Exact-eigenstates study of the disordered Hubbard model

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The static magnetic susceptibility of the disordered Hubbard model for the half-filled-band case is studied using exact eigenstates of the noninteracting disordered system. The macroscopic renormalization-group method is applied to study the critical behavior in the vicinity of the noninteracting (Anderson) fixed point. A weak Hubbard interaction is found to stabilize the metallic phase.

I. INTRODUCTION

In this paper we have studied some aspects of the Hubbard model with disorder (random on-site energies) using exact eigenstates of the noninteracting part of the Hamiltonian. We have calculated the static magnetic susceptibility of the noninteracting system in one and three dimensions (1D and 3D) and have studied the nature of the magnetic instability in this disordered, interacting system within the random-phase approximation (RPA). We have also applied the macroscopic renormalization-group (MRG) method to first order in interaction strength to study the nature of the phase diagram in the vicinity of the noninteracting fixed point which describes the Anderson transition.

The presence of strong interaction leads to a development of spin alignment, and spin fluctuations which play an important role in the mechanism of the metal-insulator transition are generated in the system.[1-3] To this end it is thus of interest to study the nature of spin fluctuations (e.g., their q dependence[3]). The q dependence of the impurity (static) susceptibility for the parabolic band case was studied recently[4] in the (impurity) ladder approximation. However, it appears difficult to go beyond this approximation and calculate, in particular, the localization contribution to the static susceptibility. Here we have used the exact eigenstates to evaluate the static impurity susceptibility, and so the localization part of the problem is treated exactly. By substituting this in a RPA-type formula for the magnetic susceptibility of the interaction system we find that the appearance of magnetic instability is still signaled by the Stoner criterion. The magnetic response is maximum for $\mathbf{q}=\pi$, suggesting a disordered antiferromagnetic arrangement in the magnetic phase. The response is, however, very broad, implying that the spin fluctuations which are generated are extremely localized in space.

A surprising aspect of the interplay of disorder and short-ranged interaction is that the correction due to interaction, to first order as well as to higher orders [(but all being of $O(1/N)$ within a $1/N$ expansion scheme)], to the dc conductivity, $d\sigma/\sigma$, is positive.[3] (The $1/N$ expansion scheme was introduced by Wegner[5] and Oppermann and Wegner[6] who showed that by introducing N species of electrons with N large, it is possible to use $1/N$ as an expansion parameter.) This implies that the metallic phase is stabilized by short-ranged interaction. By using the MRG method we have verified that this is true even when the disorder part is treated exactly. However, one significant difference is that in our exact-eigenstates analysis of the *lattice system* this stabilization effect is present in the $O(1)$ contribution of interaction. This stabilization of the metallic phase can be understood within a self-consistent Hartree picture in which the Hubbard interaction is seen to screen disorder.[7,8] The self-consistent treatment of short-range interaction within the exact-eigenstates analysis is, in fact, nontrivial for the lattice system and is, we have indicated in Sec. III C, quite important.

II. MAGNETIC SUSCEPTIBILITY

We consider the disordered Hubband model on a lattice. The Hamiltonian is

$$H=\sum_{i,\sigma}\epsilon_i a_{i\sigma}^{\dagger}a_{i\sigma}-V\sum_{i,j,\sigma}{}'(a_{i\sigma}^{\dagger}a_{j\sigma}+a_{j\sigma}^{\dagger}a_{i\sigma}) + U\sum_i a_{i\uparrow}^{\dagger}a_{i\uparrow}a_{i\downarrow}^{\dagger}a_{i\downarrow}\ . \tag{1}$$

The first term in Eq. (1) is the on-site random potential and the ϵ_i are chosen from a uniform distribution on $-W/2<\epsilon_i<W/2$. The second term is the hopping, with the prime indicating a sum over the nearest-neighbor pair of sites only. The last term is the on-site Hubbard interaction.

Information about the nature of spin fluctuations in the disordered system is contained in the magnetic susceptibility, $\chi(\mathbf{q},\omega)$; therefore, it is of interest to study the effect of disorder on χ. We first study the configuration-averaged static impurity susceptibility, $\overline{\chi^0}$ of the noninteracting part of the system, H^0. In the pure system, for the half-filled band case, $\chi^0(\mathbf{q})$ exhibits a peak at $\mathbf{q}=\pi/a$, indicating that the magnetic instability in the interacting system is antiferromagnetic in nature. Disorder will lead, quite generally, to a broadening of this peak. Through the Kramers-Kronig relation, which connects the real and imaginary parts of the dynamical susceptibility, this then indicates a spatial lo-

calization of spin fluctuations. However, we expect that spin-independent impurity scattering will not remove this feature[4] and that the system will still exhibit a (disordered) antiferromagnetic arrangement in the magnetic phase. Spin-dependent impurity scatterings (e.g., due to spin-orbit interaction, or magnetic impurities) lead to a competition between different types of orderings, and it is possible that a system with such impurities exhibits a qualitatively different kind of arrangement of spins in the "magnetic" phase.

