Surface critical behavior of tricritical systems

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We study the surface critical behavior of semi-infinite systems with an n-component order parameter and short-range interactions at a bulk *tricritical* point. The special surface transition is investigated in detail for space dimensionalities $d=3-\epsilon\leq 3$ using field-theoretic renormalization-group methods. For $\epsilon>0$ its infrared-stable fixed point is described by a Hamiltonian which contains a $|\vec{\phi}|^4$ surface interaction of strength $O(\epsilon^{1/2})$ in addition to the familiar $|\vec{\phi}|^6$ bulk interaction of strength $O(\epsilon)$. For $d\equiv 3$, logarithmic corrections of a non-power-law form appear. The renormalization of a proper continuum model is expounded and the implications of the results are discussed, especially with regard to the adsorption of polymers from a θ solution.

I. INTRODUCTION

In a recent letter[1] we investigated the *surface critical behavior* of macroscopic systems[2,3] at a *bulk tricritical point.* Familiar examples of systems displaying bulk tricritical behavior are[4] metamagnets, ^{3}He-^{4}He mixtures, fluid mixtures, and polymers in a θ solution.[5] If such systems are bounded by a surface, the local (tri)critical behavior is modified in the vicinity of that surface. Just as in the case of an ordinary bulk critical point, several classes of surface transitions taking place at bulk (tri)criticality can be distinguished.[6-11] For systems of the $O(n)$, short-range interaction type, tricritical analogs of the familiar ordinary, extraordinary, and special transitions[2,3] have been identified, in particular. Although there exist some studies in the literature,[6-11] these surface transitions have been investigated to a much lesser degree than their analogs for a bulk critical point.

In Ref. 1 we reported results of a detailed renormalization-group (RG) analysis of the special transition for space dimensionalities $d=3-\epsilon\leq d^*\equiv 3$, the upper critical dimension (UCD). We showed that the dimensionality expansion for $\epsilon>0$ takes the form of an expansion in powers of $\epsilon^{1/2}$, notwithstanding the fact that bulk tricritical behavior is accessible to a conventional power-series expansion in ϵ.[4] Precisely at the UCD, logarithmic corrections of an unusual form (involving exponentials of square roots of logarithms) were found. The purpose of the present paper is to give a more detailed exposition of our analysis.

The findings just mentioned distinguish the special transition considered here *from all previously studied analogous surface transitions at a bulk critical or tricritical point.* For these the dimensionality expansion about the UCD *always took the same form as for the associated bulk transition,* namely that of an expansion in powers of ϵ. Of course, there is no *a priori* reason why this should be the case. It should also be noted that expansions in powers of $\epsilon^{1/2}$ as well as logarithmic corrections of a similar unusual form were encountered before in studies of bulk critical behavior in random Ising systems.[12-14] However, the mechanism leading to an $\epsilon^{1/2}$ expansion is different in both cases.

In our case the situation is as follows. Since we assume the interactions to be of short range, an adequate framework for our analysis is a continuum model with a Hamiltonian of the generic form

$$\mathcal{H}=\int d^dx[\theta(z)\mathcal{L}_b+\delta(z)\mathcal{L}_1]\ . \tag{1.1}$$

Here $\mathcal{L}_b$ and $\mathcal{L}_1$ are functions of the n-component order parameter $\vec{\phi}(\mathbf{x})=(\phi_\alpha(\mathbf{x});\alpha=1,2,\ldots,n)$ and its derivatives, $\mathbf{x}$ is a position vector $\in\mathbb{R}^d$, and $z\geq 0$ measures the distance from the surface plane $z=0$. In $\mathcal{L}_b$ contained is in particular the usual bulk term $\propto v|\vec{\phi}|^6$ (see e.g., Refs. 4 and 15; which other bulk or surface terms have to be included in $\mathcal{L}_b$ or $\mathcal{L}_1$ will be discussed in detail in Sec. II. Here it will be sufficient to recall from Ref. 1 the *crucial importance of including a term* $\propto u_1|\vec{\phi}|^4$ in $\mathcal{L}_1$. At the Gaussian fixed point from which the nontrivial infrared-stable fixed point splits off as d drops below $d^*=3$, both v and u_1 are marginally relevant. Moreover, the associated operators *mix under the RG, so that a nonvanishing* u_1 *is generated if* $v\neq 0$, *even if originally* $u_1\equiv 0$. It is precisely this fact which entails that u_1 is of order $\epsilon^{1/2}$ at the infrared-stable fixed point and thus gives rise to an expansion in powers of $\epsilon^{1/2}$.[16]

From these remarks it is obvious that a study of the special transition is certainly very interesting from a conceptual point of view. Another attractive feature of this transition (which it shares with bulk tricritical behavior) is *that the UCD coincides with the physically interesting dimension* $d=3$. Since for $d\equiv 3$ the theory is asymptotically free in the infrared, the *asymptotic critical behavior* obtained from the perturbative RG calculations described below *is expected to be exact.*

This brings us to the natural question of whether the results predicted below are accessible to experimental

verification. As is well known, the experimental observation of bulk tricritical behavior is more difficult than that of bulk critical behavior since it requires the location of a tricritical point in a space of (up to) four thermodynamic fields.[4] A thermodynamic description of surface phases involves additional surface fields, e.g., a field c, frequently called *surface enhancement,* which measures the enhancement of the interactions at the surface.[2,3] (In the general, symmetryless case further surface fields are needed, as will be explained in Sec. II.) The special transition we are concerned with here is described by a *multicritical point* at which c takes a special value. (Additional symmetry-breaking surface fields would have to vanish.) Compared with bulk tricriticality we thus have at least one additional constraint to be met. At first sight the experimental observation of this transition therefore seems to be a rather delicate task. We nevertheless believe it to be possible.

The kind of system which seems to us *particularly well suited for experimental investigations of the special transition* is a *polymer immersed into a θ solvent.* As has been known for some time,[17] the θ point of such a bulk system may be understood as a bulk tricritical point. If such a system is bounded by a wall, the polymer may be preferentially adsorbed by the wall, provided the wall-monomer interaction is sufficiently attractive. At a critical value of the interaction strength the polymer changes over from a nonadsorbed, roughly three-dimensional state to an adsorbed, roughly two-dimensional state. Just as in the case of a good solvent,[18,19] this adsorption threshold is the polymer analog of the special transition. The advantage of using polymers is that fewer thermodynamic fields are involved: the θ point has to be located in a space of only two thermodynamic variables (e.g., temperature and polymer concentration), and to reach the adsorption threshold a single additional surface variable has to be adjusted then.

Using familiar arguments (see, e.g., Refs. 18 and 19) one can show that the wall adsorption of θ polymers is described by the $n \to 0$ limit of the semi-infinite n-vector model to be introduced below; so the problem fits well into the framework of our present analysis. Actually, one motivation for the present study was a recent Monte Carlo simulation of a simplified three-dimensional lattice model for the adsorption of a θ polymer by van Dieren and Kremer.[20] These authors modeled the polymer by a walk which was random away from, but self-avoiding at, the surface. Such a walk maps onto the $n \to 0$ limit of a semi-infinite n-vector model with $v \equiv 0$.

The remainder of this paper is organized as follows. In Sec. II a proper continuum model is introduced, our choices of the bulk and surface densities $\mathcal{L}_b$ and $\mathcal{L}_1$ are motivated, and the renormalization of the model is expounded. Owing to the larger number of marginal and relevant variables, the renormalization is somewhat more involved than for the familiar semi-infinite n-vector model describing surface transitions at an ordinary bulk critical point,[3] so a more extensive discussion is necessary. In Sec. III we give the resulting renormalization-group equations and present our results for the required renormalization functions. In Sec. IV the critical behavior at the special transition is analyzed for $d \equiv 3-\epsilon < 3$. Section V is devoted to the case $d \equiv 3$. Finally, Sec. VI contains a brief summary of our results, a discussion of the relation with van Dieren and Kremer's work,[20] and some concluding remarks. A number of computational details have been deferred to Appendices A–C.

II. CONTINUUM MODEL AND RENORMALIZATION

In order to complete the definition of our model, we must now specify the bulk and surface densities $\mathcal{L}_b$ and $\mathcal{L}_1$ in (1.1). We assume that the symmetry which is spontaneously broken is O(n). Accordingly $\mathcal{L}_b$ as well as $\mathcal{L}_1$ will have an O(n)-symmetric part and a part which breaks this symmetry. For the sake of simplicity, we will allow *in both $\mathcal{L}_b$ and $\mathcal{L}_1$ only symmetry-breaking parts that leave a remaining* O($n-1$) *symmetry* with respect to a *common* axis specified by a unit vector $\vec{e}$ *unbroken.* Given this restrictive assumption, the Hamiltonian can be constructed along standard lines (cf. Ref. 3, Sec. II B), taking into account the following points.

(i) Since we wish to study surface critical behavior at a bulk critical point, a prime requirement is that $\mathcal{L}_b$ be an adequate bulk density for analyzing bulk tricritical behavior in the vicinity of the UCD. Which terms have to be included in such a bulk density has been discussed in detail in Ref. 4, so $\mathcal{L}_b$ may be gleaned from there.

(ii) As a general rule, all local (bulk or surface) operators which are relevant or marginal at the UCD should be included in $\mathcal{L}_b$ and $\mathcal{L}_1$. By a local operator we mean a monomial in $\vec{\phi}$ and its spatial derivatives, as usual.

(iii) Within a Landau-Ginzburg approximation the special transition is described by a Hamiltonian of type (1.1) with $\mathcal{L}_b = \frac{1}{2}(\nabla\vec{\phi})^2$ and $\mathcal{L}_1 \equiv 0$. In order to decide which interaction terms are relevant or marginal at the UCD, we may study their scaling behavior at this Gaussian fixed point. This is determined by the naive dimensions of the associated coupling constants: Let $M(\vec{\phi})$ be a monomial whose naive dimension is μ^{d_M} in units of an arbitrary momentum scale μ. Then a contribution $g^{(b/1)}M(\vec{\phi})$ to $\mathcal{L}_b$ or $\mathcal{L}_1$ is relevant, marginal, or irrelevant at the UCD d^* (in this naive power-counting sense) depending on whether the μ dimension $d^{(b)} = d^* - d_M$ or $d^{(1)} = d^* - d_M - 1$ of the corresponding bulk or surface coupling constant $g^{(b/1)}$ is positive, zero, or negative, respectively.

For convenience, we have listed in Table I the relevant bulk and surface operators for our case, together with the corresponding dimensions $d^{(b/1)}$. Note that we have omitted the relevant bulk operator $\vec{e}\cdot\vec{\phi}\,|\vec{\phi}|^4$; its inclusion is unnecessary because it can be eliminated through a shift $\vec{\phi}(\mathbf{x}) \to \vec{\phi}(\mathbf{x}) + \delta\vec{\phi}$, at the expense of modifying the coupling constants of the other bulk operators.[4] For a similar reason it also is unnecessary to include terms proportional to the marginal surface operator $\vec{\phi}\cdot\partial_z\vec{\phi}$ or the relevant surface operator $\vec{e}\cdot\partial_z\vec{\phi}$ in $\mathcal{L}_1$ (cf. Ref. 3, Sec. II B). To see this one should recall that the propagator belonging to the aforementioned Gaussian fixed point is given by[3]

TABLE I. (a) and (b). Relevant bulk (surface) operators $M_i^{(b)}$, ($M_j^{(1)}$), associated dimensionless coupling constants $g_i^{(b)}$, ($g_j^{(1)}$) and μ dimensions $d_i^{(b)}$, ($d_i^{(1)}$) of the respective dimensionful coupling constants.

