omega_{\mathrm{LO}} M_q^2 \delta(E_{\mathbf{k}-\mathbf{q},f}-E_{\mathbf{k},i}+\hbar\omega_{\mathrm{LO}}) , \quad (1)$$

where the matrix element M_q is given by

$$M_q^2=\frac{4\pi e^2}{q^2}\frac{\hbar\omega_{\mathrm{LO}}}{2}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right]=\frac{4\pi\alpha(\hbar\omega_{\mathrm{LO}})^{3/2}}{(2m)^{1/2}}\frac{1}{q^2} , \quad (2)$$

$\hbar\omega_{\mathrm{LO}}$ is the LO-phonon energy, ϵ_∞ and ϵ_0 are the optical and static dielectric constants of the semiconductor, m is the electron effective mass, α is the dimensionless coupling constant ("Fröhlich coupling"), and i and f denote, respectively, the initial and final electronic states. Summing over all phonon wave vectors q we have

$$\frac{dE}{dt}=2\frac{(\hbar\omega_{\mathrm{LO}})^{5/2}\alpha}{\hbar}\frac{1}{\sqrt{E}}\sinh^{-1}\left[\left(\frac{E}{\hbar\omega_{\mathrm{LO}}}-1\right)^{1/2}\right] , \quad (3)$$

for unit volume. If the electrons have a classical

Maxwell-Boltzmann distribution, the power loss per electron (P) is given by

$$P=\frac{4(\hbar\omega_{\rm LO})^{5/2}\alpha\beta^{3/2}}{\sqrt{\pi}\hbar}\times\int_{\hbar\omega_{\rm LO}}^{\infty}dE\,\sinh^{-1}\left[\left[\frac{E}{\hbar\omega_{\rm LO}}-1\right]^{1/2}\right]e^{-\beta E}\;, \tag{4}$$

where $\beta=(k_BT)^{-1}$. If we consider only the temperatures for which $\beta\hbar\omega_{\rm LO}\gg1$, we can expand the integrand in powers of $1/\beta\hbar\omega_{\rm LO}$ and keep only the first term. This leads to the "classical" expression for the power loss:

$$P\simeq\frac{\hbar\omega_{\rm LO}}{\tau}e^{-\beta\hbar\omega_{\rm LO}}\;, \tag{5}$$

where the relaxation time τ is given by

$$\frac{1}{\tau}=2\alpha\omega_{\rm LO}\;. \tag{6}$$

For GaAs $\alpha=0.07$ and the numerical value of this theoretical relaxation time is 0.13 ps. Equation (5) is the limiting form of the well-known formula for power loss due to Stratton for zero lattice temperature and $\beta\hbar\omega_{\rm LO}\gg1$.[4,9] It predicts a linear relationship between $1/T$ and $\ln(P)$ which is observed experimentally in a certain range of electron temperatures (between 50 and 150 K). However, the measured relaxation times show departures from (6), suggesting that (5) should be improved by including quantum effects. We discuss this issue in Sec. II.

Formula (5) is simple enough to give us a qualitative insight into the importance of the phenomenon of plasmon-phonon coupling in this context. In the presence of free electrons the LO-phonon frequency is renormalized due to plasmon-phonon coupling, giving rise to two coupled modes with frequencies ω_+ and ω_- which lie close to the uncoupled modes away from the mode-crossing region. Thus there are two phonons at ω_+ and ω_- as the phonon spectral function has peaks at these two frequencies. We call the phonon with the frequency close to $\omega_{\rm LO}$ the phononlike mode, and the phonon with the frequency close to ω_p the plasmonlike mode. We now show that even though the phonon spectral weight in the plasmonlike mode is small, its presence may give rise to a significant modification of the power loss. The electronic power is now dissipated into two channels and (5) can be modified to

$$P=\hbar\omega_+C_+e^{-\beta\hbar\omega_+}+\hbar\omega_-C_-e^{-\beta\hbar\omega_-}\;, \tag{7}$$

