Magnetic exchange in disordered metals

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We consider the indirect exchange between magnetic moments placed in a metallic environment that is weakly disordered due to the presence of randomly distributed nonmagnetic impurities. We find that the even moments of the isotropic exchange interactions are long-ranged, falling off as a power law in the spin separation distance just as in the pure metal. Depending on the experimental quantity of interest, the effective coupling between spins is given by either the first or the second moment of the distribution of exchange couplings in the system. The effect of spin-orbit scattering on the interaction between local moments is considered. In the case of isotropic spin-orbit scattering in both bulk materials and thin films, the isotropic exchange interactions are exponentially suppressed at large distances. In strictly two dimensions, however, there are long-range anisotropic spin-exchange couplings that have power-law fall-off with the square of the distance. In this case the interaction becomes effectively Ising at large distances. The distribution of couplings is seen to be broad, reflecting the sensitivity of the interactions to changes in the impurity configuration. Correlations of the couplings between sites are also found to have power-law fall-off with distance; however, these are higher order in the perturbation expansion. The relevance to experiments is discussed.

I. INTRODUCTION

We consider magnetic interactions between local moments situated in a weakly disordered metal. The indirect Ruderman-Kittel-Kasuya-Yosida (RKKY) exchange interaction between two moments mediated by the conduction electrons of the host metal is well known in the case of the pure metal to be an oscillatory function that falls off with the cube of the distance.[1] In the presence of disorder, which we assume here to arise due to elastic scattering of the electrons from nonmagnetic impurities, the magnetic exchange is modified from that of the pure metal because of the existence of a finite mean free path. The interaction between spins situated a given distance apart is now taken from a distribution over the values of the exchange coupling corresponding to different impurity configurations. A knowledge of the moments of this distribution[2] would help determine such quantities as the typical strength of the exchange coupling in a disordered metal, and hence an estimate of the critical temperature if the sample were to undergo a magnetic transition. We compute the moments in the weak disorder limit, where it is assumed that the spin-separation distance is much greater than the electron mean free path, and the dominant modification of the large distance behavior arises from the diffusive motion of conduction electrons in the disordered medium. Thus the diffusion-propagator ladder graphs may be considered to give the most important contributions. It is seen that the first moment of the distribution is short ranged,[3] reflecting the sensitivity of the phase of the RKKY interaction to changes in the impurity potential. However, averaging the square of this oscillating potential, one finds that it decays with the same power law as in the pure metal.[4,5] Thus although the average coupling decays exponentially with the distance, this is the consequence of performing an incoherent phase average that obscures the fact that the interactions are in fact long-ranged for any given impurity configuration. We estimate higher moments of the distribution and show that it is non-Gaussian and fairly broad. In addition, we investigate magnetic correlations due to a given local moment between two sites that are arbitrarily far apart. These do not decay exponentially as might have been expected, but have power-law fall-off with the distance. We discuss briefly the implications for spin glasses and amorphous metallic alloys.

In an early paper[3] de Gennes found the exponential decay of the average RKKY interaction. His result has often been used to analyze experiments on metallic spin glasses, for example. Later, de Châtel[2] pointed out the possible importance of fluctuations of the RKKY interaction, and noted that the interactions in disordered metals could retain the long-ranged oscillatory character of the pure RKKY interaction provided that the electron wave functions are extended and have smoothly varying phase as a function of energy.[6] Recently, Zyuzin and Spivak,[4] and Bulaevskii and Panyukov[5] have given the correct interpretation of the de Gennes result and have pointed out that the spin-glass transition temperature depends on typical values of the RKKY interaction and that these are long-ranged. The transition temperature in a spin glass is therefore not expected to exhibit any significant dependence on the mean free path (over a range of values of l). Indeed, unsuccessful attempts to fit experimental results to the exponential form[7-10] led to the conclusion[10] that the dependence on mean free path was relatively slight.

More generally, let us regard the interactions in the disordered metal as arising from some probability distribution whose moments may be calculated. In the absence of ferromagnetism the transition temperature may be taken in mean-field theory to be the second moment of the distribution, which, as we shall see, is long-ranged. However, if one admits a possible ferromagnetic ordering in the system, the transition temperature would then be

given in mean field by the first moment of the distribution and in this case one would expect exponential suppression of the transition temperature as disorder is increased.