(a)

	$i=4$	$i=3$	$i=2$	$i=1$
$g_i^{(b)}$	u	w	τ	h
$d_i^{(b)}$	$1+\epsilon$	$\frac{3}{2}+\frac{\epsilon}{2}$	2	$\frac{5}{2}-\frac{\epsilon}{2}$
$M_i^{(b)}$	$\frac{1}{4!}\lvert\vec{\phi}\rvert^4$	$\frac{1}{3!}(\vec{e}\cdot\vec{\phi})\vec{\phi}^2$	$\frac{1}{2}\vec{\phi}^2$	$-(\vec{e}\cdot\vec{\phi})$

(b)

	$j=3$	$j=2'$	$j=2$	$j=1$
$g_j^{(1)}$	w_1	c'	c	h_1
$d_j^{(1)}$	$\frac{1}{2}+\frac{\epsilon}{2}$	1	1	$\frac{3}{2}-\frac{\epsilon}{2}$
$M_j^{(1)}$	$\frac{1}{3!}(\vec{e}\cdot\vec{\phi})\vec{\phi}^2$	$\frac{1}{2}(\vec{e}\cdot\vec{\phi})^2$	$\frac{1}{2}\vec{\phi}^2$	$-(\vec{e}\cdot\vec{\phi})$

$$G_N(\mathbf{r};z,z')\equiv\langle\phi_\alpha(\mathbf{r},z)\phi_\alpha(\mathbf{0},z')\rangle_{\text{Gauss}}$$
$$=\int_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}}\frac{1}{2p}(e^{-p\lvert z-z'\rvert}+e^{-p(z+z')})$$
$$=G_b([\mathbf{r}^2+(z-z')^2]^{1/2})+G_b([\mathbf{r}^2+(z+z')^2]^{1/2}), \tag{2.1}$$

where $\mathbf{r}\in\mathbb{R}^{d-1}$ denotes the parallel components of $\mathbf{x}=(\mathbf{r},z)$, $\int_{\mathbf{p}}\equiv(2\pi)^{1-d}\int d^{d-1}p$, and G_b means the Gaussian bulk propagator

$$G_b(\lvert\mathbf{x}\rvert)=\tfrac{1}{4}\pi^{-d/2}\Gamma(\tfrac{1}{2}d-1)\lvert\mathbf{x}\rvert^{2-d}. \tag{2.2}$$

The index N indicates that G_N satisfies the Neumann boundary condition

$$\partial_z G_N(\mathbf{r};z=0,z'>0)=0. \tag{2.3}$$

Using this together with the fact that

$$\lim_{z\to 0+}\partial_z G_N(\mathbf{r};z,0)=-\delta^{d-1}(\mathbf{r}) \tag{2.4}$$

one can convince oneself that the effect of the above-mentioned two surface operators, within a (regularized) perturbation expansion about the Gaussian fixed point, may equivalently be described through changes of the coupling constants $g_j^{(1)}$ of the surface operators $M_j^{(1)}$ listed in Table 1(b).

We are thus led to the following choice of densities:

$$\mathcal{L}_b=\tfrac{1}{2}(\nabla\vec{\phi})^2+2\pi^2 f\mu^{2\epsilon}\frac{1}{6!}v\lvert\vec{\phi}\rvert^6+\sum_{i=1}^{4}\mu^{d_i^{(b)}}g_i^{(b)}M_i^{(b)}+\mathcal{L}_b^{CT}, \tag{2.4a}$$

$$\mathcal{L}_1=2\pi f^{(1)}\mu^{\epsilon}\frac{1}{4!}u_1\lvert\vec{\phi}\rvert^4+\sum_{j=1,2,2',3}\mu^{d_j^{(1)}}g_j^{(1)}M_j^{(1)}+\mathcal{L}_1^{CT}. \tag{2.4b}$$

Here v, $g_i^{(b)}$, u_1, and $g_j^{(1)}$ are *dimensionless* bulk or surface coupling constants. The corresponding operators $M_i^{(b)}$ and $M_j^{(1)}$ as well as the dimensions $d_i^{(b)}$ and $d_j^{(1)}$ are given in Table I. The factors f and $f^{(1)}$ are power series in ϵ of the form

$$f^{(1)}=1+f_1^{(1)}\epsilon+f_2^{(2)}\epsilon^2+\cdots, \tag{2.5}$$

which we have introduced to facilitate comparison with other conventions of normalizing the renormalized coupling constants v and u_1. (The fact that the f_i and $f_i^{(1)}$ must drop out of all universal quantities provides us with a convenient means of checking explicit calculations.) Finally, $\mathcal{L}_b^{CT}$ and $\mathcal{L}_1^{CT}$ denote the bulk and surface counterterms which are required to absorb the ultraviolet (UV) singularities of the free energy

$$F=-\ln\left[\int d\{\vec{\phi}\}e^{-\mathcal{H}}\right] \tag{2.6}$$

and other thermodynamic quantities. We claim that the Hamiltonian defined by (1.1), (2.4a), and (2.4b) is renormalizable i.e., the counterterms *do not involve any bulk and surface operators other than those appearing in the rest of* $\mathcal{H}$ and explicitly given in (2.4a) and (2.4b).

To substantiate this claim, we will rely on power counting. Since we also wish to consider correlation functions, let us first include source terms in $\mathcal{H}$, making the replacements

$$g_i^{(b)}\to g_i^{(b)}(\mu\mathbf{x})=g_i^{(b)}+J_i^{(b)}(\mu\mathbf{x}),\quad i=1,2,3,4;$$
$$g_j^{(1)}\to g_j^{(1)}(\mu\mathbf{r})=g_j^{(1)}+J_j^{(1)}(\mu\mathbf{r}),\quad j=1,2,2',3. \tag{2.7}$$

Here the $J_i^{(b/1)}$ are smooth source functions that vanish at infinity. We then define the generating functional

$$\mathcal{G}(v,u_1;\{g(\cdot)\})=\ln[\mathcal{Z}(v,u_1;\{g(\cdot)\})/\mathcal{Z}(v,u_1;\{0\})],$$
$$\mathcal{Z}(v,u_1;\{g(\cdot)\})=\int d\{\vec{\phi}\}e^{-\mathcal{H}\{\vec{\phi},g(\cdot)\}}, \tag{2.8}$$

in which $\{g(\cdot)\}$ denotes the set of all spatially varying bulk and surface interaction constants on the right-hand side (rhs) of (2.7). Taking functional derivatives of $\mathcal{G}$ with respect to $\pm g(\cdot)$ and setting $g(\cdot)\equiv g$ (i.e., all sources $J_i^{(b)}$ and $J_j^{(1)}$ equal to zero) afterwards, gives us the usual (renormalized) connected correlation functions of the operators $M_i^{(b)}$ and $M_j^{(1)}$. Specifically, we consider

$$G^{(N_h,N_\tau,N_{h_1})}(\mu\mathbf{x},\mu\mathbf{X},\mu\mathbf{r};v,u_1,\{g\})=\left[\prod_{i=1}^{N_h}\frac{\delta}{\delta h(\mu\mathbf{x}_i)}\right]\left[\prod_{j=1}^{N_\tau}\frac{-\delta}{\delta\tau(\mu\mathbf{X}_j)}\right]\left[\prod_{k=1}^{N_{h_1}}\frac{\delta}{\delta h_1(\mu\mathbf{r}_k)}\right]\mathcal{G}\Big|_{g(\cdot)\equiv g}, \tag{2.9}$$

where $\mathbf{x}\equiv\{\mathbf{x}_i\}$, $\mathbf{X}$, and $\mathbf{r}$ are convenient shorthands. (Note that in order to simplify our subsequent considerations, we have refrained from including source functions that couple to the transverse components of $\vec{\phi}$, namely those perpendicular to $\vec{e}$. However, it is an easy matter to convince oneself that if the cubic fields w and w_1, as well as c' vanish, the reparametrizations required to renormalize the above correlation functions [and given in (2.14)] also suffice to make their analogs involving transverse and/or longitudinal components of $\vec{\phi}$ finite.)

We expand

$$\mathcal{G}(v,u_1;\{g(\cdot)\equiv g\})\equiv -F+\text{const} \tag{2.10}$$

as well as the other functions generated by $\mathcal{G}$ in a perturbation series about the Gaussian fixed point. [Strictly speaking, we must not expand in powers of $g(\cdot)$ unless all $g(\cdot)$ are smoothly varying functions of bounded support, because otherwise we will encounter infrared problems.[21] However, this is of no importance for the following considerations concerning the ultraviolet singularities of the theory, for we may set $g(\cdot)=g$ in the resummed theory.] Each individual term in these series may be represented as a Feynman diagram whose lines correspond to the free propagator G_N introduced in (2.1). As is explained in detail in Ref. 3, the singular behavior of G_N at short distances gives rise to nonintegrable bulk and surface UV singularities which should be absorbable by local bulk and surface counterterms. The general form of such contributions to $\mathcal{L}_b$ and $\mathcal{L}_1$ is

$$P^{(b)}(\partial_x)^N[\phi(\mathbf{x})]^{N_\phi}\prod_{i(\neq 1)}[\mu^{d_i^{(b)}}g_i^{(b)}(\mu\mathbf{x})]^{N_i^{(b)}} \tag{2.11a}$$

and

$$P^{(1)}(\partial_x)^N[\phi(\mathbf{r},0)]^{N_\phi}\prod_{i(\neq 1)}[\mu^{d_i^{(b)}}g_i^{(b)}(\mu\mathbf{r},0)]^{N_i^{(b)}}\prod_j[\mu^{d_j^{(1)}}g_j^{(1)}(\mu\mathbf{r})]^{N_j^{(1)}}, \tag{2.11b}$$

respectively. Here N, N_ϕ, $N_i^{(b)}$, and $N_j^{(1)}$ are nonnegative integers, $P^{(b)}$ and $P^{(1)}$ are power series in v and u_1 (renormalization functions), and $(\partial_x)^N$ is a symbolic notation for an arbitrary product of N derivatives with respect to any components of $\mathbf{x}$. In principle, each individual derivative may act on some (or all) factors of ϕ and/or the source functions involved. However, derivatives acting on source functions in most cases can be transferred to the fields $\vec{\phi}$ by integrating by parts; the only counterterms with spatial derivatives of source functions that we *cannot* get rid of in this manner are surface counterterms involving $\mathbf{z}$ derivatives of bulk sources.

Let us suppose, for the moment, that the UV divergences have been regulated by means of a large momentum cutoff Λ (even though we will ultimately use dimensional regularization). For dimensional reasons, $P^{(b)}$ and $P^{(1)}$ then must vary as

$$P^{(b)}\sim\Lambda^{\delta_b},\qquad \delta_b=d-N-d_\phi N_\phi-\sum_{i(\neq 1)}d_i^{(b)}N_i^{(b)}, \tag{2.12a}$$

and

$$P^{(1)}\sim\Lambda^{\delta_1},\quad \delta_1=\delta_b-1-\sum_j d_j^{(1)}N_j^{(1)}, \tag{2.12b}$$

respectively, where $d_\phi=(d-2)/2$ means the μ dimension of $\phi(\mathbf{x})$. In $d^*-\epsilon\le d^*$ dimensions it is sufficient to include all terms with superficial degrees of divergence $\delta_b(d^*)\ge 0$ and $\delta_1(d^*)\ge 0$. Terms with $\delta_b(d^*)=0$ and $\delta_1(d^*)=0$ correspond to logarithmic divergences in Λ. We will use dimensional regularization in conjunction with minimal subtraction of poles.[21] Then only the latter terms (whose divergences correspond to poles in ϵ) are needed, and the renormalization functions take the form

$$P^{(b)}=\sum_{l=1}^{\infty}\epsilon^{-l}Q_l^{(b)}(v;n), \tag{2.12a'}$$

$$P^{(1)}=\sum_{l=1}^{\infty}\epsilon^{-l}Q_l^{(1)}(v,u_1;n). \tag{2.12b'}$$

Note that while $Q_l^{(b)}$ is a power series in v (with n-dependent coefficients), $Q_l^{(1)}$ is a power series *in both marginal variables*, v and u_1. That $Q_l^{(b)}$ is independent of u_1 is an obvious consequence of the local character of the UV divergences; for the same reason, the surface sources do not enter into the bulk counterterms $\mathcal{L}_b^{CT}$.

Collecting all counterterms according to the prescrip-

tion given above, we find that $\mathcal{L}_b$ and $\mathcal{L}_1$ can be written as

$$\mathcal{L}_b=\mathcal{L}_{b,0}+\mathcal{S}_b \ , \qquad \mathcal{L}_{b,0}=\tfrac{1}{2}(\nabla\vec{\phi}_0)^2+v_0\frac{1}{6!}\,|\,\vec{\phi}_0\,|^6+\sum_{i=1}^{4} g_{i,0}^{(b)}M_{i,0}^{(b)} \ , \tag{2.13a}$$

and

$$\mathcal{L}_1=\mathcal{L}_{1,0}+\mathcal{S}_1 \ , \qquad \mathcal{L}_{1,0}=u_{1,0}\frac{1}{4!}\,|\,\phi_0\,|^4+\sum_{j=1,2,2',3} g_{j,0}^{(1)}M_{j,0}^{(1)} \ . \tag{2.13b}$$

Here we have introduced the *bare* bulk and surface quantities:

$$\begin{aligned}
&\vec{\phi}_0=Z_\phi^{1/2}\vec{\phi} \ ,\\
&v_0=2\pi^2 f\mu^{2\epsilon}Z_v v \ ,\\
&g_{1,0}^{(b)}\equiv h_0=\mu^{(5-\epsilon)/2}(Z_\phi^{-1/2}h+uwP_A) \ ,\\
&g_{2,0}^{(b)}\equiv \tau_0=\mu^2(Z_\tau\tau+u^2P_B) \ ,\\
&g_{3,0}^{(b)}\equiv w_0=\mu^{(3+\epsilon)/2}Z_w w \ ,\\
&g_{4,0}^{(b)}\equiv u_0=\mu^{1+\epsilon}Z_u u \ ,\\
&u_{1,0}=2\pi f^{(1)}\mu^{\epsilon}(u_1+P_I) \ ,\\
&g_{1,0}^{(1)}\equiv h_{1,0}\\
&\quad=\mu^{(3-\epsilon)/2}[(Z_1Z_\phi)^{-1/2}h_1+w_1^3P_J+cw_1P_K\\
&\qquad +c'w_1P_L+uw_1P_M+wP_N] \ ,\\
&g_{2,0}^{(1)}\equiv c_0=\mu(Z_c c+c'P_Q+w_1^2P_R+uP_T) \ ,\\
&g_{2',0}^{(1)}\equiv c_0'=\mu(Z_{c'}c'+w_1^2P_U) \ ,\\
&g_{3,0}^{(1)}\equiv w_{1,0}=\mu^{(1+\epsilon)/2}Z_{w_1}w_1 \ ,
\end{aligned} \tag{2.14}$$

and the shorthand $M_{i,0}^{(b/1)}\equiv M_i^{(b/1)}(\vec{\phi}_0)$. The operator-free contribution $\mathcal{S}_b$ can be expressed as

$$\mathcal{S}_b=\mu^d(P_C u^3+P_D\tau u+P_E w^2) \ ; \tag{2.15}$$

its surface analog $\mathcal{S}_1$ contains pole terms proportional to h_1w_1, w_1^4, c^2, cc', c'^2, w_1^2c, w_1^2c', cu, $c'u$, w_1w, w_1^2u, τ, u^2, and $\partial_z u$. All above renormalization functions P and $Z-1$ have the Laurent form given on the rhs of (2.12a') or (2.12b').