where the factors C_+ and C_- depend on the phonon spectral weight at the two frequencies. For high densities (when the plasmon energy exceeds the phonon energy), $\omega_+\simeq\omega_p$ and $\omega_-\simeq\omega_{\rm LO}$. Since the phonon spectral weight is dominant in the ω_- mode ($C_+\ll C_-$), most of the power is dissipated into the minus (phononlike) mode and the inclusion of the mode coupling does not alter our previous results. In contrast, for low densities (when $\omega_p<\omega_{\rm LO}$) $\omega_+\simeq\omega_{\rm LO}$ and $\omega_-\simeq\omega_p$. Now the phonon spectral weight is mainly at ω_+, and $C_-\ll C_+$. Thus the prefactors favor the phononlike mode, but the exponentials favor the plasmonlike mode. At high temperatures the exponentials are not very different and the power is dissipated mainly into the ω_+ mode. However, below a certain temperature, the exponential term dictates the power loss and ω_- becomes the dominant channel. Thus experimentally one would expect to see that the slope of $\ln(P)$ versus $(k_BT)^{-1}$ would switch from $\hbar\omega_+$ to $\hbar\omega_-$ as the temperature is decreased.

This effect can be seen experimentally only if the change from the phononlike mode to the plasmonlike mode takes place in the temperature regime at which the power loss to acoustic phonons may be ignored. At extremely low temperatures, the acoustic phonons always dominate the power-dissipation process. In order to determine the density and temperature ranges over which inclusion of the mode coupling is important, we have calculated the power loss to optical as well as to acoustic phonons. In Sec. II we study the corrections to (5) due to the quantum many-body effects. Then, we critically compare different levels of theoretical approximations so as to assess the importance of various physical effects. In Sec. III we discuss the effect of the mode-coupling phenomenon. In Sec. IV we calculate the total power loss that is expected for steady-state field-induced experiments for GaAs and $\mathrm{Ga}_x\mathrm{In}_{1-x}\mathrm{As}$ and carry out a comparison with the existing experimental data. We also predict the experimental conditions which would enable one to see the mode-coupling effects on the hot-electron energy-loss process clearly.

II. STATISTICS, SCREENING, AND HOT-PHONON EFFECTS

In order to include the quantum many-body effects, we assume that the electronic response of the electron gas is described by the standard finite-temperature random-phase approximation (RPA). Then, Eq. (5) is modified to read[8,10]

$$P=\frac{1}{N}\sum_q M_q^2\int\frac{d\omega}{\pi}\hbar\omega[n_{T_L}(\omega)-n_T(\omega)]\times{\rm Im}\chi(q,\omega){\rm Im}D(q,\omega)\;, \tag{8}$$

where $n_T(\omega)$ and $n_{T_L}(\omega)$ are the Bose occupancy factors at the electron and lattice temperatures and N is the total number of electrons. Under the usual experimental conditions ($T_L\lesssim10$ K, $T>25$ K) the dependence on T_L is irrelevant, and we will take $n_{T_L}(\omega)=0$. M_q is the bare ("Fröhlich") electron-phonon coupling strength given by (2). $\chi(q,\omega)=\chi_0(q,\omega)/\epsilon(q,\omega)$ is the finite-temperature-reducible RPA polarizability of the electron gas, where χ_0 is the irreducible Lindhard polarizability and ϵ is the dielectric function. D is the interacting LO-phonon propagator. Notice that the δ function in Eq. (1) follows from ImD in Eq. (8).

Let us first ignore the plasmon-phonon mode coupling so that

$$D(q,\omega)=D^0(q,\omega)=\frac{2\omega_q}{\hbar(\omega^2-\omega_q^2)},\quad \omega_q=\omega_{\rm LO} \tag{9}$$

and Eq. (8) reduces to

$$P=\sum_q \hbar\omega_{\rm LO}R_q n(\omega_{\rm LO}) , \tag{10}$$

where

$$R_q=-2\hbar^{-1}M_q^2{\rm Im}\chi(q,\omega_{\rm LO}) \tag{11}$$

is the scattering rate. (This has the correct units of s^{-1} as M_q^2 has the units of energy.)

