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Superfluid density tensor in unconventional superconductors

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We consider the superfluid density tensor ρ_s for two unconventional spin-triplet gaps: an A phase and a polar phase. We pay particular attention to the effect of impurity scattering and how such scattering influences the anisotropy of ρ_s.

I. INTRODUCTION

The current-carrying state in unconventional superconductors has recently been the subject of experimental[1,2] and theoretical[1-5] investigation. One particularly important quantity for theoreticians to calculate is the superfluid density tensor, ρ_s, since it is directly related to the measurable magnetic penetration depth.[1-3]

The theory of ρ_s is discussed in Refs.1, 2, and 3. In this work we discuss ρ_s using the quasiclassical[6] theory of superconductivity, paying particular attention to the effect of nonmagnetic impurities. As already noted,[2] for a spherical Fermi surface impurities have the interesting effect of enhancing the anisotropy of ρ_s. A simple way to understand this enhancement is to realize that impurities dramatically increase the number of low-energy quasiparticle states in the vicinity of gap nodes on the Fermi surface.[7] This serves to decrease ρ_s proportionately more in the direction of such nodes than in the direction perpendicular to the nodes.

We consider, as does Ref. 2, two types of gap: an A phase type with two point nodes, and a polar phase gap with a line of nodes. We consider a metal with a spherical Fermi surface in the normal state, with a concentration c of impurities.

In Sec. II we use the quasiclassical theory to drive some general expressions for ρ_s. In Secs. III and IV we examine in more detail two limits: the $T=0$ limit and the Ginzburg-Landau regime near T_c.

II. GENERAL RESULTS

To do our calculations we use the same quasiclassical formalism[6] used in our previous work.[4,5] We include scattering from randomly placed impurities of concentration c; for simplicity we take the scattering potential to be purely s wave and of strength v. We solve for the gauge transformed propagator $\bar{g}(\hat{\mathbf{k}},\epsilon)$, which is a 2×2 matrix in the particle-hole space. To compute the current $\mathbf{J}$ we need the $\hat{\tau}_3$ component of the propagator, g_3. For the A-phase gap,

$$\Delta_{\alpha\beta}(\hat{\mathbf{k}})=\delta_{\alpha\beta}\Delta_0(\hat{\mathbf{x}}+i\hat{\mathbf{y}})\cdot\hat{\mathbf{k}} , \tag{1}$$

we find

$$g_3=\frac{-\pi i(\epsilon+iq+ia_3)}{[(\epsilon+iq+ia_3)^2+(\Delta_0\hat{\mathbf{k}}\cdot\hat{\mathbf{y}}-ia_1)^2+(\Delta_0\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}-ia_2)^2]^{1/2}} .$$

For a polar gap,

$$\Delta_{\alpha\beta}(\hat{\mathbf{k}})=\delta_{\alpha\beta}\Delta_0\hat{\mathbf{k}}\cdot\hat{\mathbf{z}} , \tag{3}$$

we find

$$g_3=\frac{-\pi i(\epsilon+iq+ib_3)}{[(\epsilon+iq+ib_3)^2+(\Delta_0\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}-ib_2)^2]^{1/2}} . \tag{4}$$

In these equations, $q=p_f\hat{\mathbf{k}}\cdot\mathbf{v}_s$, and Δ_0 is the gap amplitude with impurity scattering fully taken into account. To first order in $\mathbf{v}_s$, the amplitude and structure of the gap are unaffected by the superflow, and to compute ρ_s we need g_3 only to first order in $\mathbf{v}_s$.

The quantities $a_i(\epsilon)$ (A phase) and $b_i(\epsilon)$ (polar phase) are given by ct_i, where t_i is the $\hat{\tau}_i$ component of the impurity t matrix. The t matrix is computed from

$$\bar{t}(\epsilon)=v+N_0v\int\frac{d\hat{\mathbf{k}}}{4\pi}\bar{g}(\hat{\mathbf{k}},\epsilon)\bar{t}(\epsilon) , \tag{5}$$

as in our previous work,[4,5] we used the full impurity averaged $\bar{g}$ in (5), so that our results are not limited to the dilute regime. The functions $a_3(\epsilon)$ and $b_3(\epsilon)$ may be evaluated at $\mathbf{v}_s=0$, while $a_1(\epsilon)$, $a_2(\epsilon)$, and $b_2(\epsilon)$, being odd function of $\mathbf{v}_s$, are evaluated to first order in $\mathbf{v}_s$. Hence we write $a_1(\epsilon)=\alpha_1(\epsilon)\mathbf{v}_s\cdot\hat{\mathbf{y}}$, $a_2(\epsilon)=\alpha_2(\epsilon)\mathbf{v}_s\cdot\hat{\mathbf{x}}$, and $b_2(\epsilon)=\beta_2(\epsilon)\mathbf{v}_s\cdot\hat{\mathbf{z}}$; it is straightforward to check that a_1, a_2, and b_2 are proportional, respectively, to $\mathbf{v}_s\cdot\hat{\mathbf{y}}$, $\mathbf{v}_s\cdot\hat{\mathbf{x}}$, and $\mathbf{v}_s\cdot\hat{\mathbf{z}}$.