The upshot of these considerations is that the Hamiltonian defined by (1.1), (2.4a), and (2.4b) and generalized by (2.7) to spatially varying interaction constants *is indeed renormalizable* by means of the above reparametrizations. Note that while the above *bulk* reparametrizations are in conformity with those of Refs. 4, 15, and 22 for the special case $w\equiv 0$, their required generalization to the case of nonvanishing cubic field w does not entirely agree with the one asserted in Ref. 4. In this latter work the additional term $\propto uwP_A$ in the equation for h_0 is missing. Accordingly the resulting flow equations will also differ through the contribution implied by this term.

The resulting relations between bare and renormalized correlation functions may be compactly summarized by expressing the generating functional $\mathcal{G}$ in terms of bare quantities. To this end we define $\mathcal{G}_0$, the bare analog of $\mathcal{G}$, by replacing in (2.8) $\mathcal{G}$, $\mathcal{L}_b$, $\mathcal{L}_1$, as well as all other renormalized quantities through their bare counterparts $\mathcal{G}_0$, $\mathcal{L}_{b,0}$, $\mathcal{L}_{1,0}$, etc. Combining (2.8), (2.13a), (2.13b), and (2.14) gives

$$\mathcal{G}_0(v_0,u_{1,0};\{g_0(\cdot)\})=\mathcal{G}(v,u_1;\{g(\cdot)\})+\int d^dx[\theta(z)\mathcal{S}_b+\delta(z)\mathcal{S}_1] \ , \tag{2.16}$$

which lends itself easily to a derivation of renormalization-group equations (see the following section).

III. RENORMALIZATION-GROUP EQUATIONS AND RESULTS FOR RENORMALIZATION FUNCTIONS

From Eq. (2.16) renormalization-group equations (RGE's) for correlation functions can be derived in a standard fashion by exploiting the μ independence of $\mathcal{G}_0$ and taking functional derivatives. These RGE's take a particularly simple form for the dimensionless functions $G^{(N_h,N_\tau,N_{h_1})}$ defined in (2.9), which we will write as $G^{\mathcal{N}}$, introducing the shorthand

$$\mathcal{N}=(N_h,N_\tau,N_{h_1}) \ . \tag{3.1}$$

The reason for this simplicity is that the $G^{\mathcal{N}}$ do not involve functional derivatives with respect to those sources that appear as factors of the functions $P_A,\ldots,P_U$ in (2.14). For the sake of simplicity, we will furthermore restrict attention to such values of $\mathcal{N}$ for which the operator-free parts $\mathcal{S}_b$ and $\mathcal{S}_1$ do not contribute (assuming, e.g., $N_h>0$ or $N_{h_1}>1$). Denoting μ derivatives at fixed bare-interaction constants v_0, $u_{1,0}$, and $\{g_0\}$ by $\partial_\mu\,|_0$, we define the β functions

$$\beta_\kappa(v,u_1,\{g\};\epsilon)\equiv\mu\partial_\mu\,|_0\,\kappa \ , \qquad \kappa=v,u_1,g \ , \qquad g=h,\tau,w,u,h_1,c,c',w_1 \ , \tag{3.2}$$

and

$$\vartheta_g(v,u_1,;\epsilon)=d_g(\epsilon)+\eta_g(v,u_1) \ , \qquad \eta_g(v,u_1)=\mu\partial_\mu\,|_0\ln Z_g \ , \tag{3.3}$$

where d_g means the μ dimension of g_0 and

$$Z_h\equiv Z_\phi^{-1/2}, \quad Z_{h_1}\equiv(Z_\phi Z_1)^{-1/2} \ . \tag{3.4}$$

The RGE's thus become

$$\left[\mu\partial_\mu+E_{\mathcal{N}}+\sum_{\kappa=v,u_1,\{g\}}\beta_\kappa\partial_\kappa\right]G^{\mathcal{N}}=0 \ , \tag{3.5}$$

with

$$E_{\mathcal{N}}=(d-\vartheta_h)N_h+(d-\vartheta_\tau)N_\tau+(d-1-\vartheta_{h_1})N_{h_1} \ . \tag{3.6}$$

Utilizing (2.14) the β functions can be rewritten as

$$\beta_v \equiv \beta_v(v;\epsilon) = -\frac{2\epsilon}{\partial_v \ln(vZ_v)} ,$$

$$\beta_{u_1} \equiv \beta_{u_1}(v,u_1;\epsilon) = -\frac{\epsilon u_1 + (\epsilon + \beta_v \partial_v) P_I}{1 + \partial_{u_1} P_I} , \tag{3.7}$$

$$\beta_g(v,u_1,\{g\};\epsilon) = -g\vartheta_g(v,u_1;\epsilon) - \Delta_g(v,u_1,\{g\}) ,$$

where the Δ_g represent the contributions implied by the functions $P_A,\ldots,P_U$. For a given value of g, Δ_g depends on the set of sources $\{g\}$ in just the same way as the contributions proportional to $P_A,\ldots,P_U$ do in the expression for g_0 given in (2.14). Accordingly we have, for example,

$$\Delta_w \equiv \Delta_u \equiv \Delta_{w_1} \equiv 0 ,$$
$$\Delta_h = uwB(v) , \tag{3.8}$$

and

$$\Delta_c = c'Q(v,u_1) + w_1^2 R(v,u_1) + uT(v,u_1) . \tag{3.9}$$

Here B, Q, R, and T are finite functions of v, or of v and u_1 as indicated. (Like occasionally before, and below, the dependence on n is suppressed.) We content ourselves with giving the resulting expression for T; it reads

$$T(v,u_1) = (\vartheta_c - \vartheta_u + \beta_v \partial_v + \beta_{u_1}\partial_{u_1}) P_T Z_c^{-1} . \tag{3.10}$$

The special form the renormalization functions P and $Z-1$ take within the present *minimal subtraction scheme* so far has been exploited only to conclude that the renormalization group (RG) functions η_g and Δ_g are *independent* of ϵ. Used in conjunction with our knowledge that all RG functions β_κ, ϑ_g, η_g, Δ_g, etc. remain *finite* in the limit $\epsilon \to 0$, it enables us to simplify the above expressions considerably. Let us denote the residuum of Laurent series such as P_I, $Z_v - 1, \ldots$ at $\epsilon = 0$ by $P_I^{(1)}$, $Z_v^{(1)}$, etc. Elementary considerations then yield

$$\beta_v = -2\epsilon v + 2v^2 \partial_v Z_v^{(1)} , \tag{3.11}$$

$$\beta_{u_1} = -\epsilon u_1 + B_{u_1}(v,u_1) ,$$
$$B_{u_1} = (-1 + 2v\partial_v + u_1 \partial_{u_1}) P_I^{(1)} , \tag{3.12}$$

$$\eta_g = -(2v\partial_v + u_1 \partial_{u_1}) Z_g^{(1)} , \tag{3.13}$$

and

$$T(v,u_1) = -(1 + 2v\partial_v + u_1 \partial_{u_1})(P_T Z_c^{-1})^{(1)} . \tag{3.14}$$

We proceed by giving explicit perturbative results for various renormalization functions needed in the sequel. From studies of bulk tricritical behavior[4,15] it is known that

$$Z_{\tau,\phi} = 1 + O(v^2), \quad \eta_{\tau,\phi} = O(v^2) , \tag{3.15}$$

and

$$\beta_v = -2\epsilon v + \frac{3n+22}{120} v^2 + O(v^3) . \tag{3.16}$$

In order to compute β_{u_1} we must determine the renormalization function P_I. To this end we write

$$P_I = \sum_{\substack{i,j=0 \\ [(i,j)\neq(0,0),(0,1)]}}^{\infty} A_{i,j}(\epsilon,n) v^i u_1^j \tag{3.17}$$

and

$$B_{u_1} = \sum_{i,j} B_{i,j}(n) v^i u_1^j , \tag{3.18}$$

where the latter summation is restricted to the same pairs (i,j) as in (3.17). In Appendix A we evaluate the Feynman graph expansion of $G^{(4,0,0)}$ for $\{g \equiv 0\}$ to orders u_1^3 and vu_1. From the results given there and (3.15) we may conclude that

$$A_{1,0} = \epsilon^{-1} B_{1,0}, \quad B_{1,0} = -\frac{n+4}{80} , \tag{3.19}$$

$$A_{1,1} = -\frac{(n+4)(n+8)}{480\epsilon^2} + \frac{1}{2\epsilon} B_{1,1} ,$$
$$B_{1,1} = \frac{n+4}{240}\left[\frac{62-n}{4} + (n+8)(f_1 - 2f_1^{(1)})\right] , \tag{3.20}$$

$$A_{0,2} = \epsilon^{-1} B_{0,2}, \quad B_{0,2} = \frac{n+8}{6} , \tag{3.21}$$

and

$$A_{0,3} = \frac{(n+8)^2}{36\epsilon^2} + \frac{1}{2\epsilon} B_{0,3} ,$$
$$B_{0,3} = -\frac{5n+22}{9} 2\ln 2 . \tag{3.22}$$

The surface renormalization functions Z_c and Z_1 can be determined by computing

$$\delta^3 \mathcal{G} / \delta c(\mu \mathbf{r}) \delta h(\mu \mathbf{x}_1) \delta h(\mu \mathbf{x}_2)$$

and $G^{(1,0,1)}$ for $\{g \equiv 0\}$. Our perturbative results to orders u_1^2 and v are described in Appendix B. Using these, one arrives at

$$Z_c = 1 + u_1 \frac{n+2}{6\epsilon} + u_1^2 \frac{n+2}{6\epsilon}\left[\frac{n+5}{6\epsilon} - \ln 2\right] - v\frac{(n+2)(n+4)}{5!\,8\epsilon}\left[\frac{1}{\epsilon} + \frac{1}{4} + 2f_1^{(1)} - f_1\right] + O(u_1 v, u_1^3) \tag{3.23}$$

and

$$Z_1 = 1 + v\left[\frac{(n+2)(n+4)}{5!\,\epsilon}(\tfrac{1}{32} - \tfrac{1}{9}u_1) + O(v, u_1^2)\right] . \tag{3.24}$$

Substitution of Z_c, (3.4), and Z_1 into (3.13) then yields

$$\eta_c = -u_1 \frac{n+2}{6} + u_1^2 \frac{n+2}{3} \ln 2 + v\frac{(n+2)(n+4)}{5!\,4}(\tfrac{1}{4} + 2f_1^{(1)} - f_1) + O(u_1 v, u_1^3) \tag{3.25}$$

and

$$\eta_{h_1} = -\tfrac{1}{2}(\eta_\phi + \eta_1) \tag{3.26}$$

with

$$\eta_1 = -v\left[\frac{(n+2)(n+4)}{5!}(\tfrac{1}{16} - \tfrac{1}{3}u_1) + O(v,u_1^2)\right] . \tag{3.27}$$

In closing this section we briefly turn to the special case $v \equiv 0$. As shown in Ref. 1, one has

$$Z_1(v \equiv 0, u_1, \epsilon) \equiv 1 . \tag{3.28}$$

It will be helpful to recall how this result can be understood within the framework of our present analysis. Consider the Feynman graph expansion for $G^{(0,0,N_{h_1})}$ for vanishing bulk interaction constants $v \equiv g_i^{(b)} \equiv 0$. Since only *surface* vertices are present, this expansion involves the propagator G_N [defined in (2.1)] *only between surface points;* the corresponding Fourier transform is given by

$$\hat{G}^{(0,0,N_{h_1})} = p^{-1} . \tag{3.29}$$

Consequently the above Feynman graphs may be viewed as those of a $d-1$ *dimensional bulk system* with a free propagator given by (3.29) in momentum space. To be precise, let

$$\vec{\psi}(\mathbf{r}) \equiv \vec{\phi}_0(\mathbf{r},0) \tag{3.30}$$

and set $c' \equiv w_1 \equiv 0$, for simplicity. (The generalization to the case $c' \neq 0$ and $w_1 \neq 0$ should be obvious.) Then, order by order in perturbation theory, the Feynman graph expansion of $G^{(0,0,N_{h_1})}$ is identical to that of the N_{h_1}-point correlation function of the bulk model defined by the Hamiltonian

$$\mathcal{H}_{LR}\{\vec{\psi}\} = \int d^{d-1}r \left[\frac{1}{2}\left[\int_p (p + c_0) e^{i\mathbf{p}\cdot\mathbf{r}}\right]\vec{\psi}(\mathbf{r})^2 + \frac{1}{4!}u_{1,0}\,|\,\vec{\psi}(\mathbf{r})\,|^4 - h_{1,0}\vec{e}\cdot\vec{\psi}(\mathbf{r})\right] . \tag{3.31}$$

The renormalization factor (3.28) may be recognized as the amplitude renormalization factor of $\vec{\psi}$. Power counting shows that $\vec{\psi}$ does not require an amplitude renormalization, so (3.28) holds.