In order to quantify the importance of various physical effects, we calculate the power loss using different levels of theoretical approximation. In Fig. 1 we show our results for two values of the carrier density ($n=10^{18}$ and 10^{17} cm^{-3}) and for temperatures between 50 and 150 K. In each plot, power loss per carrier is given as a function of the inverse of the electron temperature. As a reference we also give the classical results [Eq. (5)] denoted by curve 1. We have adopted the following parameters, appropriate for GaAs: $m=0.07m_e$, $\hbar\omega_{\rm LO}=36.8$ meV, $\epsilon_\infty=10.91$, $\epsilon_0=12.91$.

Neglecting the screening of the electron gas we set $\chi(q,\omega_{\rm LO})\simeq\chi_0(q,\omega_{\rm LO})$ in formula (11) and obtain curves labeled 2. Comparing with curves 1, we see that the effect of quantum statistics is non-negligible, except for low densities, and reduces the power loss because it introduces a restriction in phase space for the electron scattering. Obviously, the inclusion of static screening $[\chi(q,\omega_{\rm LO})\simeq\chi_0(q,\omega_{\rm LO})/\epsilon(q,0)]$ provides a further reduction of the power loss (curves 3). In the dynamically screened case (curves 4) we take into account the ω dependence of the dielectric function ϵ and obtain results rather close to the unscreened values at low densities, and very close to the statically screened values at high densities. At low densities [Fig. 1(b)] and low temperatures, the dynamically screened result for power loss is actually slightly in excess of the unscreened results (the so-called "antiscreening" effect), but the effect is too small to be seen in the logarithmic plot of the figure. Also, in light of the present work, its relevance becomes questionable, as it works in the same direction as the quantitatively much more pronounced phonon self-energy effect (which we discuss below).

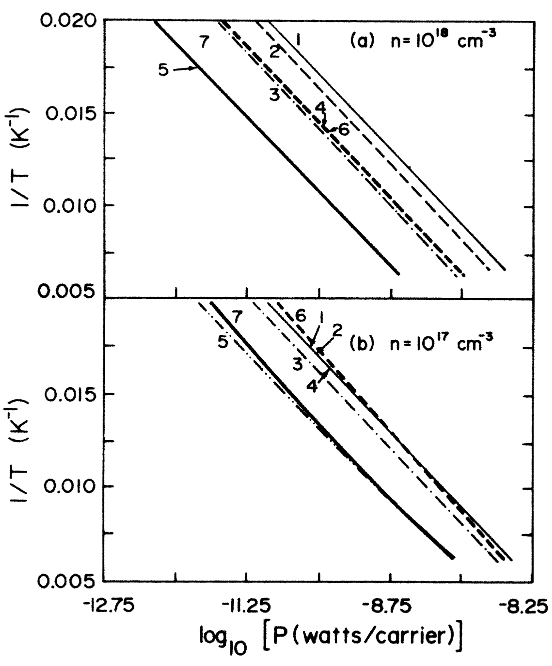

FIG. 1. Power loss by hot electrons to LO phonons in GaAs as a function of the inverse of the carrier temperature T for a number of different approximations: (1) classical statistics, (2) quantum statistics without screening, (3) static screening, (4) dynamical screening, (5) dynamical screening including hot-phonon effects, (6) dynamical screening including plasmon-phonon coupling, and (7) dynamical screening including both plasmon-phonon coupling and hot-phonon effects. Results for two densities, (a) 10^{18} and (b) 10^{17} cm^{-3}, are shown.

So far we have assumed that the LO phonons decay instantaneously into acoustic phonons. This is, strictly speaking, incorrect, since the LO phonons have a finite lifetime, and in steady-state experiments there is a finite number of phonons present in the system. This allows the possibility of phonon reabsorption, which, in certain situations, translates into a considerable reduction of the power loss. This is termed the hot-phonon effect.[5] We will assume that the thermal relaxation of the LO phonons is completely described by the phonon lifetime $\tau_{\rm ph}$ whose measured value for bulk GaAs is 7 ps. Following the usual procedure,[2] if the phonon occupation number is N_q, then