We have so far discussed processes of elastic impurity scattering that do not flip electron spin. The addition of impurities with spin-orbit interactions will modify the spin-exchange couplings at large distances. We find, as expected, that isotropic spin-spin interactions are limited in range by the spin-orbit diffusion length $L_{s.o.}$. This result holds only for spatially isotropic spin-orbit scattering. When the motion of the electrons is restricted to lie in the plane, however, we find that typical interactions remain long ranged and exchange couplings corresponding to anisotropic Dzyaloshinskii-Moriya–type interactions fall off with the inverse square of the distance. In two dimensions the residual spin interactions at sufficiently large separation become Ising in character.

In this paper we consider noninteracting electrons throughout, since Coulomb interactions between the electrons produce effects[11] of higher order than those we consider here.

II. THE RKKY INTERACTION IN A DISORDERED METAL

The interaction energy of two spins $\mathbf{S}_1(\mathbf{r}_1)$ and $\mathbf{S}_2(\mathbf{r}_2)$ can be expressed in terms of the zero frequency nonlocal electronic susceptibility as

$$H_{\text{int}} = -J^2 \mathbf{S}_1 \cdot \mathbf{S}_2 \chi(\mathbf{r}_1,\mathbf{r}_2) \ , \tag{1}$$

where J is the local moment-to-conduction electron coupling, and the susceptibility is written in terms of Matsubara Green's functions for the conduction electron as

$$\chi(\mathbf{r}_1,\mathbf{r}_2) = \frac{2}{\beta} \sum_{\omega_n} G_{\omega_n}(\mathbf{r}_1,\mathbf{r}_2) G_{\omega_n}(\mathbf{r}_2,\mathbf{r}_1) \ , \tag{2}$$

(ω_n are the discrete fermion frequencies). In the pure metal, at zero temperature, Eq. (2) can be evaluated to obtain the RKKY exchange interaction,

$$\chi_0(R) = -\frac{2mk_F \cos(2k_F R)}{(2\pi)^3 R^3} \ , \tag{3}$$

where $R = |\mathbf{r}_1 - \mathbf{r}_2|$.

We note that this is long-ranged, falling off as R^{-3}. It exhibits Friedel oscillations, making possible the two features of competition and frustration that are responsible for spin glass behavior.

In the metal with disorder, we calculate first the average over random impurity configurations of the susceptibility (denoted by $[F]_{\text{av}}$) and find

$$[\chi(R)]_{\text{av}} = \chi_0(R) e^{-R/l} \ , \tag{4}$$

where l is the mean free path. This is computed in the weak disorder limit (the perturbation parameter being $1/k_F l$) and is valid at distances R exceeding the mean free path l. The result in Eq. (4), first written by de Gennes, has been interpreted to mean that the range of magnetic interactions in disordered metals is limited by the mean free path l, being exponentially attentuated at large distances. It was pointed out however by de Châtel[2] that it is necessary to investigate higher moments of the distribution of $\chi(R)$ to determine the importance of fluctuations, and hence the range and strength of the interactions in a particular sample.

The second moment of the susceptibility has leading long-distance contributions from the processes shown in Fig. 1,[4] and is given by

$$\chi^2(R) = \frac{4}{\beta^2} \sum_{\substack{\omega_1 > 0 \\ \omega_2 < 0}} [[G_{\omega_1}(R) G_{\omega_2}(R)]^2]_{\text{av}} \ . \tag{5}$$

The averages over the electron Green's functions taken pairwise can be evaluated in the diffusion-propagator approximation to be (omitting exponentially small terms)

$$[G_{\omega_1}(R) G_{\omega_2}(R)]_{\text{av}} = \frac{3m^2}{4\pi^2 l R} \exp[-(3|\omega_1 - \omega_2|\tau)^{1/2} R/l] \ , \tag{6}$$

for ω_1 and ω_2 of opposite sign ($\tau = l/v_F$). Replacing the sums over frequency in Eq. (5) by integrals in the limit of zero temperature, and using the approximation above, one finds

$$[\chi^2(R)]_{\text{av}} = 3 \left(\frac{mk_F}{(2\pi)^3} \right)^2 \frac{1}{R^6} \ . \tag{7}$$