We now expand g_3 to first order in $\mathbf{v}_s$, and use the result to compute $\mathbf{J}$. We then obtain, for the A phase,

$$\rho^s_{ij}=2N_0v_f\Delta_0\pi T\sum_{\epsilon}\int\frac{d\hat{\mathbf{k}}}{4\pi}\frac{\Delta_0p_f\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2]+(\epsilon+ia_3)[\alpha_1(\hat{\mathbf{k}}\cdot\hat{\mathbf{y}})^2\hat{\mathbf{y}}_i\hat{\mathbf{y}}_j+\alpha_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2\hat{\mathbf{x}}_i\hat{\mathbf{x}}_j]}{\{(\epsilon+ia_3)^2+\Delta_0^2[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2]\}^{3/2}} \tag{6}$$

and, for the polar phase,

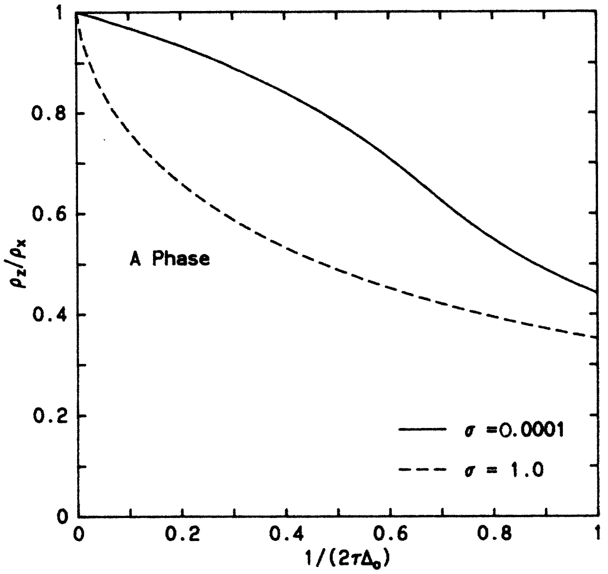

FIG. 1. Plot of ρ^s_{zz}/ρ^s_{xx} versus $1/(2\tau\Delta_0)$ for the A phase at $T=0$. The solid line is for the Born limit ($\sigma=0.0001$), while the dashed line is for the unitarity limit ($\sigma=1.0$).

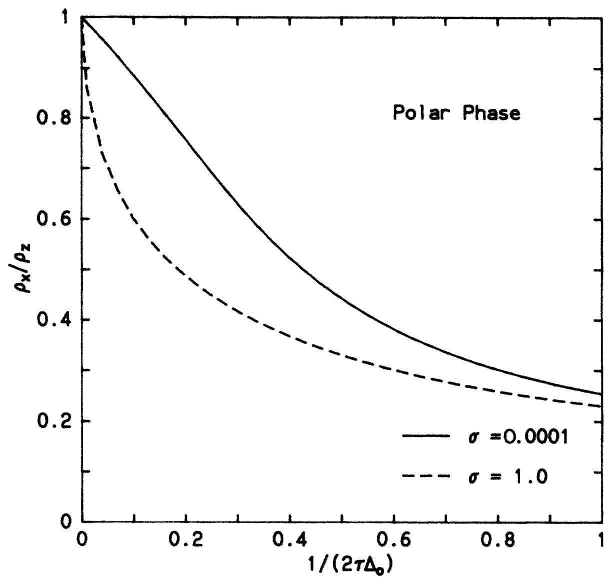

FIG. 2. Plot of ρ^s_{xx}/ρ^s_{zz} versus $1/(2\tau\Delta_0)$ for the polar phase at $T=0$. Born and unitarity limits are shown, as in Fig.1.

$$\rho^s_{ij}=2N_0 v_f \Delta_0 \pi T \sum_{\epsilon} \int \frac{d\hat{\mathbf{k}}}{4\pi} \frac{\Delta_0 p_f(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2\hat{\mathbf{k}}_i\hat{\mathbf{k}}_j+\beta_2(\epsilon+ib_3)(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2\hat{\mathbf{z}}_i\hat{\mathbf{z}}_j}{[(\epsilon+ib_3)^2+\Delta_0^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})^2]^{3/2}} . \tag{7}$$

Results (6) and (7) are valid for any temperature T. We now discuss two limiting cases.