$$\frac{dN_q}{dt}=R_{\rm em}(N_q+1)-R_{\rm abs}N_q-\frac{N_q}{\tau_{\rm ph}} , \tag{12}$$

which must be zero in the steady state. Here $R_{\rm em}$ and $R_{\rm abs}$ are the emission and absorption scattering rates, respectively, which are related to R_q according to $R_{\rm em}=n_T(\omega)R_q$ (fluctuation-dissipation theorem) and $R_{\rm abs}-R_{\rm em}=R_q$ (principle of detailed balance). The dissipated power, i.e., $\sum_q\hbar\omega_q N_q/\tau_{\rm ph}$, is then given by

$$P=\sum_q \hbar\omega_{\rm LO}\frac{R_q}{1+\tau_{\rm ph}R_q}n(\omega_{\rm LO}) , \tag{13}$$

which is identical to (10) for vanishing phonon lifetime ($\tau_{\rm ph}=0$). The inclusion of hot-phonon effects (curves 5 in Fig. 1) gives a further reduction of the power loss with respect to the dynamically screened case, as is also clear from Eq. (13).

Thus we see that the inclusion of quantum statistics, screening, and hot-phonon effects reduces the power loss but conserves the approximate linearity of the plot. Thus the relaxation time τ still has a well-defined (although weakly-temperature-dependent) value, which is no longer given by (6) and must be calculated from the full theory. To summarize our results of this section we plot, in Fig. 2, the numerically obtained relaxation time τ for various densities using different theoretical approximations. From this plot we see that to neglect screening is a good

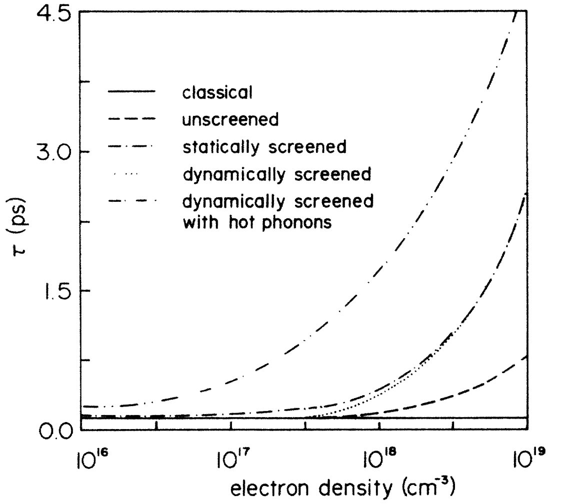

FIG. 2. Relaxation time of hot electrons in the temperature range 50–150 K as a function of the carrier density n for a number of different approximations.

approximation up to densities of the order of 5×10^{17} cm^{-3}, while static screening becomes a good approximation above 10^{18} cm^{-3}. The crossover from the unscreened to the statically screened regimens occurs in a narrow region of densities around 7×10^{17} cm^{-3} when the plasma frequency is equal to the LO-phonon frequency.

An important conclusion that we extract from Fig. 2 is that the hot-phonon effect is sensitive to the electron density even for very low densities. Without including hot phonons, the low-density electron gas can be treated as nondegenerate and unscreened, which yields a density-independent result for power loss per carrier. However, after including the hot-phonon effect, the power loss per carrier depends on the density; the relaxation time goes up by a factor of 2 as the density changes from 10^{16} to 10^{17} cm^{-3}.

III. MODE-COUPLING EFFECTS

As we mentioned in the Introduction, the inclusion of the mode-coupling phenomenon gives rise to a second frequency in the problem and the results of the preceding section are considerably altered for low electron densities. In the language of many-body theory, we say that in the presence of the electron gas the phonon propagator is renormalized and has two poles at the coupled plasmon-phonon frequencies. We use the finite-temperature plasmon-pole approximation for χ in order to calculate the renormalized D (see the Appendix for details), given by

$$D(q,\omega)\simeq\frac{2\omega_{\rm LO}(\omega^2-\tilde{\omega}_p^2)}{\hbar(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)}\ , \qquad (14)$$