We note, firstly, that the interaction falls off as a power law in the spin-separation distance, and also that it is independent of the mean free path. This result is in agreement with the results reported in Refs. 4 and 5. One interpretation of Eqs. (4) and (7) is perhaps readily seen in the model considered by Bulaevskii and Panyukov,[5] in which the electrons move in a slowly varying weak random potential. Starting from a path integral representation of the electronic susceptibility defined in Eq. (2) they evaluate it in a semiclassical approximation. This yields an expression for $\chi(R)$, the unaveraged susceptibility, in terms of a functional of the random scattering potential. The result thus obtained for the susceptibility is an RKKY-like expression that has an R^{-3} prefactor multiplying an oscillatory cosine term. The effect of the impurity distribution is to introduce a random phase shift in the oscillations of $\chi(R)$. This is an explicit illustration of a model in which it is the phase rather than magnitude of the oscillatory potential that is sensitive to disorder effects. Averaging randomly phase-shifted oscillations results in the exponential decay of the average susceptibility. For a given sample, of course, the strength of the interactions must be measured by the amplitude rather than phase of the exchange potential, and the quantity of relevance in a number of measurements to be described is the typical strength of the

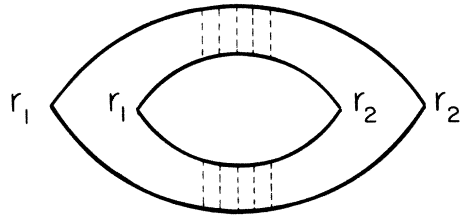

FIG. 1. Diagram for leading large-distance contribution to $[\chi^2(R)]_{\text{av}}$. The dashed lines denote impurity ladders.

coupling between magnetic moments. This is given by $J_{\rm eff}=J^2[\chi^2(R)]_{\rm av}]^{1/2}$, which is therefore seen to fall off like R^{-3} as in the pure metal.

We note that the result for $[\chi^2(R)]_{\rm av}$ in Eq. (7) might have been inferred from taking the interaction in dirty systems to be given by Eq. (3) modified to include a random impurity-dependent phase shift (as suggested by the approach of Ref. 5). Squaring this interaction and averaging it over impurity configurations would result in Eq. (7) up to an overall numerical prefactor that would depend on the choice of the distribution assumed for the random parameters. In particular, if we made the simplifying assumption that the random phase shift $\phi\{\mathbf{R}_i\}$ is a variable uniformly distributed in the interval $[0,2\pi]$, we would obtain a prefactor of 2 in Eq. (7) rather than the somewhat enhanced value that we obtain upon summing the ladder graphs.

This oversimplifies the true situation (as does Ref. 5). In this random phase model, the fourth cumulant of $\chi(R)$ is negative. We show below that this quantity is actually positive. As pointed out by C. Henley (private communication), this implies that there are appreciable amplitude fluctuations in $\chi(R)$.

Since $[\chi^2(R)]_{\rm av}$ gives us information on the strength of the interaction between spins a distance R apart, a question of interest arises concerning the fluctuations of the interactions around the average value given by Eq. (7). This question relates to the width of the distribution of the coupling corresponding to a given distance, and determines whether the estimate of $J_{\rm eff}$ determined from $([\chi^2]_{\rm av})^{1/2}$ is a good estimate of the "typical" exchange coupling in a particular sample. We compute therefore the fourth moment, or the corresponding cumulant

$$\langle\chi^4\rangle_c=[\chi^4(R)]_{\rm av}-3[[\chi^2(R)]_{\rm av}]^2 ,$$

which represents the deviation from a Gaussian distribution (the first and third moments can be ignored in the asymptotic region since they decay exponentially). The leading contribution to this is shown in Fig. 2. We can easily generalize the analysis to higher moments. The leading contribution to the connected part of $[\chi^{2n}]_{\rm av}$ is

$$\langle\chi^{2n}\rangle_c=2^{2n}(2n-1)!\left[\frac{mk_F}{(2\pi)^3}\right]^{2n}\frac{I_{2n}}{R^{6n}} , \tag{8}$$

where

$$I_{2n}=\int_0^\infty\cdots\int_0^\infty dx_1\cdots dx_{2n}\exp\left[-(x_{2n}+x_1)^{1/2}-\sum_{i=1}^{2n-1}(x_i+x_{i+1})^{1/2}\right] . \tag{9}$$

In particular, the fourth cumulant is

$$\langle\chi^4\rangle_c=2^4 3!I_4\left[\frac{mk_F}{(2\pi)^3}\right]^4\frac{1}{R^{12}} , \tag{10}$$

where I_4 is evaluated numerically to be ≈ 0.4. Taking the square root of $\langle\chi^4\rangle_c$, we find that it is comparable in magnitude to the second moment given by Eq. (7). The distribution of $\chi(R)$ is thus seen to be fairly broad. This is an indirect confirmation of the statement made earlier, namely that changes in the disorder configuration have the effect of phase shifting the oscillations in χ, and leads to a broad, flat distribution of width proportional to R^{-3}.