III. $T=0$ CASE

Firstly, we consider $T=0$. We must then numerically solve for the functions $\alpha_1(\epsilon)$, $\alpha_2(\epsilon)$, $a_3(\epsilon)$, $\beta_2(\epsilon)$, and $b_3(\epsilon)$. In Figs.1 and 2 we show results which illustrate the anisotropic behavior of ρ_s at $T=0$. For the A phase we plot ρ^s_{zz}/ρ^s_{xx} as a function of $1/(2\tau\Delta_0)$, while for the polar phase we plot ρ^s_{xx}/ρ^s_{zz}. Here, $1/\tau$ is the normal state impurity scattering rate, and Δ_0 is the order parameter magnitude with impurity scattering taken into account. Hence, the parameter $1/(2\tau\Delta_0)$ ranges from zero to infinity as the impurity concentration increases.

The parameter σ is defined as

$$\sigma=\frac{(N_0 v\pi)^2}{1+(N_0 v \pi)^2} .$$

We show results for $\sigma=0.0001$ (Born limit) and $\sigma=0.9999$ (unitarity limit). In the Born limit, the A phase becomes gapless when $1/(2\tau\Delta_0)=2/\pi$, whereas in the unitarity limit the A phase is gapless for any nonzero value of $1/(2\tau\Delta_0)$.[7] The polar phase is gapless whenever $1/\tau$ is nonzero, regardless of the value of σ.[7]

Figures 3 and 4 show how the ratios ρ^s_{xx}/ρ, ρ^s_{zz}/ρ decrease from unity as impurities are added.

IV. GINZBURG-LANDAU LIMIT

In this limit, we work to lowest order in Δ_0^2, and more analytic progress is possible. We expand (6) and (7) to first order in Δ_0^2; we note that $a_3(\epsilon)$ and $b_3(\epsilon)$ can then be evaluated in the normal state, where we have

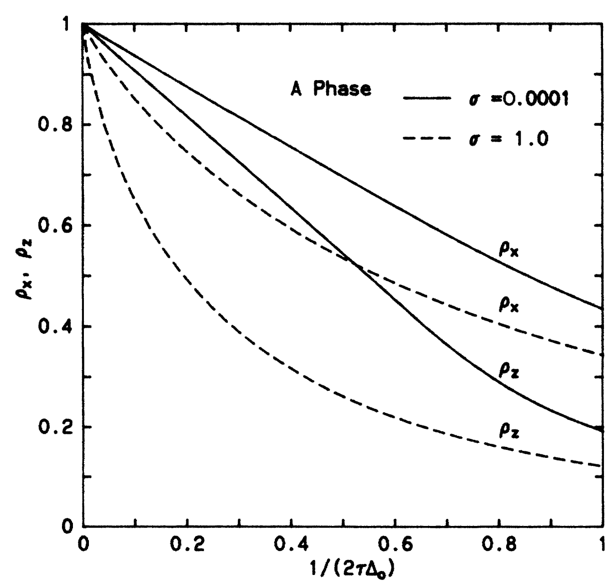

FIG. 3. Plot of ρ^s_{xx}/ρ and ρ^s_{zz}/ρ versus $1/(2\tau\Delta_0)$ for the A phase at $T=0$. Born ($\sigma=0.0001$) and unitarity ($\sigma=1.0$) limits are shown.

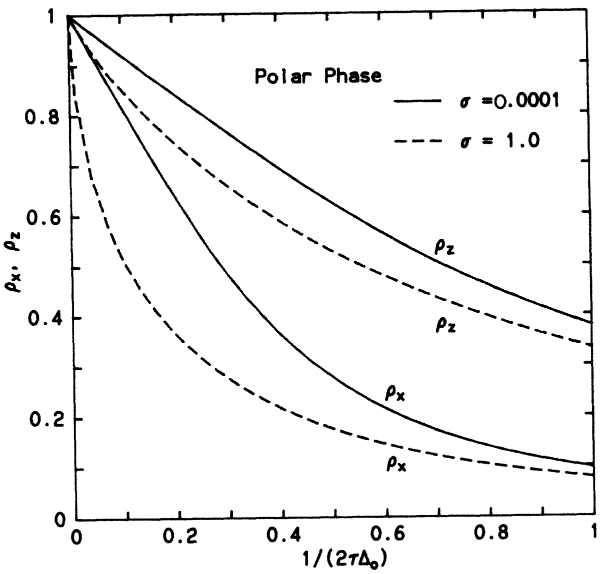

FIG. 4. Plot of ρ^s_{xx}/ρ and ρ^s_{zz}/ρ versus $1/(2\tau\Delta_0)$ for the polar phase at $T=0$. Born ($\sigma=0.0001$) and unitarity ($\sigma=1.0$) limits are shown.