where $\tilde{\omega}_p$ is the effective plasma frequency and $\omega_\pm$ are the frequencies of the two phonon branches, i.e., the phonon spectral function $[-(1/\pi){\rm Im}D]$ has two δ-function peaks at $\omega_\pm$. (We emphasize that we use plasmon-pole approximation only in calculating the coupled modes.) Therefore, Eq. (8) reduces to

$$P=\sum_q[\hbar\omega_+R_q^+n_T(\omega_+)+\hbar\omega_-R_q^-n_T(\omega_-)]\ , \qquad (15)$$

$$R_q^\pm=\frac{\omega_{\rm LO}}{\omega_\pm}\left[\frac{|\omega_\pm^2-\tilde{\omega}_p^2|}{\omega_+^2-\omega_-^2}\right][-2\hbar^{-1}M_q^2{\rm Im}\chi(q,\omega_\pm)]\ . \qquad (16)$$

We include the effects of Landau damping by using the bare phonon propagator in the Landau-damped region (where the plasmon is not a well-defined RPA mode).

Thus, due to phonon renormalization, the power is lost to two phonon modes instead of one, as was the case with bare phonons. This is reflected by the presence of two terms in Eq. (15). The physics of each of these terms is essentially the same as that of (7) since quantum statistics and screening merely provide an overall renormalization of τ. In order to illustrate this point we plot in Fig. 3 the power dissipated into each mode, as well as the total power, for a high density (10^{18} cm^{-3}) and a low density (10^{17} cm^{-3}). Each mode shows its characteristic energy by the slope of $(k_BT)^{-1}$ versus $\ln(P)$. For the phononlike mode the slope is $\hbar\omega_{\rm LO}$, for the plasmonlike mode the slope is $\hbar\omega_p$ (in fact, slightly greater than $\hbar\omega_p$ since the plasma frequency has a slight dispersion in q).

As previously mentioned, for high densities the power dissipated into the plasmonlike mode is negligible and all the power is dissipated into the phononlike mode. The

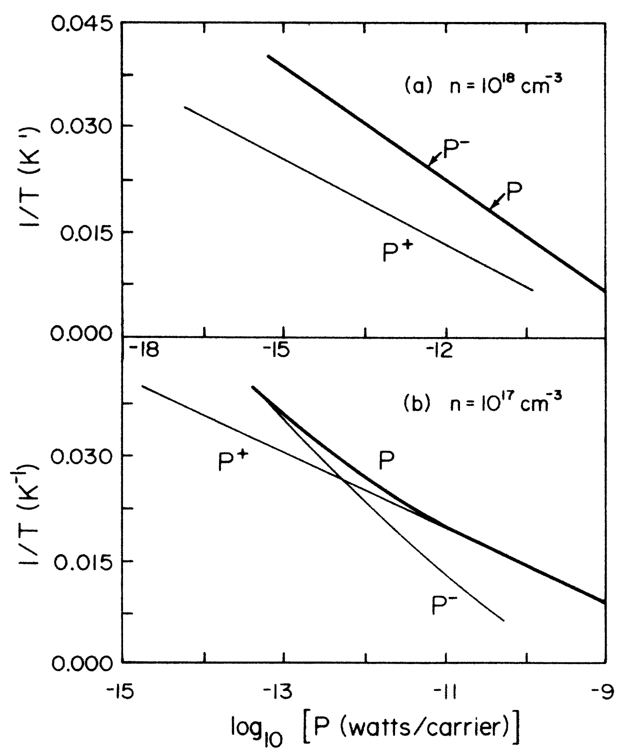

FIG. 3. Power loss by hot electrons to LO phonons, showing the contribution from each mode. Results are shown for two electron densities, (a) $n=10^{18}$ and (b) 10^{17} cm^{-3}.

mode coupling is therefore not important [Fig. 3(a)]. For low densities, however, we see in Fig. 3(b) a crossover from one mode to the other at about 40 K. Thus it is essential to keep the physics of mode coupling below this temperature for low densities.