In two dimensions in a pure system, $\chi_0(R)=-(m/2\pi^2R^2)\sin 2k_Fr$ and in the disordered system, $[\chi(R)]_{\rm av}$ decreases exponentially. In order to calculate higher moments Eq. (6) is replaced in $d=2$ by

$$[G_{\omega_1}(R)G_{\omega_2}(R)]_{\rm av}=\frac{m\tau}{\pi l^2}K_0((R/l)\sqrt{2|\omega_1-\omega_2|\tau}) , \tag{11}$$

where K_0 is a Bessel function. Using this result we find

$$[\chi^2(R)]_{\rm av}=\frac{m^2}{6\pi^4R^4} . \tag{12}$$

Again, this is of the same order of magnitude as in the pure system and is independent of the mean free path.

In the canonical spin-glass systems such as CuMn, one may obtain for sufficiently low Mn concentrations, a sample for which the conditions $R\gg l$, $k_Fl\gg 1$ can be met, where l is the mean free path in the sample due to scattering off nonmagnetic impurities added to the alloy. This system is a candidate for transition to a possible spin-glass state. Assuming a Gaussian distribution, the transition temperature is given in mean field theory by the second moment of the exchange couplings.[12,13] From Eq. (7) we find

$$T_c\sim\left[\int d\mathbf{R}\rho(\mathbf{R})[\chi^2(R)]_{\rm av}\right]^{1/2}=\left[c\int_{r_0}^\infty d\mathbf{R}\frac{1}{R^6}\right]^{1/2}\sim c , \tag{13}$$

assuming a constant of spins c, and using a lower cutoff $r_0\sim c^{-1/3}$.

This scaling of T_g with concentration is a consequence of the power-law dependence of the interactions and would be expected to hold although, as shown by Eq. (10), the actual distribution of exchange interactions is not Gaussian and the transition temperature, even in the mean-field approximation will not be determined by the second moment $[\chi^2]_{\rm av}$ alone. As in the pure metal, furthermore, the effective coupling $J_{\rm eff}$ corresponding to the average spin-separation distance is proportional to the concentration of magnetic moments, c. The same scaling laws that have been proposed and experimentally verified for the canonical spin-glass alloys[14] are therefore expected to hold for the weakly disordered alloys formed, for example, by the addition of a nonmagnetic impurity. We note

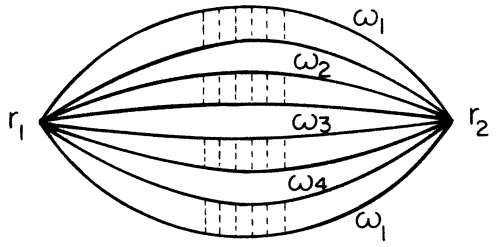

FIG. 2. Connected part of the fourth moment of $\chi(R)$.

also that Eq. (7) predicts that T_g should not exhibit any dependence on the mean free path in the sample provided firstly that l is large enough compared to the lattice spacing to justify the approximation of weak disorder, and secondly that the concentration of moments c is small enough that we may use our results obtained in the limit of $R/l \gg 1$. Most experiments that have been performed to investigate disorder effects on T_g in spin glasses cannot be considered to satisfy this criterion of being doubly dilute.[8,9,15,16] However Vier and Schultz[10] have measured T_g as a function of l by varying the percentage of added impurities to CuMn and AgMn. As the mean free path decreases from its value in the intrinsic alloy, they find that the transition temperature appears to level off and become independent of the mean free path for values of l comparable to the distance between the Mn ions. This is the region in which we expect Eq. (13) to begin to hold, and indeed on the basis of their data Vier and Schultz correctly concluded that the "interaction was not only mean free path independent but long ranged as well." A similar tendency can be seen in the higher resistivity data on Au-Fe collected by Larsen.[15]