The model (3.31) is a special case of the class of bulk systems with long-range interactions studied by Fisher, Ma, and Nickel.[23] The fact that the surface critical behavior of our model, in the special case $v \equiv 0$, can be mapped onto the above bulk model implies that the associated surface critical exponents can be expressed in terms of the corresponding bulk exponents.[1] Our pertinent results, which will be given in the next section, are fully consistent with those of Ref. 23.

IV. RENORMALIZATION-GROUP FLOW AND CRITICAL EXPONENTS FOR $d < 3$

We now turn to a discussion of the RGEs (3.5). Solving (3.5) by characteristics one finds that a homogeneous dilatation of the length scale by a factor $e^\lambda > 1$ can be expressed as

$$G^{\mathcal{N}}(\mu\mathbf{x},\mu\mathbf{X},\mu\mathbf{r};\{\kappa\}) = G^{\mathcal{N}}(e^{-\lambda}\mu\mathbf{x},e^{-\lambda}\mu\mathbf{X},e^{-\lambda}\mu\mathbf{r};\{\bar{\kappa}(\lambda)\}) \times \exp\left[-\int_0^\lambda d\lambda'\, E_{\mathcal{N}}(\bar{v}(\lambda'),\bar{u}_1(\lambda'))\right] . \tag{4.1}$$

In accordance with our previous conventions [cf. Eq. (3.2)], $\{\kappa\}$ here means the set of all marginal and relevant variables,

$$\{\kappa\} \equiv \{v,u_1\} \cup \{g = h, \tau, w, u, h_1, c, c', w_1\} . \tag{4.2}$$

The running coupling constants $\bar{\kappa}(\lambda)$ are defined by

$$\frac{d}{d\lambda}\bar{\kappa}(\lambda) = -\beta_\kappa(\bar{v}(\lambda),\bar{u}_1(\lambda),\{\bar{g}(\lambda)\};\epsilon)$$

with (4.3)

$$\bar{\kappa}(0) \equiv \kappa .$$

Our aim is to determine the fixed points of these flow equations that split off from the trivial Gaussian fixed point G ($\kappa \equiv 0$, for all κ) as ϵ grows beyond zero. At G the variables g are all (strongly) relevant. Hence, at least for small $\epsilon > 0$, they must also be relevant at the fixed points we are looking for, so we may confine ourselves to the RG invariant subspace

$$\mathcal{M} = \{(v,u_1,\{g \equiv 0\})\} . \tag{4.4}$$

The coordinates $(v,u_1) \equiv (v^*,u_1^*)$ of fixed points $\in \mathcal{M}$ are given by the zeros of the corresponding β functions

$$\beta_v(v^*;\epsilon) \equiv 0 \equiv \beta_{u_1}(v^*,u_1^*;\epsilon) . \tag{4.5}$$

Upon insertion of (3.16) into the first of these equations we recover the two familiar roots[4,15]

$$v^* = v_G^* \equiv 0 , \qquad v^* = v_t^* \equiv \frac{240}{3n+22}\epsilon + O(\epsilon^2) . \tag{4.6}$$

Substituting these into β_{u_1}, and using (3.12) in conjunction with (3.18)–(3.22) the $o(\epsilon)$ roots of β_{u_1} can be easily determined. [We ignore any fixed points for which v^* and u_1^* are not both $o(\epsilon)$; i.e., fixed points that do not merge into G as $\epsilon \to 0+$.] In addition to G, we thus find the following fixed points. Two fixed points

$$T:\ v=v_t^*,\ u_1=(u_1^*)_T\ ,$$

$$(u_1^*)_T=\left[\frac{18(n+4)\epsilon}{(3n+22)(n+8)}\right]^{1/2}+\frac{3}{n+8}\epsilon\left[1+\frac{n+4}{3n+22}\left[\frac{n-62}{4}+(2f_1^{(1)}-f_1)(n+8)+4(\ln 2)\frac{5n+22}{n+8}\right]\right]+O(\epsilon^{3/2})\ , \tag{4.7a}$$

and

$$T':\ v=v_t^*, u_1=(u_1^*)_{T'}\ ,$$

$$(u_1^*)_{T'}=-\left[\frac{18(n+4)\epsilon}{(3n+22)(n+8)}\right]^{1/2}+O(\epsilon)\ , \tag{4.7b}$$

at which *the bulk is tricritical,* and a fixed point

$$LR:\ v=0,\ u_1=(u_1^*)_{LR}\ ,$$

$$(u_1^*)_{LR}=\frac{6\epsilon}{n+8}+48\epsilon^2\frac{5n+22}{(n+8)^3}\ln 2+O(\epsilon^3)\ , \tag{4.7c}$$

with a *Gaussian bulk interaction.* The latter fixed point, *LR* (= long range), *is identical to the infrared-stable fixed point which governs the critical behavior of the* $d-1$ *dimensional bulk system described by the Hamiltonian* $\mathcal{H}_{LR}$ in (3.31).

In Fig. 1 a qualitative picture of the RG flow in the subspace $\mathcal{M}$ is given. As one sees, T attracts all points in the half-plane $v>0$ above a separatrix σ passing through G and T'.[24] On the other hand, LR and T' only attract the boundary of this region, namely the half-line $v=0$, $u_1>0$, and the line σ, respectively. Finally, G is completely infrared unstable.

In order to understand the origin of the above results, one should note that, as a result of the mixing of the $|\vec{\phi}|^6$ bulk operator and the $|\vec{\phi}|^4$ surface operator under renormalization, the bare interaction constant $u_{1,0}$ contains a *term linear in* v. (At lowest order this is due to the diagram $F^B_{1,0}$ in Fig. 4.) Owing to this term, β_{u_1} also gets a piece linear in v. This in turn has the following two important consequences.

(i) On the whole half-line $u_1=0$, $v>0$, the flow *has a nonvanishing upward component.* Thus *there is no fixed point with coordinates* $(v_t^*, u_1\equiv 0)$, contrary to the assumption underlying Speth's work[8] on the special transition.

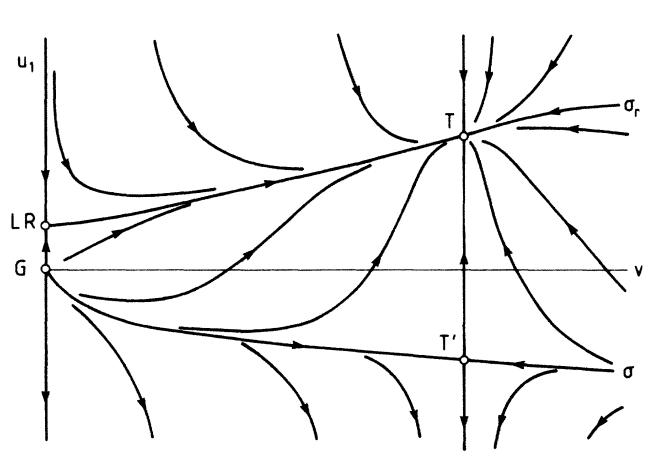

FIG. 1. Flow diagram for $d<3$, showing the fixed points T, T', LR (long range), and G (Gaussian), the separatrix σ, and the line σ_r (which is a separatrix for the reversed flow).

(ii) The coordinate $(u_1^*)_T$ of the stable tricritical fixed point T *has an expansion in powers of* $\epsilon^{1/2}$ *rather than* ϵ. This implies that the *dimensionality expansion of the associated surface critical exponents also takes the form of an* $\epsilon^{1/2}$ *expansions.*[25] On the other hand, the fixed point value $(u_1^*)_{LR}$ as well as the surface exponents of LR *do have an expansion in integer powers of* ϵ.[23]

Having found the fixed points, we are ready to derive the asymptotic behavior of the correlation functions $G^{\mathcal{N}}$. We assume that v and u_1 are in the domain of attraction of one of the fixed points T, T', or LR. The desired asymptotic scaling behavior of $G^{\mathcal{N}}$ then follows from (4.1) by substituting the appropriate fixed-point values for the running coupling constants $\{\bar{\kappa}\}$ on the rhs. Since this is a well-known procedure, leading to the scaling forms anticipated on the basis of phenomenological scaling, it will be sufficient to consider a representative example. Choosing $\mathcal{N}=(0,0,2)$, we find

$$G^{(0,0,2)}(\mu\mathbf{r};v,u_1,\tau,c)=|\mu\mathbf{r}|^{-(d-2+\eta_\parallel)}X(\mu r\tau^\nu, c\tau^{-\phi})\ , \tag{4.8}$$

where all bulk and surface interaction constants suppressed have been set to zero. We have introduced the *surface correlation exponent*

$$\eta_\parallel=-d+2+E^*_{(0,0,2)}=-2\eta^*_{h_1}\ , \tag{4.9}$$

the *bulk correlation-length exponent*

$$\nu=1/\vartheta_\tau^*=(2+\eta_\tau^*)^{-1}\ , \tag{4.10}$$

and the *surface crossover exponent*

$$\phi=\vartheta_c^*/\vartheta_\tau^*=\nu(1+\eta_c^*)\ . \tag{4.11}$$

Here all functions marked by an asterisk are to be taken at the fixed point in question.

Of the fixed points considered, T has the largest domain of attraction. The dimensionality expansion of its surface exponents can be obtained from Eqs. (3.3), (3.6), (3.25)–(3.27), (4.6), (4.7a), (4.9), and (4.11); it reads

$$\eta_\parallel^{(T)}=-\frac{(n+2)(n+4)}{8(3n+22)}\epsilon\left[1-16\left[\frac{2(n+4)\epsilon}{(3n+22)(n+8)}\right]^{1/2}\right]+O(\epsilon^2) \tag{4.12}$$

and

$$\phi^{(T)}=\frac{1}{2}-\frac{n+2}{4}\left[\frac{2(n+4)\epsilon}{(3n+22)(n+8)}\right]^{1/2}+\frac{(n+2)\epsilon}{(3n+22)(n+8)}\left[\frac{29n}{8}+12+\frac{(n+4)(1-n)}{n+8}2\ln 2\right]+O(\epsilon^{3/2}) . \quad (4.13)$$

Notice that the nonuniversal numbers f_1 and $f_1^{(1)}$ appearing in (3.23) and (4.7a) have indeed cancelled out to give a universal exponent ϕ.

If $v\equiv 0$ and $u_1>0$, the asymptotic behavior is governed by the fixed point LR. Using (3.28) and (4.7c), together with some of the previously mentioned equations, we find that the corresponding surface exponents are given by

$$\eta_{\parallel}^{(LR)}\equiv 0 \quad (4.14)$$

and

$$\phi^{(LR)}=\frac{1}{2}-\frac{n+2}{2(n+8)}\epsilon -2(\ln 2)\frac{(n+2)(7n+20)}{(n+8)^3}\epsilon^2+O(\epsilon^3) . \quad (4.15)$$

Just as (3.28), *the result (4.14) holds to all orders in perturbation theory.* Furthermore, both (4.14) and (4.15) are *completely consistent with the results of Fisher, Ma, and Nickel*[23] *for the $d-1$ dimensional bulk model (3.31).* If we denote the bulk exponents obtained by these authors by $\eta_{\mathrm{FMN}}(d-1,n)$ and $\nu_{\mathrm{FMN}}(d-1,n)$, a straightforward comparison with our results shows that these exponents do indeed satisfy the required relations, namely

$$\eta_{\mathrm{FMN}}(d-1,n)=\eta_{\parallel}^{(LR)}(d,n)+1 \quad (4.16)$$

and

$$\nu_{\mathrm{FMN}}(d-1,n)=[2\phi^{(LR)}(d,n)]^{-1} . \quad (4.17)$$

We leave it to the reader to work out the dimensionality expansion for the surface exponents of T'.