Hot-phonon effects are included by replacing $R_q^{\pm}$ by $R_q^{\pm}(1+\tau_{ph}R_q^{\pm})^{-1}$ in (15), which produces, as before, an overall shift towards lower power losses. In principle, we should use a different lifetime for the plasmonlike mode, but for simplicity we take the same lifetime for both the modes. This is not crucial because we find that the dependence of the power loss on the phonon lifetime is not strong for $\tau_{ph} \gtrsim 1$ ps. In addition, for the low densities, for which the mode coupling is relevant, the hot-phonon effects are not very important.

IV. TOTAL POWER LOSS

In order to carry out a direct comparison with the experimental data, we must also include power loss to acoustic phonons. Since the coupling between plasmons and acoustic phonons is not important, the power loss of the hot electrons to the acoustic phonons is still given by (8), but with the frequency dispersion $\omega_q = cq$, and the matrix element[7,11]

$$M_q^2 \simeq \left[\frac{\hbar}{2\rho\omega_q}\right](q^2D^2+156e^2e_{14}^2/\epsilon_0^2) , \quad (17)$$

which includes both the deformation potential and the piezoelectric coupling to acoustic phonons. For GaAs, the deformation constant $D = 7$ eV, the sound velocity $c = 3.51\times10^5$ cm s^{-1}, the piezoelectric modulus[12] $e_{14} = 4.8\times10^4$ g$^{1/2}$ cm$^{-1/2}$ s^{-1}, and the sample density $\rho = 5.36$ g cm^{-3}.

For the electron densities that we are interested in, the Fermi velocity is much greater than the sound velocity c and the dynamically screened results are very close to the statically screened ones for the acoustic-phonon relaxation. In Fig. 4 we show separately the power loss per electron of the hot electrons to LO phonons (P_{LO}) and to acoustic phonons (P_{ac}), as well as the total power loss (P), for an electron density of 5×10^{16} cm^{-3}. It can be seen that the acoustic phonons dominate the dissipation process below 20 K, from 20 to 50 K the mode coupling is important, and above 50 K the only active channel is that of LO phonons. We also include in Fig. 4 measurements of a photoexcited experiment[4] where the photoexcited carrier density in the sample was estimated to be $(5\pm2)\times10^{16}$ cm^{-3} at the highest pump power. These experiments measure the pump intensity; the exact power given to the electron system is not known. We have shifted the horizontal axis (power per carrier) so as to have agreement between theory and experiment in the high-temperature LO-phonon-dominated region. Clearly the complete theory provides a quantitative agreement over the whole temperature range. The agreement becomes worse if the plasmon-phonon coupling is not included.

We must point out here that a comparison of our numerical results and the experimental data becomes meaningless if there are a sizable number of holes present in

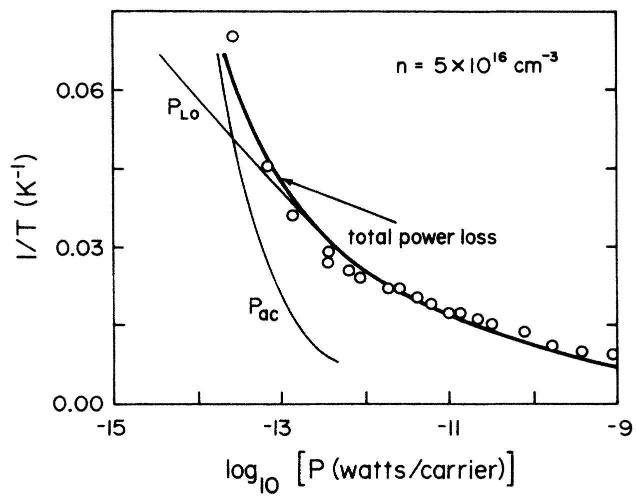

FIG. 4. Total power loss by hot electrons is shown, which is a sum of the power loss to LO phonons (P_{LO}) and the power loss to acoustic phonons (P_{ac}). Electron density $n = 5\times10^{16}$ cm^{-3}. The experimental results of Shah *et al.* (from Refs. 3 and 4) are also shown (see text for discussion).

the experiment, because then the holes, not included in our calculations, may dominate the power-loss process. Should this be the case in Fig. 4, the agreement between the theory and experiment is merely fortuitous, and even misleading, and must be disregarded. We, therefore, stress the need for steady-state-photoexcitation or electric-field-heating experiments on *n-doped* semiconductors to which the present work directly relates. The problem of power loss by holes can also be studied along similar lines, but we feel that it would be useful and appropriate to first understand fully the much simpler problem of power loss by hot electrons since the holes have much more complicated band structure and electron-phonon couplings.