Nuclear magnetic resonance in such systems on the other hand, should yield information on both the average value of the exchange interactions, and the higher moments. The line shift, being proportional to the first moment of the interactions should exhibit the exponential dependence on l given by the de Gennes result. The inhomogeneous linewidth of the host metal NMR, on the other hand, should be determined by the quantity $[\chi^2]_{\rm av}$, and in sufficiently dilute alloys be independent of l. The 1966 data of Heeger, Klein, and Tu,[6] who measured the change of linewidth in Cu-Mn as a function of added Al, have been considered[17] to be evidence of the exponential dependence given by Eq. (4). Unfortunately, the mean free paths were much larger than typical Mn separations so that the samples cannot be considered to satisfy the asymptotic limit in which Eqs. (4) and (7) are valid. Thus, the observed decrease of linewidth with disorder remains unexplained.

A different class of systems exists in which the de Gennes result for $[\chi]_{\rm av}$ might be expected to apply. If the disordered metal has a transition to a low-temperature phase with ferromagnetic or antiferromagnetic order, mean-field theory gives the result that the transition temperature depends on the first moment of the interactions. Accordingly, the critical temperature T_c is expected to be exponentially suppressed with disorder. However, the de Gennes result is valid only asymptotically ($R/l \gg 1$) and would not be expected to hold well in the concentrated alloys or magnetic compounds such as those studied, for example, in Refs. 18 and 19.

In connection with the exponential decay of $[\chi(R)]_{\rm av}$, and experimental measurements of this quantity, it should be noted that Eq. (4) is valid only asymptotically in R and therefore does not violate the sum rule on susceptibility

$$\int d\mathbf{R}\,\chi(R) = \int d\mathbf{R}\,\chi_0(R) = N(\epsilon_F) \tag{14}$$

which continues to hold for the averaged susceptibility $[\chi]$. [In Eq. (14) it has been assumed that the density of states is not appreciably changed upon addition of the impurities.] There have been several discussions in the literature[3,20] on this point. We emphasize again that the de Gennes formula [Eq. (4)] is valid at $R \gg l$ for the average susceptibility but not for distances less than or comparable to the mean free path.

III. SPIN-ORBIT SCATTERING

We consider next the generalization of our results to include situations in which spin-orbit scatterers are present. It is assumed that these are sufficiently dilute so that the spin-orbit diffusion length $L_{\rm s.o.} \gg l$, where $L_{\rm s.o.} = \sqrt{3D\tau_{\rm s.o.}}$. For spin separation distances R such that $L_{\rm s.o.} \gg R \gg l$, we expect that Eq. (7) will continue to describe the typical interaction strength between spins. For distances exceeding $L_{\rm s.o.}$ the RKKY interaction must be suppressed due to spin-flip scattering. This is found to be the case, when we compute the contribution to the second moment from the diagrams in Fig. 2. We assume the spin-orbit scattering is isotropic (i.e., $\tau^x_{\rm s.o.} = \tau^z_{\rm s.o.} = 3\tau_{\rm s.o.}$) and take $R \gg L_{\rm s.o.}$. We find the second moment of the interaction to be

$$\sum_{i,j,k,l} [\chi_{ij}\chi_{kl}]_{\rm av} S^i_1 S^j_2 S^k_1 S^l_2 = A_1(R)\mathbf{S}_1^2\mathbf{S}_2^2 + A_2(R)[(\mathbf{S}_1\times\mathbf{S}_2)^2 + (\mathbf{S}_1\cdot\mathbf{S}_2)^2 - \mathbf{S}_1^2\mathbf{S}_2^2] + A_3(R)[2(\mathbf{S}_1\cdot\mathbf{S}_2)^2 + \mathbf{S}_1^2\mathbf{S}_2^2 - 2(\mathbf{S}_1\times\mathbf{S}_2)^2], \tag{15}$$

where the coefficients are given first for the case of three dimensions

$$A_1(R) = \frac{3}{2^6}\left(\frac{mk_F}{(2\pi)^3}\right)^2 \frac{1}{R^6},$$

$$A_2(R) = \frac{3}{2}\left(\frac{mk_F}{(2\pi)^3}\right)^2 \frac{e^{-2R/L_{\rm s.o.}}}{R^6}, \tag{16}$$

$$A_3(R) = \frac{1}{2^4}\left(\frac{mk_F}{(2\pi)^3}\right) \frac{1}{R^4 L^2_{\rm s.o.}} e^{-4R/L_{\rm s.o.}}.$$

The terms in the root-mean-square interaction which decay exponentially with a characteristic length $L_{\rm s.o.}$ include the isotropic RKKY and some anisotropic Dzyaloshinskii-Moriya (DM)[21] couplings. However, a term decaying as R^{-3} survives; it originates from an anisotropic interaction of the DM type. The result, Eq. (16) is in agreement with that of Ref. 4.