$$ia_3=ib_3=\frac{N_0 v^2\pi c\ \mathrm{sgn}(\epsilon)}{1+(N_0 v\pi)^2}\equiv\frac{1}{2\tau}\ \mathrm{sgn}(\epsilon)\ . \tag{8}$$

The functions α_1, α_2, and β_2 are evaluated to lowest order in Δ_0. The final results for ρ_s are as follows.

A phase:

$$\rho^s_{ij}=\frac{N_0 v_f p_f \Delta_0^2}{15\pi^2 T_c^2}[\hat{\mathbf{z}}_i\hat{\mathbf{z}}_j S_3+(\hat{\mathbf{x}}_i\hat{\mathbf{x}}_j+\hat{\mathbf{y}}_i\hat{\mathbf{y}}_j)(2S_3+\tfrac{5}{6}wS_5)]\ . \tag{9}$$

Polar phase:

$$\rho^s_{ij}=\frac{N_0 v_f p_f \Delta_0^2}{2\pi^2 T_c^2}[\hat{\mathbf{z}}_i\hat{\mathbf{z}}_j(\tfrac{1}{5}S_3+\tfrac{1}{3}wS_5)+\tfrac{1}{15}(\hat{\mathbf{x}}_i\hat{\mathbf{x}}_j+\hat{\mathbf{y}}_i\hat{\mathbf{y}}_j)S_3]\ . \tag{10}$$

Here, we define $w\equiv 1/(4\pi\tau T_c)$, and the two functions S_3 and S_5 are defined as

$$S_3=\sum_{n\geq 0}\frac{1}{(n+\frac{1}{2}+w)^3}\ , \tag{11}$$

$$S_5=\sum_{n\geq 0}\frac{1}{(n+\frac{1}{2})(n+\frac{1}{2}+w)^3}\ . \tag{12}$$

For small w, we have

$$S_3\approx 7\zeta(3)-\frac{\pi^4}{2}w+\cdots\ , \tag{13}$$

$$S_5\approx\frac{\pi^4}{6}+\cdots\ , \tag{14}$$

while for large w, we have

$$S_3\approx\frac{1}{2w^2}-\frac{1}{8w^4}+\cdots\ , \tag{15}$$

$$S_5\approx\frac{\ln(w)}{w^3}-\frac{1}{w^3}[\psi(\tfrac{1}{2})+\tfrac{3}{2}]+\frac{1}{4w^5}+\cdots\ . \tag{16}$$

Here $\psi(x)$ is the digamma function. So, if we examine the anisotropy ratios, we find, for the *A* phase,

$$\rho^s_{xx}/\rho^s_{zz}\approx 2+1.608w+\cdots\ ,\quad w\ll 1 \tag{17}$$

$$\rho^s_{xx}/\rho^s_{zz}\approx\tfrac{5}{3}\ln(w)-\tfrac{1}{2}-\tfrac{5}{3}\psi(\tfrac{1}{2})+\cdots\ ,\quad w\gg 1 \tag{18}$$

and, for the polar phase,

$$\rho^s_{zz}/\rho^s_{xx}\approx 3+3.215w+\cdots\ ,\quad w\ll 1 \tag{19}$$

$$\rho^s_{zz}/\rho^s_{xx}\approx\tfrac{10}{3}\ln(w)-2-\tfrac{10}{3}\psi(\tfrac{1}{2})+\cdots\ ,\quad w\gg 1\ . \tag{20}$$

We note that the anisotropy ratio can be made as large as we want, by increasing w. The quantity w varies from zero to infinity as c increase from zero to the critical concentration which makes T_c vanish.

V. DISCUSSION

Our work, following that of Gross *et al.*,[2] illustrates how, for a superconductor with an anisotropic gap and a spherical Fermi surface, the presence of impurities can enhance the anisotropy of ρ_s. It is of interest to include the effect of a more realistic Fermi surface, and we hope to turn to this problem in the future. If ρ_s is anisotropic even in the pure limit, because of an anisotropic Fermi surface, the effect of impurities could well be more complicated.

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