In terms of the eigenfunctions $\{|\phi_l\rangle\}$ and eigenvalues $\{\mathcal{E}_l^0\}$ of H^0, obtained with periodic boundary conditions in all directions, the susceptibility matrix χ_{ij}^0 is given by

$$\chi_{ij}^0 = i\int \frac{d\omega}{2\pi} G_{ij}^0(\omega) G_{ji}^0(\omega) = 2 \sum_{\substack{\mathcal{E}_l^0 (>\mathcal{E}_F) \\ \mathcal{E}_m^0 (<\mathcal{E}_F)}} \frac{\phi_l^i \phi_m^i \phi_l^j \phi_m^j}{\mathcal{E}_l^0 - \mathcal{E}_m^0} . \quad (2)$$

We have considered the half-filled-band case and the position of the Fermi energy $\mathcal{E}_F$ is determined accordingly. After ensemble averaging over several configurations of the random potential we find that $\overline{\chi_{ij}^0}$ depends roughly only on the separation between sites i and j and has the following features.

(i) The oscillations in $\overline{\chi^0}(r)$ become increasingly damped with increasing disorder ending up with an overdamped form.

(ii) $\overline{\chi_{ij}^0}$ is extremely local in 3D. [For $W/V=15$, $\overline{\chi^0}(|\mathbf{i}-\mathbf{j}|=1)/\overline{\chi^0}(0)$ is $\approx \frac{1}{30}$]. This suggests that spin fluctuations in the interacting system are extremely localized. This local nature of the susceptibility is not a consequence of configuration averaging resulting from a cancellation between terms of different sign. It is rather, a general feature of every configuration.

Fourier transformation leads to

$$\overline{\chi^0}(\mathbf{q}) = \sum_{\mathbf{r}/a=|\mathbf{i}-\mathbf{j}|} \cos(\mathbf{q}\cdot\mathbf{r})\overline{\chi_{ij}^0} . \quad (3)$$

The sign change in $\overline{\chi^0}(r)$ locally is what leads to $\overline{\chi^0}(\mathbf{q})$ exhibiting a maximum at $\mathbf{q}=\pi/a$. In Fig. 1 we plot

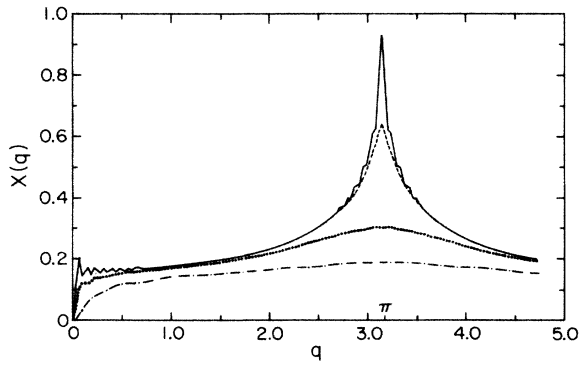

FIG. 1. Plot of $\overline{\chi}(q)$ vs q for the $n=150$, 1D case for different values of W/V: solid line, $W/V=0$; dashed line, $W/V=1$; dotted line, $W/V=3$; chain, $W/V=5$.

$\overline{\chi^0}(q)$ versus q for the (1D) system with $n=150$ and different values of W/V. The vanishing of $\overline{\chi^0}$ $(q=0)$ is an artifact of the finite size of the system. $\overline{\chi^0}$ $(q=0)$ is the response to an infinitesimal uniform magnetic field, and since a finite system the energy levels have a finite spacing, the response vanishes. Mathematically, it is seen to be a consequence of orthogonality of the wave functions [Eqs. (2) and (3)]. Particle conservation requires that the $q=0$ static susceptibility in an infinite system must equal the density of states per spin.

We can now write an RPA-type expression for the susceptibility of the interacting system in terms of the configuration-averaged impurity susceptibility $\overline{\chi^0}(\mathbf{q})$,

$$\bar{\chi}(\mathbf{q}) = \frac{\overline{\chi^0}(\mathbf{q})}{1-U\overline{\chi^0}(\mathbf{q})} . \quad (4)$$