For purposes of comparison with experiment, it is important to note that the exponential decay and hence exponential scaling with concentration predicted by Eqs. (15) and (16) is valid only in the limit of large spin separations. To take a specific example, we consider metallic alloys such as $CuMn_xA_{y_{\rm s.o.}}$. When the concentration x of local moments is much greater than that of the spin-orbit scatterers $y_{\rm s.o.}$, we expect that interactions will decay with a power law. The conclusions of Sec. II will continue to hold, and additionally, due to the presence of the long-range Dzyaloshinskii-Moriya interactions we expect linear scaling of the macroscopic anisotropy field H_A with con-

centration $y_{s.o.}$.[22] For $x \geq y_{s.o.}$ we expect a crossover to exponential scaling. The spin-glass transition temperature should decrease exponentially with $y_{s.o.}$ and so also the anisotropy field associated with remanent magnetization.

In thin films (thickness small compared to the diffusion length) similar results are obtained, with the coefficients $A_i(R)$ of the spin couplings now being replaced by

$$B_2(R) = \frac{2^6 m^2}{(2\pi)^4 R^4} K_0(2R/L_{s.o.}) \sim \frac{1}{R^4 \sqrt{R/L_{s.o.}}} e^{2R/l_{s.o.}} ,$$

$$B_3(R) = \frac{2^4 m^2}{(2\pi)^4} \frac{1}{R^2 L_{s.o.}^2} K_0^2(2R/L_{s.o.}) \sim \frac{1}{R^3 L_{s.o.}} e^{-4R/L_{s.o.}} , \tag{17}$$

(the functions have been evaluated for distances $R \gg L_{s.o.}$). Now we consider the case of strictly two spatial dimensions, setting $1\tau_{s.o.}^x = 1/\tau_{s.o.}^y = 0$. Restricting the spin-orbit scattering to lie strictly in the plane must now result in a highly anisotropic interaction between the spins $\mathbf{S}_1$ and $\mathbf{S}_2$ owing to the fact that electrons with spins oriented along the z-direction are not flipped by the spin-orbit scattering. Indeed one now finds long-range spin-spin couplings, as seen in the following expression for the second moment:

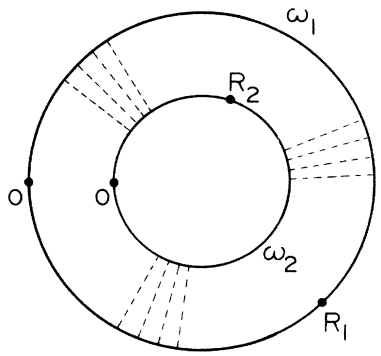

FIG. 3. Leading contribution to three site correlations [Eq. (16)].

$$\sum_{i,j,k,l} [\chi_{ij}\chi_{kl}]_{av} S_1^i S_2^j S_1^k S_2^l = C_1(R)[\mathbf{S}_{1z}^2 \mathbf{S}_{2z}^2 + (\tilde{\mathbf{S}}_1 \cdot \tilde{\mathbf{S}}_2)^2 + (\tilde{\mathbf{S}}_1 \times \tilde{\mathbf{S}}_2)^2]$$
$$+ C_2(R)[2(\mathbf{S}_1 \cdot \mathbf{S}_2)^2 - 2\mathbf{S}_{1z}^2 \mathbf{S}_{2z}^2 - 2(\tilde{\mathbf{S}}_1 \cdot \tilde{\mathbf{S}}_2)^2] + C_3(R)[\mathbf{S}_{1z}^2 \mathbf{S}_{2z}^2 + (\tilde{\mathbf{S}}_1 \cdot \tilde{\mathbf{S}}_2)^2 - (\tilde{\mathbf{S}}_1 \times \tilde{\mathbf{S}}_2)^2] . \tag{18}$$