V. LOGARITHMIC CORRECTIONS IN THREE DIMENSIONS AND APPLICATION TO θ POLYMERS

In the limit $d\to 3-$, the fixed points T, T', and LR all merge into the Gaussian one G. A schematic picture of the resulting flow diagram for $d=3$ is shown in Fig. 2. There still is a line σ, which, however, now separates points flowing into G from those running away to large negative values of u_1. Explicit expressions for the asymptotic behavior of the running coupling constants $\bar{v}$ and $\bar{u}_1$ in the infrared limit $\lambda\to\infty$ can be deduced from Eqs. (3.12), (3.16), (3.18)–(3.22), and (4.3). We will not consider initial values (v,u_1) belonging to points below the separatrix σ. Thus we may distinguish three types of limiting behavior, corresponding to trajectories starting (i) inside the region bounded by σ and the positive u_1 axis, (ii) on the positive u_1 axis, or (iii) on σ. The respective asymptotic form of $\bar{v}$ is

$$\bar{v}(\lambda)\simeq C_1\lambda^{-1} \text{ if } v>0 \text{ [cases (i) and (iii)]} , \quad (5.1)$$

while, of course,

$$\bar{v}(\lambda)\equiv 0 \text{ if } v=0 \text{ [case (ii)]} ; \quad (5.2)$$

that of $\bar{u}_1$ is

$$\bar{u}_1(\lambda)\simeq C_2\lambda^{-1/2}+C_3\lambda^{-1} \text{ if } u_1>u_1(\sigma) \text{ [case (i)]} , \quad (5.3)$$

$$\bar{u}_1(\lambda)\simeq B_{0,2}^{-1}\lambda^{-1} \text{ if } v=0 \text{ and } u_1>0 \text{ [case (ii)]} , \quad (5.4)$$

and

$$\bar{u}_1(\lambda)\simeq -C_2\lambda^{-1/2}+O(\lambda^{-1}) \text{ if } (v,u_1)\in\sigma \text{ [case (iii)]} . \quad (5.5)$$

Here $u_1(\sigma)$ means the coordinate of a point on σ. The limiting expressions are correct to order $o(\lambda^{-1})$ for $\lambda\to\infty$. The coefficients are found to be

$$C_1=\frac{120}{3n+22}, \quad C_2=(-B_{1,0}C_1/B_{0,2})^{1/2} , \quad C_3=(\tfrac{1}{2}-B_{1,1}C_1+B_{0,3}B_{1,0}C_1/B_{0,2})/(2B_{0,2}) , \quad (5.6)$$

in which the $B_{i,j}$ mean the numbers given in (3.19)–(3.22). Note that these coefficients are independent of the initial values v and u_1. This is a reflection of the fact that all trajectories starting inside the region bounded by σ and the positive u_1 axis asymptotically approach the same limiting curve σ_r. As can be seen from the parametric representations (5.1), (5.3), and (5.4), both

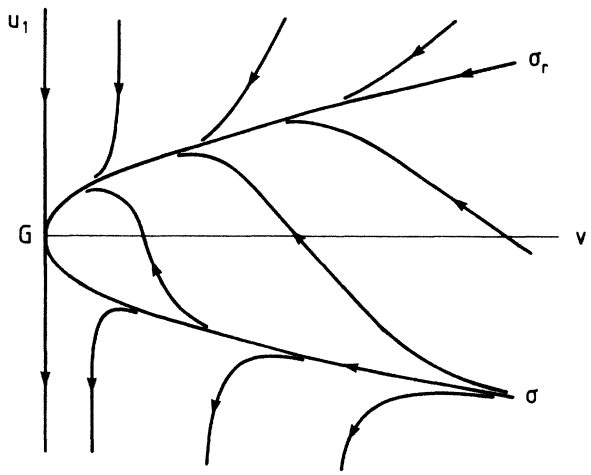

FIG. 2. Flow diagram for $d=3$.

σ and σ_r display a square-root behavior in the vicinity of G; one has

$$u_1(\sigma)\simeq -C_2C_1^{-1/2}v^{1/2}\simeq -u_1(\sigma_r) \tag{5.7}$$

as $v\to 0+$.

In order to obtain from (4.1) the critical behavior of the correlation functions $G^{\mathcal{N}}$, we must also determine the asymptotic form of the exponential factor on the rhs of (4.1), as well as the behavior of all nonvanishing relevant variables. For the sake of simplicity, we will allow at most τ and c to be nonzero. In that case, the running variables may be written as

$$\begin{aligned}\bar{\tau}(\lambda)&=\mathcal{E}_\tau(\lambda;v,u_1)\tau\ ,\\ \bar{c}(\lambda)&=\mathcal{E}_c(\lambda;v,u_1)c\ ,\end{aligned} \tag{5.8}$$

where $\mathcal{E}_\tau$ and $\mathcal{E}_c$ are trajectory integrals defined by

$$\mathcal{E}_g\equiv\exp\left[\int_0^\lambda \vartheta_g[\bar{v}(\lambda'),\bar{u}_1(\lambda')]d\lambda'\right]\ . \tag{5.9}$$

Let us introduce the expansion coefficients $\eta_1^{[i,j]}$ and $\eta_c^{[i,j]}$ by

$$\eta_{1,c}=\sum_{i,j}\eta_{1,c}^{[i,j]}v^iu_1^j\ . \tag{5.10}$$

A straightforward analysis then shows that, provided $v>0$ and $u_1>u_1(\sigma)$ [case (i)],

$$\mathcal{E}_\tau\simeq\text{const}\times e^{2\lambda}\ , \tag{5.11}$$

$$\mathcal{E}_c\simeq\text{const}\times\exp\{\lambda+\lambda^{1/2}2\eta_c^{[0,1]}C_2 +(\ln\lambda)(\eta_c^{[0,1]}C_3+\eta_c^{[0,2]}C_2^2+\eta_c^{[1,0]}C_1)\}\ , \tag{5.12}$$

and

$$\exp\left[-\int_0^\lambda E_{\mathcal{N}}d\lambda'\right]\simeq\text{const}\times\exp\{-\lambda[N_\tau+\tfrac{1}{2}(N_h+N_{h_1})] -\tfrac{1}{2}(\ln\lambda)C_1N_{h_1}\eta_1^{[1,0]}\} \tag{5.13}$$

in the limit $\lambda\to\infty$. Here the constants denoted by const mean nonuniversal factors depending on v and u_1. Note, however, that the coefficients multiplying λ, $\lambda^{1/2}$, and $\ln\lambda$ in (5.12) and (5.13) are *universal;* in particular, the nonuniversal term $2f_1^{(1)}-f_1$ appearing in $\eta_c^{[1,0]}$ and C_3 drops out from the coefficient of $\ln\lambda$.

Substituting these results, together with those for $\bar{v}$ and $\bar{u}_1$, into (4.1), one can easily work out the critical behavior of $G^{\mathcal{N}}$. As a particularly simple first example consider $G^{(0,0,2)}$, the pair correlation of surface spins, at the multicritical point $c=\tau=0$. Choosing λ such that $e^\lambda\mu r\equiv 1$, and assuming again that $v>0$ and $u_1>u_1(\sigma)$, we find for $r\to\infty$ the asymptotic behavior

$$G^{(0,0,2)}(\mu\mathbf{r};v,u_1)\simeq\text{const}\times\frac{1}{\mu r}[\ln(\mu r)]^{(n+2)(n+4)/[16(3n+22)]}\ . \tag{5.14}$$

On the other hand, if $v\equiv 0$ and $u_1>0$, $G^{(0,0,2)}$ decays as $(\mu r)^{-1}$ without any logarithmic correction; this follows from the fact that $\eta_{h_1}(v=0,u_1)\equiv 0$.

Apart from a missing factor of $\frac{1}{2}$ in the exponent the result (5.14) agrees with that of Speth in Ref. 8. Thus, *in this special case,* Speth's omission of the u_1 term does not affect the above logarithmic correction. However, the logarithmic corrections of many other quantities [as well as subleading corrections to the result (5.14)] *are severely affected,* as we will see below. [That the result (5.14) remains unaffected is due to the fact that $\eta_1(\bar{v},\bar{u}_1)$ does not have a term $\propto\bar{u}_1$, so that its leading contribution for $\lambda\to\infty$ is given by the term $\propto\bar{v}$.]

The effects of the slow vanishing $\sim\lambda^{-1/2}$ of $\bar{u}_1$ show up clearly in quantities involving $\mathcal{E}_c$, such as derivatives with respect to c. As pointed out in Ref. 1 and discussed in the Introduction, we believe that *polymer systems* are particularly well suited for experimental and numerical investigations of the resulting unusual logarithmic corrections. For this reason, the examples we are going to discuss are taken from polymer physics.

We consider a single polymer chain consisting of L repeat units which is immersed into a θ solution bounded by a wall. Following common practice,[5] we will model this polymer by an interacting walk. Instead of specifying directly the Gibbs factor for polymer configurations, we prefer to do this indirectly, utilizing the well-known relationship between the statistics of such walks and the $n\to 0$ limit of n-component spin models.[5,15,17–19,26] In this way one easily realizes that the $n\to 0$ limit of the Hamiltonian given by (1.1), (2.4a), and (2.4b), with $h=w=h_1=c'=w_1\equiv 0$ [i.e., with all interactions breaking the $O(n)$ invariance set to zero], defines for us a proper model for the above polymer system.[1,20] In the polymer picture, the bulk coupling constants u and v measure the strength (in units of k_BT) of the two- and three-body interactions of monomers in the bulk solution, respectively, where the special value $u=0$ corresponds to the critical strength of the two-body interaction at the θ point.[15] (For simplicity, we will therefore set $u=0$ henceforth.) Likewise, the spin interactions $\propto c$ and $\propto u_1$ correspond to one- and two-body interactions localized at the wall. Finally, the critical value $c=0$ corresponds to the adsorption threshold mentioned in the Introduction.[18,19]

To be specific, let us consider a chain whose one end is attached to a point on the surface, but whose other end may be anywhere in the half-space $z>0$. We denote the corresponding chain partition function (defined as a sum

over walks starting at $z=0$ and ending in the region $z>0$) by $Z_1(L)$; it is related to the $n\to 0$ limit of the surface spin susceptibility

$$\chi_1=\int d^d(\mu x)\,\theta(z)G^{(1,0,1)}(\mu\mathbf{x},\mathbf{0}) \tag{5.15}$$

via a Laplace transform, i.e.,

$$\int_0^\infty dL\, e^{-L\tau}Z_1(L,c,v,u_1)=\lim_{n\to 0}\chi_1(\tau,c,v,u_1)\ . \tag{5.16}$$

Since c is the thermodynamic field conjugate to the number of monomers at the surface, it is clear that, apart from a normalization factor,

$$\mathcal{M}_1(L,c,v,u_1)\equiv-\frac{\partial}{\partial c}\ln Z_1(L,c,v,u_1) \tag{5.17}$$

is the average number of monomers at the surface. Information about the critical behavior of $\mathcal{M}_1$, Z_1, or other polymer quantities may be conveniently gained by exploiting the RG directly for these polymer quantities.[27] First, note that the running variable $\bar{L}(\lambda)$ into which L transforms under RG transformations is given by

$$\bar{L}(\lambda)=L/\mathcal{E}_\tau(\lambda;v,u_1)\ . \tag{5.18}$$

This follows from the fact that L is the Laplace variable conjugate to τ and (5.8). Solving the RG of $\mathcal{M}_1$ by characteristics then gives

$$\mathcal{M}_1(L,c,v,u_1)=\mathcal{E}_c(\lambda;v,u_1)\mathcal{M}_1(\bar{L},\bar{c},\bar{v},\bar{u}_1)\ . \tag{5.19}$$

We now choose λ such that $\bar{L}(\lambda)=1$ and use (5.1), (5.3), and (5.4) to conclude that as $L\to\infty$,

$$\mathcal{M}_1(L,c=0,v,u_1)\simeq\text{const}L^{1/2}\times\exp[-(\tfrac{1}{22}\ln L)^{1/2}](\ln L)^{(12+\ln 2)/88}\ , \tag{5.20}$$

if $v>0$ and $u_1>u_1(\sigma)$, while

$$\mathcal{M}_1(L,c=0,v,u_1)\simeq\text{const}L^{1/2}(\ln L)^{-1/4} \tag{5.21}$$

if $v=0$ and $u_1>0$. The latter result (5.21) which applies to the case simulated by van Dieren and Kremer[20] was derived independently in Ref. 28.