In the RPA or ladder approximation the susceptibility matrix $[\chi]_{ij}$ is given by $([\chi^0]/[1-U\chi^0])_{ij}$. Instead of impurity averaging the resultant matrix, if we average separately the numerator and denominator, we obtain Eq. (4) upon Fourier transformation. Performing the averaging separately amounts to ignoring, in a diagrammatic scheme, graphs with vertex corrections and crossed impurity and interaction lines. Another way to look upon this approximation is that it ignores the fluctuation terms in the impurity-susceptibility matrix, $[\delta\chi^0]\equiv[\chi^0]-[\overline{\chi^0}]$. Any power of $[\chi^0]$ can be expressed, after impurity averaging, in terms of powers of $[\overline{\chi^0}]$ and (even) moments of the fluctuation part $[\delta\chi^0]$. Now, as far as the spatial behavior of the susceptibility is concerned, especially the oscillatory nature, it is contained in $[\overline{\chi^0}]$. Therefore, the neglect of the second and higher moments of $[\delta\chi^0]$ does not lead to any change in the qualitative nature of the magnetization in a disordered system with spin-independent scattering impurities, which is what we are concerned with here.

From Eq. (4) we see that a magnetic instability first occurs for $\mathbf{q}=\pi$ when $U=U^*\equiv 1/\overline{\chi^0}(\pi)$. Within this approximation a transition thus occurs at this value of U^* between a disordered paramagnetic phase and a disordered antiferromagnetic phase. The value of U^* is fairly independent of the number of lattice sites in the range of disorder values studied. The variation in $\overline{\chi^0}$ $(\mathbf{q}=\pi)$ is within 6% in 1D in going from $n=50$ to $n=150$ for the range of disorder values studied $(1\le W/V\le 5)$ and is within 3% in 3D in going from $n=4$ to $n=6$ for disorder values in the range $7.5\le W/V\le 15$. In Figs. 2 and 3 (insert) we have shown the phase boundaries by plotting the critical interaction strength versus disorder for the 1D $(n=150)$ and 3D $(n=6)$ cases, respectively. It is clear that the Stoner criterion is still valid in the 3D case. The density of states per spin goes roughly as $1/W$ when $W\gg V$ and U^*/W is very close to 1.

III. MACROSCOPIC RENORMALIZATION GROUP

We now set up the MRG method in which we preserve certain macroscopic physical properties of the system as the lattice parameter is varied. We consider specifically two systems with different lattice spacings

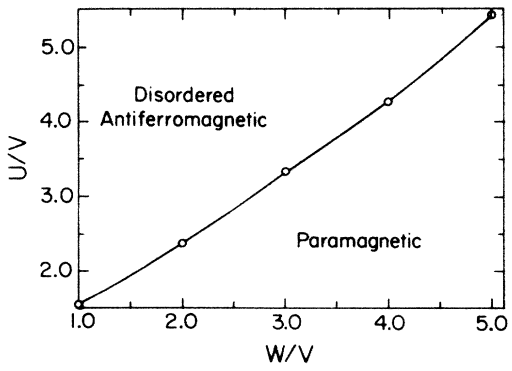

FIG. 2. Phase diagram for the 1D system.

but the same physical size. The first one has n^3 sites, with lattice spacing a and length $L=na$, and the Hamiltonian parameters are W,V,U. The second system has n'^3 sites with lattice spacing a', the same length $L=n'a'=na$, and the Hamiltonian parameters are W',V',U'. We require that the two systems represent the same physical problem with different microscopic length scales a and a' and, therefore, demand that the Hamiltonian parameters of these two systems be so related that the physical properties are preserved. The physical properties we choose to preserve under the renormalization group (RG) are the one-electron density of states near the Fermi energy $N(0)$, the antisymmetric Landau interaction parameter F^a, and the width of the phase-sensitivity-energy distribution, $\tilde{\Theta}$. Fermi-liquid theories of very dirty metals[9–11] indicate that the Landau interaction parameters are indeed relevant physical quantities. The justification for using F^a, the antisymmetric part, as an appropriate MRG variable has been discussed by Singh.[12] $\tilde{\Theta}$ measures the sensitivity of quasiparticle energies to changes in boundary conditions and was first used as a measure of localization by McMillan.[13] We have used this formulation because it is more convenient for the interacting fermion problem being in

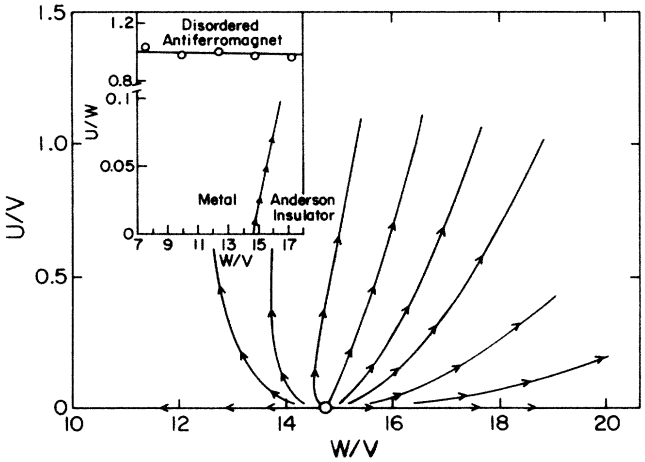

FIG. 3. RG flow in the (w,u) parameter space in the vicinity of the noninteracting fixed point. Inset: Phase diagram for the 3D system.