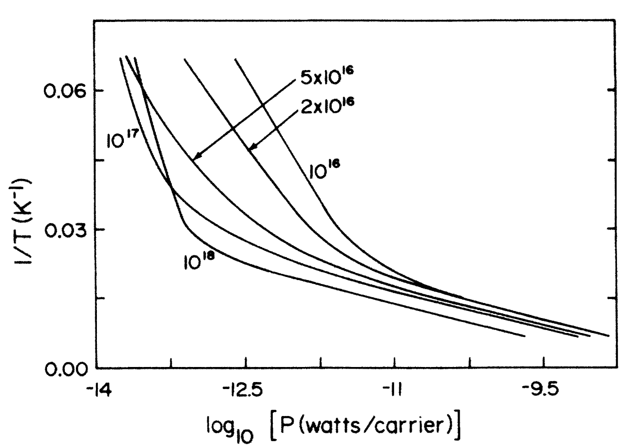

FIG. 5. Total power loss by hot electrons in GaAs for various carrier densities shown near the curves (in units of cm^{-3}).

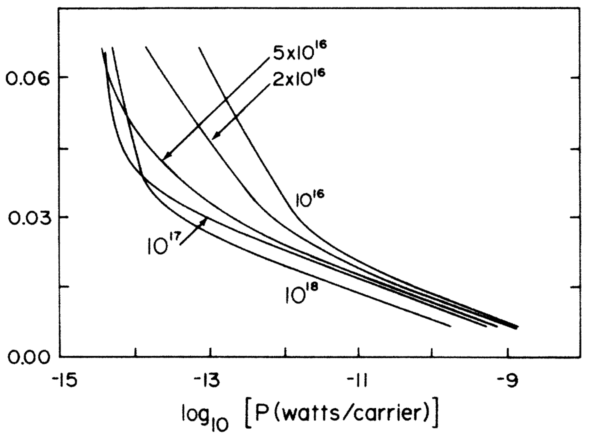

FIG. 6. Total power loss by hot electrons in $Ga_{0.47}In_{0.53}As$ for various carrier densities shown near the curves (in units of cm^{-3}).

There have been a number of suggestions in the literature[13] of the existence of "missing" energy relaxation mechanisms at low temperatures, since the calculations including only the acoustic and bare LO phonons could not account for the total observed power loss. We have provided here one such mechanism, namely, the plasmonlike phonon, which owes its existence to the phonon self-energy correction due to its coupling with the electron gas. To see what experiment would provide a clearer and unambiguous confirmation of this theory, in Fig. 5 we plot the total power loss for various densities. For high densities (10^{18} cm^{-3}) the mode coupling is not important and the transition from the ω_{LO} mode to the acoustic mode is abrupt. As we go to lower densities the effect of the mode coupling becomes progressively more important and the power loss to the ω^- mode can be seen in a wider range of temperatures. Thus we predict that for densities $n \lesssim 10^{16}$ cm^{-3} in GaAs, because of plasmon-phonon coupling, there will be a range of temperatures when the power loss will be given by $(\hbar\omega_-/\tau)\exp(-\beta\hbar\omega_-)$, with $\omega_- \simeq \omega_p$.

In Fig. 6 we present the result of the calculation done for bulk $Ga_{0.47}In_{0.53}As$ using the parameters of Ref. 14 with one effective phonon frequency of 32.7 meV and assuming a phonon lifetime $\tau_{ph}=5$ ps. In agreement with Lobentanzer *et al.*,[15] we find similar relaxation times as in bulk GaAs with the same electronic density. The power dissipated to acoustic phonons is reduced with respect to the GaAs because of weaker couplings (deformation as well as piezoelectric). The importance of the mode-coupling phenomenon for low densities is again evident.