Finally, we investigate how the average number of monomers per layer varies as a function of the distance from the surface. To this end we define a layer density of monomers at distance z by

$$\mathcal{M}(\mu z,L)=Z^{-1}\lim_{n\to 0}\mathcal{L}^{-1}_{L\to\tau}\mathcal{N}(\mu z,\tau)\ , \tag{5.22}$$

in which the operator $\mathcal{L}^{-1}_{L\to\tau}$ denotes the inverse of the Laplace transform introduced on the left-hand side of (5.16), and the dimensionless renormalized spin quantity on the rhs is given by

$$\mathcal{N}(\mu z,\tau)=Z_\tau Z_\phi^{-1}Z_1^{-1/2}\mu^3\mathcal{N}_0 \tag{5.23}$$

with

$$\mathcal{N}_0=\int d^dx'\,\theta(z')\left\langle\phi_{0,\alpha}(\mathbf{0})\phi_{0,\alpha}(\mathbf{x}')\tfrac{1}{2}\int d^{d-1}r\vec{\phi}_0^{\,2}(\mathbf{r},z)\right\rangle^{\text{conn}}\ , \tag{5.24}$$

where $\langle\cdot\rangle^{\text{conn}}$ means a connected thermodynamic average. This definition is in conformity with that of $\mathcal{M}_1$: To get $\mathcal{M}_1$ from (5.22), we just have to substitute $\mathcal{N}(\mu z,\tau)$ by its $z=0$ analog, namely

$$\mathcal{N}_1(\tau)=Z_cZ_\phi^{-1}Z_1^{-1/2}\mu^3\mathcal{N}_0(z=0)\ . \tag{5.25}$$

By analogy with (5.19), the RG yields the relation

$$\mathcal{M}(\mu z,L,c,v,u_1)=e^{-\lambda}\mathcal{E}_\tau(\lambda;v,u_1)\mathcal{M}(e^{-\lambda}\mu z,\bar{L},\bar{c},\bar{v},\bar{u}_1)\ . \tag{5.26}$$

To proceed, it is convenient to define dimensionless polymer and spin correlation lengths ξ_L and ξ_τ by

$$\bar{L}(\ln\xi_L)=1\quad\text{and}\quad\bar{\tau}(\ln\xi_\tau)=1\ , \tag{5.27}$$

respectively. From Eqs. (5.8), (5.11), and (5.18) we see that these lengths behave as

$$\xi_L\simeq\text{const}L^{1/2},\quad\xi_\tau\simeq\text{const}\tau^{-1/2}\ , \tag{5.28}$$

as $L\to\infty$ or $\tau\to 0$. We now substitute our choice of $e^\lambda=\xi_L$ into Eq. (5.26). Below the upper critical dimension, we then could replace both running coupling constants $\bar{v}$ and $\bar{u}_1$ on the rhs by their fixed-point values to obtain the scaling form of $\mathcal{M}(\mu z,L)$—the logic being that deviations from these would only give rise to corrections to scaling. However, in the present case more care is necessary. Since we are primarily interested in the behavior near the adsorption threshold, let us first set $c=0$, for simplicity. [The subsequent results remain valid provided that, for given large L, c is so small that $\bar{c}(\ln\xi_L)\ll 1$.] From (5.26) and (5.28), we then have for $L\to\infty$,

$$\mathcal{M}(\mu z,L,c=0,v,u_1)\simeq L^{1/2}F(\mu z/\xi_L,\xi_L)\ , \tag{5.29}$$

with

$$F(\zeta,\xi_L)=\text{const}\mathcal{M}(\zeta,L=1,c=0,\bar{v}(\ln\xi_L),\bar{u}_1(\ln\xi_L))\ . \tag{5.30}$$

If $\zeta\equiv\mu z/\xi_L\gtrsim 1$, we may expand the rhs of (5.30) in powers of $\bar{v}$ and $\bar{u}_1$: This expansion is well behaved because each power of $\bar{v}$ or $\bar{u}_1$ softens the infrared singularity in L by a factor of $(\ln\xi_L)^{-1}$ or $(\ln\xi_L)^{-1/2}$, respectively. Hence, in this regime, $F(\zeta,\xi_L)$ may be replaced by $F(\zeta,\infty)$, the mean-field scaling function. The explicit form of the latter function is given in Appendix C; it is in accordance with the obvious phenomenological expectation that $\mathcal{M}(\mu z,L)$ decays exponentially as $\zeta\to\infty$.

On the other hand, if $\zeta\ll 1$, this expansion *breaks down because of short-distance singularities.* To understand this, note first that the exponential prefactor of $\mathcal{M}(\mu z,L)$ on large length scales simply varies as $e^\lambda\sim L^{1/2}$ [cf. (5.11)], whereas the amplitude of $\mathcal{M}_1(L)$ has the nontrivial scale dependence of $\mathcal{E}_c$ given in (5.12). Since for $1\ll\mu z\ll\xi_L$, $\mathcal{M}(\mu z,L)$ should have the same L dependence as $\mathcal{M}_1(L)$, it is clear that (short-distance) singularities must develop in the limit $L\to\infty$, with z large but fixed. Furthermore, since $\lim_{L\to\infty}\mathcal{M}(\mu z,L)/\mathcal{M}_1(L)$ therefore exists, the z dependence of $\mathcal{M}(\mu z,L)$ in the re-

gime $1 \ll \mu z \ll \xi_L$ should simply be given by the scale dependence $\sim e^{-\lambda}\mathcal{E}_\tau/\mathcal{E}_c$ of $\mathcal{M}/\mathcal{M}_1$, with $\lambda = \ln(\mu z)$.

These considerations can be put on a firm basis by means of the short-distance expansion (cf. Ref. 3, Sec. III.9). Applied to $\mathcal{N}(\mu z, \tau)$, the short-distance expansion yields

$$\mathcal{N}(\mu z, \tau, c, v, u_1) \simeq C(\mu z; v, u_1)\mathcal{N}_1(\tau, c, v, u_1) \quad (5.31)$$

for $\mu z \ll \xi_\tau$. The validity of this relation (which is expected to hold beyond perturbation theory) is verified perturbatively to first order in u_1 in Appendix C, where we also explicitly give the RGEs of both $\mathcal{N}(\mu z, \tau)$ and $\mathcal{N}_1$. From these RGEs and (5.31) it follows that $C(\mu z)$ satisfies the RGE

$$(\mu\partial_\mu + \beta_v\partial_v + \beta_{u_1}\partial_{u_1} + \eta_c - \eta_\tau)C(\mu z, v, u_1) = 0 , \quad (5.32)$$

whose solution in terms of characteristics

$$C(\mu z, v, u_1) = C(e^{-\lambda}\mu z, \bar{v}, \bar{u}_1) \times e^{-\lambda}\mathcal{E}_\tau(\lambda; v, u_1)/\mathcal{E}_c(\lambda; v, u_1) \quad (5.33)$$

indeed gives us the anticipated scale dependence. Making the choice $e^\lambda = \mu z$, we thus see that on large length scales, i.e., for $\mu z \gg 1$ (where μ^{-1} is an adequate microscopic scale on the order of the lattice constant), $C(\mu z)$ behaves as

$$C(\mu z, v, u_1) \sim [\ln(\mu z)]^{-(12+\ln 2)/88} \exp\{[\tfrac{1}{11}\ln(\mu z)]^{1/2}\} , \quad (5.34)$$

if $v > 0$ and $u_1 > u_1(\sigma)$, but as

$$C(\mu z, 0, u_1) \sim [\ln(\mu z)]^{1/4} , \quad (5.35)$$

if $v = 0$ and $u_1 > 0$. By insertion of (5.31) into (5.22) the short-distance expansion of $\mathcal{N}(\mu z, L)$ obviously carries over to $\mathcal{M}(\mu z, L)$. Consequently, the right-hand sides of (5.34) and (5.35) also describe the z dependence of $\mathcal{M}(\mu z, L, c = 0)$ for $L \to \infty$ in the regime $1 \ll \mu z \ll \xi_L$. Hence in both cases, the monomer density *decreases* as z decreases at fixed large L.

The behavior of $\mathcal{M}(\mu z, L, c = 0)$ in the proximal region $1 \ll \mu z \ll \xi_L$ is also discussed in the recent paper by Wang,[28] though only for the special case $v = 0$, $u_1 > 0$. In contrast to our result (5.35), Wang finds a z dependence $\sim [\ln(1/\mu z)]^{-1/4}$. His derivation seems to be based on two assumptions, namely (i) that $\mathcal{M}(\mu z, L, c = 0)$ has the scaling form $L^{1/2} f(\mu z/\xi_L)$, and (ii) that the scaling function $f(\zeta)$ becomes singular as $\zeta \to 0$ in such a way that the correct L dependence (5.21) of $\mathcal{M}_1(L)$ is recovered. However, a glance at (5.29) and (5.30) immediately shows us that if (i) were correct, then $f(\zeta)$ would be given by the mean-field scaling function $F(\zeta, \infty)$. But the latter function approaches a finite and nonvanishing constant as $\zeta \to 0$, so the equality contradicts (ii).

The important point to realize is that if $\zeta \ll 1$, the function $F(\zeta, \xi_L)$ in (5.30) *cannot be computed by an expansion in powers of* $\bar{v}$ *and* $\bar{u}_1$. In other words, *the dependence on the second variable,* ξ_L, is *crucial* and *must not be neglected.* In a perturbative treatment of $F(\zeta, \xi_L)$, the short-distance singularities manifest themselves in the usual way as powers of $\ln\zeta$. In Appendix C we demonstrate by explicit computation of $F(\zeta, \xi_L)$ to first order in $\bar{u}_1$ and zeroth order in $\bar{v}$ the presence of a short-distance singularity $\sim \bar{u}_1 \ln\zeta$. Even though $\bar{u}_1(\ln\xi_L) \sim (\ln L)^{-1/2}$ (if $v > 0$) or $\sim (\ln L)^{-1}$ (if $v = 0$) becomes arbitrarily small in the limit of interest ($L \to \infty$ with fixed $\mu z > 1$), this contribution behaves as

$$\sim (\ln L)^{-1/2} \ln\zeta \sim (\ln L)^{1/2}$$

or as $\sim (\ln L)^{-1}\ln\zeta$ and hence *is much larger than,* or *of the same order as, the mean-field term.* At higher order of perturbation theory, higher powers of $\ln\zeta$ will appear, causing a complete breakdown of this perturbation expansion.

VI. SUMMARY AND CONCLUSIONS

For convenience we present here a brief overview of our findings, referencing the most important expressions. We will also comment on possible extensions of our work.

First, we have constructed an appropriate continuum model for analyzing the surface critical behavior of systems near a bulk tricritical point. The resulting Hamiltonian $\mathcal{H}$ is given by Eqs. (1.1), (2.4(a), and (2.4(b). The main difference between this semi-infinite model and those previously studied is the presence of the surface nonlinearity $\propto u_1 |\vec{\phi}|^4$ in $\mathcal{H}$. In addition, we also included a variety of relevant bulk and surface terms breaking $O(n)$ symmetry.

In order to show that the analysis of this model falls well into the realm of the field-theoretic RG approach, we then proceeded by explaining its renormalization. The reparametrizations required to absorb the ultraviolet singularities at and below the upper critical dimension $d = 3$ are given in (2.14). Knowing the form of these reparametrizations, we were able to derive renormalization group equations; those of the $(N_h + N_{h_1})$-point functions with N_τ insertions of $\phi^2(\mathbf{r}, z > 0)$, namely the functions $G^{\mathcal{N}}$ with $\mathcal{N} = (N_h, N_\tau, N_{h_1})$, are explicitly given in (3.5). Next we computed various surface renormalization functions (P_I, Z_c, and Z_1) to obtain the desired renormalization-group functions β_{u_1}, η_c, and η_{h_1}. The results are contained in Eqs. (3.11)–(3.28).

The inclusion of the surface term $\propto u_1$ turned out to be absolutely *crucial:* If not included from the outset, such a term is generated under the renormalization group, owing to the bulk interaction $\propto v |\phi|^6$. It was precisely due to this mixing of the $|\vec{\phi}|^4$ surface term and the $|\vec{\phi}|^6$ bulk term under renormalization that the flow equation of the running coupling constant $\bar{u}_1$ picked up a term linear in $\bar{v}$. This in turn implied that $(u_1^*)_T$, the fixed-point value of the infrared-stable fixed point T describing the special transition in the presence of a tricritical bulk—and hence the surface critical exponents—have an expansion in powers of $\epsilon^{1/2}$ if $\epsilon > 0$, and led to the unusu-

al logarithmic corrections discussed in Sec. V at the UCD. The resulting RG flow equations are given in Eq. (4.3); the flow diagrams for $d<3$ and $d=3$ are shown in Figs. 1 and 2, respectively.