The spin components are written $(\tilde{\mathbf{S}}, \mathbf{S}_z)$ where $\tilde{\mathbf{S}}$ are the planar components. The coefficients $C_i(R)$ are

$$C_1(R) = \frac{4m^2}{3(2\pi)^4 R^4}, \quad C_2(R) = \frac{2^8 m^2}{(2\pi)^4 R^4} K_0(2R/L_{s.o.}) \sim \frac{1}{R^4 \sqrt{R/L_{s.o.}}} e^{-2R/L_{s.o.}} ,$$
$$C_3(R) = \frac{2^5 m^2}{(2\pi)^4 R^2 L_{s.o.}^2} K_0^2(2R/L_{s.o.}) \sim \frac{1}{R^3 L_{s.o.}} e^{-4R/L_{s.o.}} . \tag{19}$$

For large R, the leading contributions are given by $C_1(R)$, and the two other functions decay exponentially. The coefficient of the term $C_1(R)$ can also be written $(S_{1z}^2 S_{2z}^2 + \tilde{\mathbf{S}}_1^2 \tilde{\mathbf{S}}_2^2 - S_{1z} S_{2z})$ which shows that the important long-range part of the spin interactions involves only the z components of the spins. Thus we expect that addition of spin-orbit scatterers in $d = 2$ will lead to a crossover from Heisenberg to Ising behavior in a dilute spin glass.

To understand the result for the second moment given by Eq. (18) we consider the long-ranged part proportional to $C_1(R)$. The spin couplings that survive over large distances in the disordered medium will of course be random. The mean square of these interaction strengths is nonvanishing at large distances and is given by the function $C_1(R)$. The random interactions are therefore long ranged, falling off as R^{-2}. In strictly two dimensions, even in the presence of spin-orbit impurity scattering, spin-exchange interactions survive at long distances. This would presumably only be relevant for strictly two-dimensional cases such as semiconductor inversion layers. In thin metallic films, on the other hand, the usual fits to magnetoresistance data[23] indicate that the spin-orbit interactions are effectively isotropic.

IV. THREE SITE CORRELATIONS

Finally, we consider the problem of correlations between the various "random" couplings in the disordered metal. It might be expected that the spin polarizations at two distant points due to a single spin at the origin would be uncorrelated (falling off exponentially with the distance between the two sites). We compute the correlation function $[\chi(R_1)\chi(R_2)]_{av}$ where $\chi(R_1)$ and $\chi(R_2)$ are essentially the spin polarization fields induced at sites R_1 and R_2 by a spin placed at the origin. This represents the correlation of the couplings between the origin and the two sites. The leading contribution to such correlations is illustrated in Fig. 3 and gives

$$[\chi(R_1)\chi(R_2)]_{av} = \frac{72}{(2\pi)^6} \frac{m^2}{l} \frac{1}{(|\mathbf{R}_1| + |\mathbf{R}_2| + |\mathbf{R}_1 - \mathbf{R}_2|)^4 |\mathbf{R}_1||\mathbf{R}_2||\mathbf{R}_1 - \mathbf{R}_2|} . \tag{20}$$

For sites R_1 and R_2 equidistant from the origin and from each other this correlation falls off with distance as R^{-7}. However, comparing it to the typical exchange couplings given by Eq. (7), these correlations are smaller by a factor of

$(k_F l)^{-2}(l/R)$ and can be neglected in the weak scattering limit.

V. CONCLUSION

Typical RKKY interactions in disordered metals with purely elastic scattering retain their long-range character, although there are large disorder-induced fluctuations about the mean value. The higher-order moments of the distribution of interactions show it to be fairly broad, reflecting the sensitive dependence of the interaction on the disorder. In a spin glass the scaling of thermodynamic quantities with the concentration of magnetic moments should however continue to hold as in the pure system. Spin-orbit scattering suppresses the spin-exchange couplings at large distances, except when the electron motion is restricted to two spatial dimensions. In this case the exchange interactions persist at long range, falling off as the inverse square of the distance between local moments. We expect further that the addition of the spin-orbit scatterers will result in a crossover from Heisenberg to Ising behavior. Correlations between couplings involving three spins do not decay exponentially but are small and can be neglected in the weak scattering limit.

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