V. CONCLUSIONS

We have investigated the influence of the plasmon-phonon mode-coupling phenomenon on hot-electron relaxation. We have also studied various levels of approximations to determine their region of validity. Our conclusions are as follows. (i) The effects of phonon reabsorption, quantum statistics, and screening are more important at high densities and provide a reduction of the power loss to LO phonons compared to the classical formula in Eq. (5). The power loss is still quite well described by the classical formula but with a renormalized τ. Our results predict a factor of 5–6 reduction of the cooling rate at low densities ($\sim 10^{17}$ cm^{-3}) compared with the classical result, which is in agreement with the experiment of Leheny *et al.*[3] At high densities ($\sim 10^{18}$ cm^{-3}) we find a reduction by a factor of 15–20, also consistent with the experiment.[3,4] (ii) It is important to include LO-phonon self-energy corrections due to its coupling with plasmons when the plasmon energy is lower than the LO-phonon energy. Now the power loss is approximately described by Eq. (7) and as the temperature is reduced, the dominant mechanism for the power loss switches from the phononlike mode (ω^+) to the plasmonlike mode (ω^-). We predict experimental densities which would enable a clear observation of this phenomenon by showing two distinct slopes of ω^+ and ω^- in the plot of $\ln(P)$ versus inverse temperature.

Note added in proof. We have recently discovered[18] that, in addition to plasmon-phonon coupling, the coupling of quasiparticle excitations to LO phonons also plays an important role in the physics of electron energy loss at low electron temperatures.

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APPENDIX

The renormalized phonon propagator is given by[11] ($\hbar=1$)

$$D(q,\omega)=\frac{2\omega_{LO}}{\omega^2-\omega_{LO}^2-2\omega_{LO}M_q^2\chi_0(q,\omega)/\epsilon(q,\omega)}\,, \tag{A1}$$

where

$$\epsilon=1-V_q\chi_0,\quad V_q=4\pi e^2/\epsilon_\infty q^2\ . \tag{A2}$$

We use plasmon-pole approximation[16,17] for χ_0:

$$\chi_0(q,\omega)=\frac{1}{V_q}\frac{\omega_p^2}{\omega^2-\beta^2q^2},\quad \omega_p^2=\frac{4\pi n e^2}{\epsilon_\infty m} \tag{A3}$$

which is chosen as to satisfy the f sum rule

$$\int_0^\infty d\omega\,\omega\,\mathrm{Im}\epsilon(q,\omega)=\frac{\pi}{2}\omega_p^2\ .$$

The quantity β [not to be confused with $(k_BT)^{-1}$ here] is determined from the zero-frequency Kramers-Kronig relation

$$\int_0^\infty \frac{d\omega}{\omega}\mathrm{Im}\left[\frac{1}{\epsilon(q,\omega)}-1\right]=\frac{\pi}{2}\left[\frac{1}{\epsilon(q,0)}-1\right]$$

to be

$$\beta^2=\frac{n}{m\,|\chi_0(q,\omega=0)|}\ . \tag{A4}$$

Substituting these in (A1) one gets

$$D=\frac{2\omega_{\rm LO}(\omega^2-\tilde{\omega}_p^2)}{(\omega^2-\omega_{\rm LO}^2)(\omega^2-\tilde{\omega}_p^2)-2\omega_{\rm LO}\frac{M_q^2}{V_q}\omega_p^2}\ , \tag{A5}$$

which yields Eq. (13) of the text with

$$\tilde{\omega}_p^2=\omega_p^2+\beta^2 q^2\ , \tag{A6}$$

and

$$\omega_\pm^2=\frac{1}{2}\left[\omega_{\rm LO}^2+\tilde{\omega}_p^2\pm\left[(\omega_{\rm LO}^2-\tilde{\omega}_p^2)^2+8\omega_{\rm LO}\omega_p^2\frac{M_q^2}{V_q}\right]^{1/2}\right]\ . \tag{A7}$$

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