For $\epsilon>0$, we found in addition to T and the Gaussian fixed point G, two other fixed points, T' and LR. Both have basins of attraction of lower dimension than T, so their observation requires that more thermodynamic constraints are met. The fixed point LR describes the special transition in the special case where the interactions are entirely *Gaussian* in the bulk, though *nonlinear* at the surface. As shown in Ref. 1, and confirmed here, the surface critical behavior of this transition belongs to the universality class of a $d-1$ dimensional bulk system with long-range interactions studied by Fisher, Ma, and Nickel.[23] The ϵ expansion of the respective critical exponents is given by Eqs. (4.14)–(4.17). The $\epsilon^{1/2}$ expansion of the surface exponents of T may be found in (4.12) and (4.13).

For $n=0$, the fixed point LR describes the long-distance behavior of walks which are completely random in the bulk, but self-avoiding at, and interacting with, the surface. Thus our results for $v=0$ directly apply to the simplified model for the adsorption of θ polymers recently simulated by van Dieren and Kremer.[20] Yet, inclusion of the bulk term $\propto v$, i.e., of a three-body bulk interaction between monomers, *is important,* as we have seen, because the logarithmic corrections in the two cases $v=0$ and $v>0$ are *qualitatively and quantitatively different.* This is clearly borne out by our results (5.20) and (5.21) for the mean number of monomers at the surface. A computer simulation of the case $v>0$ would therefore be very valuable. We also investigated in some detail how $\mathcal{M}(\mu z,L)$, the mean number of monomers per layer of a single polymer chain, varies as a function of the distance z from the surface, showing that the distinct L dependence of $\mathcal{M}(\mu z,L)$ close to and away from the surface entails the nontrivial z dependence (5.34) or (5.35) in the short-distance regime, where z is small compared to the radius of gyration of the polymer.

In light of the findings mentioned above it is natural to ask whether inclusion of the u_1 surface term might also change the results for the ordinary transition. Clearly, the inclusion of this term is indispensable if one wishes to study the crossover from special to ordinary surface critical behavior. However, we do not expect dramatic changes as far as the *asymptotic* behavior at the ordinary transition is concerned, so that the corresponding results of Speth[8] essentially should be correct. This question could be clarified by combining our above analysis with an expansion in powers of c_0^{-1}, along lines similar to those taken in Secs. 3.6 and 3.7 of Ref. 3. As another possible extension of our work, one might study the behavior in the case where (v,u_1) belongs to the region between the stability line and the separatrix σ (cf. Fig. 1 and Ref. 24). This could be done by evaluating the fluctuation correction to the surface free energy. Finally, it should be mentioned that we have ignored completely the possibility that the wall-monomer interaction (i.e., c) varies in a random fashion. Arguments similar to those used in Refs. 12–14 reveal that quenched surface randomness of this kind is marginal in three dimensions. The influence of such randomness on the critical behavior studied above will be discussed in a forthcoming paper.

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APPENDIX A: EVALUATION OF P_I

In this Appendix we compute the renormalization function P_I to the order vu_1 and u_1^3. To this end let us introduce the parallel Fourier transform

$$\phi_{\mathbf{p},\alpha}(z)\equiv\int d^{d-1}r\, e^{-i\mathbf{p}\cdot\mathbf{r}}\phi_\alpha(\mathbf{r},z) \tag{A1}$$

and consider the connected four-point correlation function

$$\left\langle\prod_{k=1}^{4}\phi_{\mathbf{p}_k,\alpha_k}(z_k)\right\rangle^{\text{conn}}$$

for $\{g=0\}$, i.e., for $h=\tau=w=u=h_1=c=c'=w_1=0$. From the remark below (2.9) it should be clear that, for such values of $\{g\}$, the counterterms introduced by Eqs. (2.13a), (2.13b), and (2.14) are sufficient to renormalize this function, regardless of whether α_k specifies a transverse or longitudinal component.

We compute the above correlation function perturbatively, using the Neumann propagator G_N given in (2.1) as the free propagator. Expanding all counterterms in powers of v and u_1, and defining

$$\Delta(\mathbf{p})\equiv(2\pi)^{d-1}\delta^{d-1}(\mathbf{p}) \tag{A1'}$$

we may write

$$\left\langle\prod_{k=1}^{4}\phi_{\mathbf{p}_k,\alpha_k}(z_k)\right\rangle^{\text{conn}}=\Delta\left[\sum_{k=1}^{4}\mathbf{p}_k\right]2\pi f^{(1)}\mu^{\epsilon}\sum_{i,j}F_{i,j}v^i u_1^j\ . \tag{A2}$$

The terms contributing to $F_{i,j}$ with $(i,j)=(0,1)$, (0,2), (0,3), (1,0), and (1,1) are shown graphically in Figs. 3 and 4. The square with a label (i,j) attached denotes the term of order $v^i u_1^j$ of the surface vertex $(u_1+P_I)Z_\phi^2\,|\vec{\phi}\,|^4$. Aside from this vertex, only the term of first order in v of the $|\vec{\phi}\,|^6$ bulk vertex contributes to the order we are calculating. This term is denoted by a full circle. Graphs containing lines with both ends connected to the same square have been omitted because they correspond to contributions $\propto G_b(0)$, a quantity which vanishes in dimensional regularization.

According to Figs. 3 and 4 we have

$$F_{0,2}=F_{0,2}^{A}+F_{0,2}^{B}\ , \tag{A3a}$$

$$F_{0,3}=\sum_{k=A}^{E}F_{0,3}^{k}\ , \tag{A3b}$$

$$F_{1,0}=F_{1,0}^{A}+F_{1,0}^{B}\ , \tag{A3c}$$

$$E$$

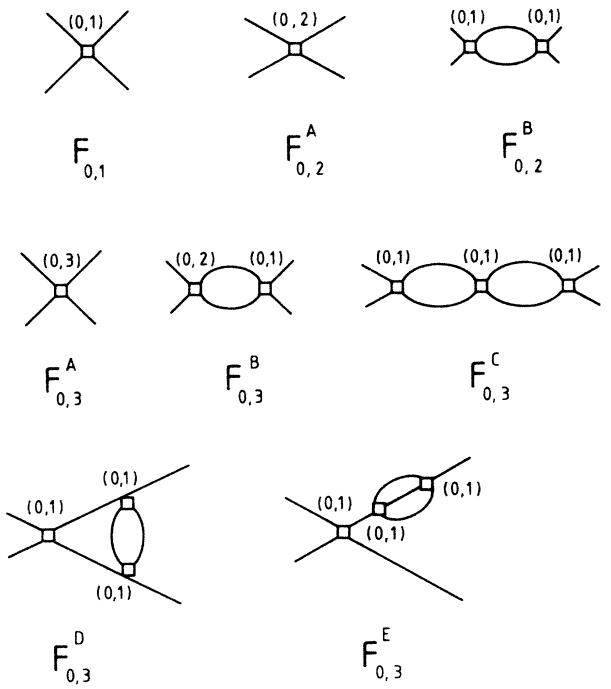

FIG. 3. Graphs contributing to the coefficients $F_{0,j}$, with $j=1,2,3$. For further explanation see text.

$$F_{1,1}=\sum_{k=A} F^{k}_{1,1} \ . \tag{A3d}$$

Introducing the shorthands

$$L=\prod_{k=1}^{4} p_k^{-1} e^{-p_k z_k} \ , \tag{A4}$$

$$T=\delta_{\alpha_1,\alpha_2}\delta_{\alpha_3,\alpha_4} \ , \tag{A5}$$

and

$$S=\tfrac{1}{3}(T+[1\leftrightarrow 3]+[1\leftrightarrow 4]) \ , \tag{A6}$$

where here and below $[1\leftrightarrow 3]$ means an interchange of the subscripts on the variables $(\alpha_k, \mathbf{p}_k, z_k)$, one obtains the explicit expressions:

$$F_{0,1}=-LS \ , \tag{A7a}$$

$$F^{A}_{i,j}=-A_{i,j}LS \ ,$$

$$\text{for } (i,j)=(0,2),(0,3),(1,0),(1,1) \ . \tag{A7b}$$

The remaining contributions in Figs. 3 and 4 can be compactly written in the form

$$F^{k}_{i,j}=\tfrac{1}{3}L\hat{F}^{k}_{i,j}+[1\leftrightarrow 3]+[1\leftrightarrow 4]+O(\epsilon^0) \ , \tag{A8}$$

where the $\hat{F}$ are given by

$$\hat{F}^{B}_{0,2}=\frac{n+8}{6\epsilon}S \ , \tag{A9a}$$

$$\hat{F}^{B}_{1,0}=-\frac{n+4}{80\epsilon}S \ , \tag{A9b}$$

$$\hat{F}^{B}_{i,j}=(\epsilon^{-1}+Q)A_{i,j-1}\left[\frac{n+2}{3}T+2S\right] \ ,$$

$$\text{for } (i,j)=(0,3),(1,1) \ , \tag{A9c}$$

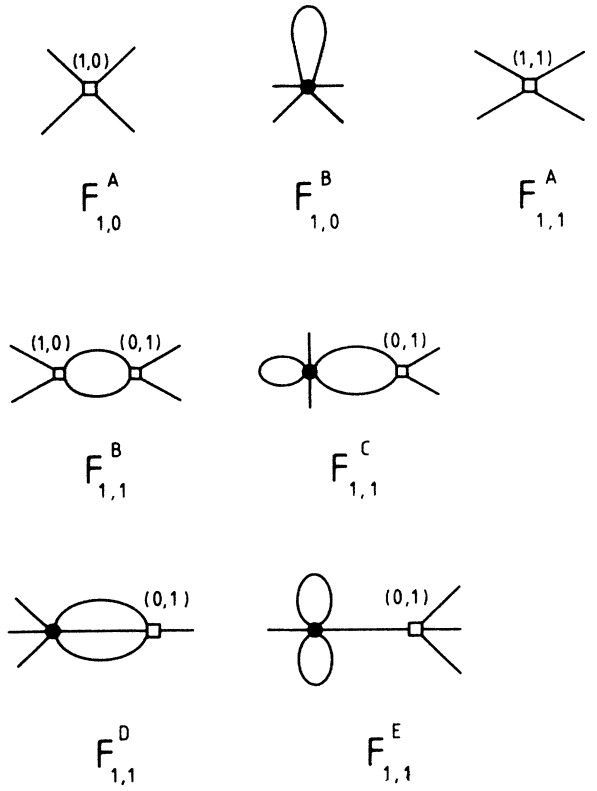

FIG. 4. Graphs contributing to $F_{1,0}$ and $F_{1,1}$.

$$\hat{F}^{C}_{0,3}=-\tfrac{1}{3}(\epsilon^{-2}+2\epsilon^{-1}Q)\left[\frac{T}{12}(n+2)(n+4)+S\right] \ , \tag{A9d}$$

$$\hat{F}^{D}_{0,3}=-\tfrac{1}{3}[\epsilon^{-2}+2\epsilon^{-1}(Q+\ln 2)]\left[\frac{n+2}{3}T+\frac{n+6}{2}S\right] \ , \tag{A9e}$$

$$\hat{F}^{E}_{0,3}=0 \ , \tag{A9f}$$

$$\hat{F}^{C}_{1,1}=[\epsilon^{-2}+\epsilon^{-1}(2Q+f_1-2f_1^{(1)})]$$

$$\times\frac{n+4}{160}\left[\frac{n+2}{3}T+2S\right] \ , \tag{A9g}$$

$$\hat{F}^{D}_{1,1}=\frac{n+4}{30\epsilon}S \ , \tag{A9h}$$

$$\hat{F}^{E}_{1,1}=-\frac{(n+4)(n+2)}{1920\epsilon}S \ . \tag{A9i}$$

Here

$$Q=3\ln 2+\tfrac{1}{2}\ln\pi-(C_E/2)+f_1^{(1)}-\ln(|\mathbf{p}_1+\mathbf{p}_2|/\mu) \ , \tag{A10}$$

and $C_E=0,577\ldots$ is Euler's constant.

Using these results, the coefficients $A_{i,j}$ given in (3.19)–(3.22) can be derived in a straightforward fashion.

APPENDIX B: EVALUATION OF Z_c AND Z_1

We proceed as in Appendix A. We write

$$Z_c = 1 + \sum_{i,j} C_{i,j} v^i u_1^j \ , \tag{B1}$$

$$Z_1^{-1/2} = 1 + \sum_{i,j} D_{i,j} v^i u_1^j \ , \tag{B2}$$

set $\{g=0\}$, and determine the coefficients $C_{i,j}$ and $D_{i,j}$ from the requirement that the functions

$$\int d^{d-1}r\, e^{-i\mathbf{p}\cdot\mathbf{r}} \langle \phi_{\mathbf{p}_1,\alpha_1}(z_1)\phi_{\mathbf{p}_2,\alpha_2}(z_2)\tfrac{1}{2}Z_c Z_\phi \vec{\phi}^2(\mathbf{r},0)\rangle^{\rm conn} = \Delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p})\delta_{\alpha_1,\alpha_2}\sum_{i,j} H_{i,j} v^i u_1^j \tag{B3}$$

and

$$\langle Z_1^{-1/2}\phi_{\mathbf{p}_1,\alpha_1}(0)\phi_{\mathbf{p}_2,\alpha_2}(z_2)\rangle^{\rm conn} = \Delta(\mathbf{p}_1+\mathbf{p}_2)\delta_{\alpha_1,\alpha_2}\sum_{i,j} J_{i,j} v^i u_1^j \tag{B4}$$

be finite. The contributions to the coefficients H_{ij}, with $(i,j)=(0,0)$, (0,1), (0,2), and (1,0) are shown in Figs. 5 and 6; those to $J_{i,j}$ with $(i,j)=(0,0)$, (1,0), (1,1), are shown in Fig. 7. The graphical notation is analogous to that in Appendix A. The broken line in Figs. 5 and 6 with the label (i,j) attached denotes the coefficient of $v^i u_1^j$ of the surface vertex $Z_c Z_\phi \vec{\phi}^2(\mathbf{r},0)$. Likewise, the empty circle in Fig. 7 with (i,j) attached means the coefficient of $v^i u_1^j$ of the surface vertex $Z_1^{-1/2}\phi_{\alpha_1}(\mathbf{r},0)$. In both cases, the label (0,0) is suppressed.

From Figs. 5 and 6 one finds

$$H_{0,1} = H^A_{0,1} + H^B_{0,1} \ , \tag{B5a}$$

$$H_{0,2} = \sum_{k=A}^{E} H^k_{0,2} \ , \tag{B5b}$$

$$H_{1,0} = \sum_{k=A}^{D} H^k_{1,0} \ , \tag{B5c}$$

with

$$H_{0,0} = K \ , \tag{B6a}$$

$$H^A_{i,j} = C_{i,j} K \quad \text{for } (i,j)=(0,1),(0,2),(1,0) \ , \tag{B6b}$$

where

$$K \equiv (p_1 p_2)^{-1} e^{-(p_1 z_1 + p_2 z_2)} \ . \tag{B7}$$

The remaining contributions have the form

$$H^k_{i,j} = K\hat{H}^k_{i,j} + O(\epsilon^0) \tag{B8}$$

with

$$\hat{H}^B_{0,1} = -\frac{n+2}{6\epsilon} \ , \tag{B9a}$$

$$\hat{H}^B_{0,2} = -(\epsilon^{-1}+Q')\frac{n+2}{6} C_{0,1} \ , \tag{B9b}$$

$$\hat{H}^C_{0,2} = -(\epsilon^{-1}+Q')\frac{n+2}{6} A_{0,2} \ , \tag{B9c}$$

$$\hat{H}^D_{0,2} = \tfrac{1}{4}(\epsilon^{-2}+2\epsilon^{-1}Q')\left[\frac{n+2}{3}\right]^2 , \tag{B9d}$$

$$\hat{H}^E_{0,2} = \tfrac{1}{4}[\epsilon^{-2}+2\epsilon^{-1}(Q'+\ln 2)]\frac{n+2}{3} \ , \tag{B9e}$$

$$\hat{H}^B_{1,0} = -(\epsilon^{-1}+Q')\frac{n+2}{6} A_{1,0} \ , \tag{B9f}$$

$$\hat{H}^C_{1,0} = -\frac{(n+2)(n+4)}{5!\,8} \times [\epsilon^{-2}+\epsilon^{-1}(f_1 - 2f_1^{(1)} + 2Q')] \ , \tag{B9g}$$

$$\hat{H}^D_{1,0} = \frac{(n+2)(n+4)}{5!\,32}\epsilon^{-1} \ . \tag{B9h}$$

Here the $A_{i,j}$ are given by Eqs. (3.19)–(3.22) and Q' is the quantity Q defined in (A10), with $|\mathbf{p}_1+\mathbf{p}_2|$ replaced by p. From these results Eq. (3.23) follows in a straightforward fashion.

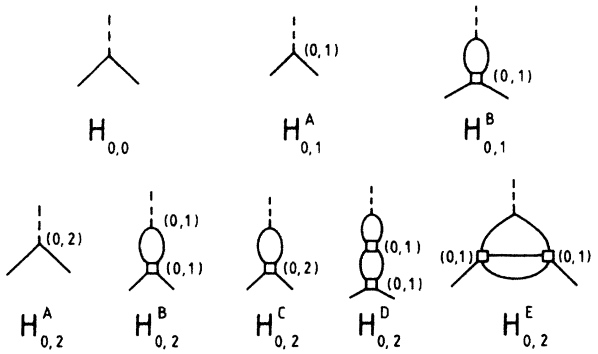

FIG. 5. Graphs contributing to $H_{0,0}$, $H_{0,1}$, and $H_{0,2}$.

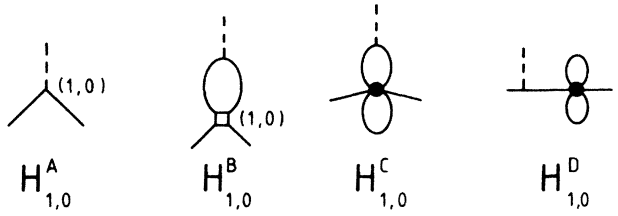

FIG. 6. Graphs contributing to $H_{1,0}$.

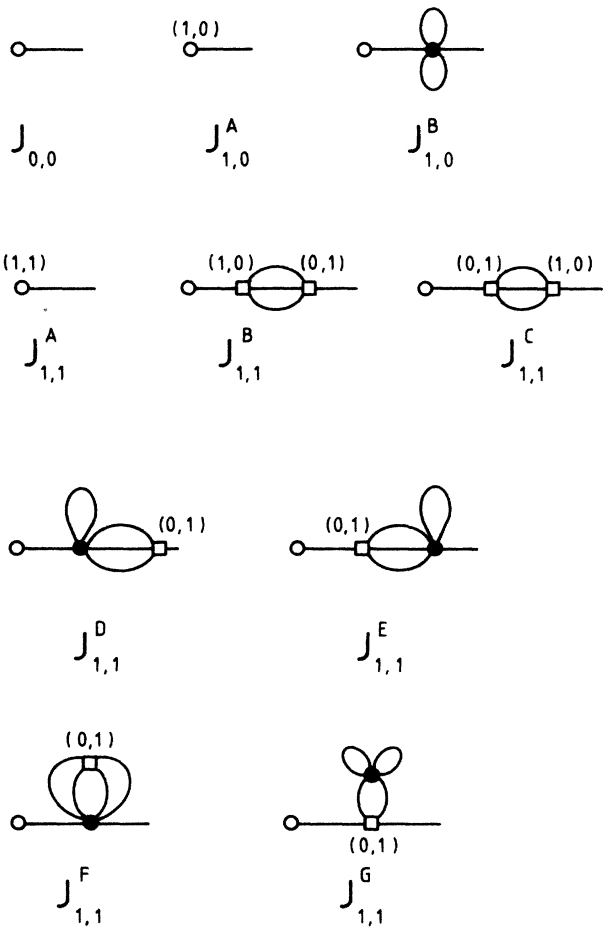

FIG. 7. Graphs contributing to $J_{0,0}$, $J_{1,0}$, and $J_{1,1}$.

Turning to Fig. 7, we recall from our discussion at the end of Sec. III that

$$D_{0,j}=0 \quad \text{all } j \ , \tag{B10}$$

i.e., the $J^{k}_{0,j}$ do not contribute to final subtractions. With the exception of $J_{0,0}$, they are, therefore, not shown. For the other $J_{i,j}$ we have

$$J_{0,0}=M\equiv\frac{1}{p_2}e^{-p_2 z_2} \ , \tag{B11a}$$

$$J_{1,0}=J^{A}_{1,0}+J^{B}_{1,0} \ , \tag{B11b}$$

$$J_{1,1}=\sum_{k=A}^{G} J^{k}_{1,1} \ , \tag{B11c}$$

with

$$J^{A}_{i,j}=D_{i,j}M \quad \text{for } (i,j)=(1,0),(1,1) \ , \tag{B12a}$$

$$J^{G}_{1,1}=0 \ , \tag{B12b}$$

$$J^{k}_{i,j}=\frac{(n+2)(n+4)}{\epsilon}M\hat{J}^{k}_{i,j}+O(\epsilon^0)$$

$$\text{for } (i,j)=(1,0), \quad k=B \ ,$$

$$\text{and } (i,j)=(1,1), \quad k=B,C,D,E,F \ , \tag{B12c}$$

where

$$\hat{J}^{B}_{1,0}=(5!\,64)^{-1} \ , \tag{B13a}$$

$$\hat{J}^{B+C}_{1,1}=(720)^{-1} \ , \tag{B13b}$$

$$\hat{J}^{D}_{1,1}=-(5\times 6^3)^{-1} \ , \tag{B13c}$$

$$\hat{J}^{E}_{1,1}=\tfrac{3}{4}\hat{J}^{D}_{1,1} \ , \tag{B13d}$$

$$\hat{J}^{F}_{1,1}=-(20\times 6^3)^{-1} \ . \tag{B13e}$$

Using these results, Eq. (3.24) can be derived in a straightforward fashion.

APPENDIX C: SHORT-DISTANCE EXPANSION OF $\mathcal{N}(\mu z,\tau)$

Here we wish to verify to first order in u_1 that the short-distance expansion (5.31) holds within perturbation theory. Except for τ, we take all relevant fields $\{g\}$ to be zero. At order u_1, there are three graphs contributing to $\mathcal{N}$: One is the analog of the diagram $H^{B}_{0,1}$ in Fig. 5, where the broken line now denotes an insertion of $\vec{\phi}^2$ away from the surface; the other two differ from this one in that the broken line, instead of being attached to the bubble, is connected to the left or right external leg. Evaluating these graphs and adding the mean-field contribution, we find

$$\mathcal{N}(\mu z,\tau)=\tau^{-3/2}e^{-\zeta}-\frac{\pi}{3}f^{(1)}u_1(n+2)\tau^{-3/2}I(\zeta)$$

$$+f^{(1)}u_1\frac{n+2}{6}\tau^{-3/2}(e^{-2\zeta}+e^{-\zeta})$$

$$+O(\epsilon)+O(u_1^2,v) \ , \tag{C1}$$

with

$$I(\zeta)\equiv\mu^{\epsilon}\int d^{d-1}r[G_N(\mathbf{r};z,0)]^2$$

$$=\begin{cases}\dfrac{1}{2\pi}E_1(2\zeta)+O(\epsilon) & \text{for } \zeta>0\\ \Gamma(\epsilon/2)(4\pi)^{-1+\epsilon/2}\tau^{-\epsilon/2} & \text{for } \zeta=0\end{cases} \ , \tag{C2}$$

where E_1 means an exponential integral in the notation of Ref. 29. The result (C1) also gives us $\mathcal{N}_1$ to the same order; we only must add the contribution from the counterterm $\propto Z_c-1$ (cf. graph $H^{A}_{0,1}$ in Fig. 5) to cancel the pole of $I(0)$.

Using these results, together with the familiar limiting behavior of $E_1(\zeta)$ for $\zeta\to 0$, one easily verifies that Eq.

(5.31) holds to this order, with

$$C(\mu z)=1+u_1\frac{n+2}{6}[\ln(\mu z)+f_1^{(1)}+\tfrac{1}{2}\ln(16\pi)+\tfrac{1}{2}C_E] +O(u_1\epsilon,u_1^2,v)\ . \tag{C3}$$

The RGE of $\mathcal{N}(\mu z,\tau)$ and $\mathcal{N}_1(\tau)$ read

$$(\mu\partial_\mu+\beta_v\partial_v+\beta_{u_1}\partial_{u_1}-\vartheta_\tau\tau\partial_\tau -3+\tfrac{1}{2}\eta_1+\eta_\phi-\eta_\tau)\mathcal{N}(\mu z,\tau)=0 \tag{C4}$$

and

$$(\beta_v\partial_v+\beta_{u_1}\partial_{u_1}-\vartheta_\tau\tau\partial_\tau-3+\tfrac{1}{2}\eta_1+\eta_\phi-\eta_c)\mathcal{N}_1(\tau)=0\ . \tag{C5}$$

From these equations and (5.31) the RGE (5.32) of $C(\mu z)$ follows in the usual manner.

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