terms of energies. Equating these three physical quantities for our two systems we obtain the recursion relations for the Hamiltonian parameters implicitly.

$$N_{n'}(W',V',U')=N_n(W,V,U)\ , \tag{5}$$

$$F^a_{n'}(W',V',U')=F^a_n(W,V,U)\ , \tag{6}$$

$$\tilde{\Theta}_{n'}(W',V',U')=\tilde{\Theta}_n(W,V,U)\ . \tag{7}$$

In Sec. III A we consider Eqs. (5)–(7) to first order in U. In Sec. III C we discuss some possible consequences of treating these relations within the bubble approximation in which the quantities are evaluated by summing all the bubble graphs.[12]

A. Weak interaction limit

In this subsection were are concerned with the behavior of the system for weak interaction in the vicinity of the noninteracting fixed point describing the Anderson transition ($W/V\approx 15$). We already know that the Hubbard on-site interaction is relevant within first order[14] and that the noninteracting fixed point is unstable. The fixed point for the interacting system, which we feel also describes the transition between the disordered paramagnetic and antiferromagnetic phases, is not accessible within a finite-order perturbation theory as seen from the discussion in Sec. II. However, it is possible to study the effect of interaction on the stability of the metallic or the insulator phase near the noninteracting fixed point.

This issue is of current interest as it appears that the correction due to interaction [which is of $O(1/N)$ within a $1/N$ expansion scheme] to a dc conductivity, $d\sigma/\sigma$, is positive.[3] This suggests that the metallic phase is stabilized by short-ranged interaction. In fact, as Béal-Monod points out, in the strong interaction limit $[UN(0)\approx 1]$ the interaction contribution can overwhelm the localization contribution and thus preempt the metal-insulator transition. In these calculations disorder is always treated at the lowest level and the results are valid in the metallic regime $(k_F l \gg 1)$. Rainbow diagrams are summed for the one-particle Green's function, yielding a result identical to the one obtained using a large $k_F l$ approximation,[15] and ladder diagrams are summed for the two particle Green's function (ladder approximation).[15] We have used the exact eigenstates for the disorder problem and find that to first order in U [this term is of $O(1)$ within a $1/N$ expansion scheme] the metallic phase is indeed stabilized.

The result of interaction is to change the quasiparticle energies by the self-energies and this modifies the density of states as

$$N(0)=N^{(0)}(0)\left[1-\frac{\langle\Delta\Sigma^{(1)}\rangle_{\rm av}}{\langle\Delta\mathcal{E}^{(0)}\rangle_{\rm av}}\right]\ , \tag{8}$$

where $\langle\Delta\mathcal{E}^{(0)}\rangle_{\rm av}$ is the mean level spacing near the band center for the noninteracting system and

$$\langle\Delta\Sigma^{(1)}\rangle_{\rm av}=(\Sigma^{(1)}_{l+1}-\Sigma^{(1)}_l)_{\rm av}\ , \tag{9}$$

$$\Sigma^{(1)}_l=U\sum_{\mathcal{E}_m\ (<\mathcal{E}_F)}\sum_i(\phi^i_l)^2(\phi^i_m)^2\ . \tag{10}$$

The Landau interaction parameter is given (to first order) by

$$-F^a = UN^{(0)}(0)\left\langle \sum_i (\phi_l^i)^4 \right\rangle_{\rm av} . \tag{11}$$

The averaging is done over one-four of the quantum states in the middle of the band and over several configurations of the random potential, typically 10 000 samples in all. The Fermi energy is chosen to lie at the center of the band, the precise location being unimportant (for the purpose of this section) due to presence of disorder.

We now turn to the calculation of the width $\tilde{\Theta}$ of the phase sensitivity energy distribution. Essentially, the idea is to determine the sensitivity of quasiparticle energies of the system to changes in boundary condition as one goes from a case with periodic boundary condition on the wave function in all three directions to one with periodic boundary condition in two directions but having, in the third (say z) a periodic boundary condition supplemented by a small phase shift θ. The boundary condition in the z direction is, then

$$\phi_l(x,y,z+L) = e^{i\theta}\phi_l(x,y,z) , \tag{12}$$

and this is achieved by replacing the hopping matrix elements V in H_0 by $Ve^{i\theta}$ and $Ve^{-i\theta}$ at the appropriate boundaries. We calculate the sensitivity of the unperturbed energies $\mathcal{E}_l^{(0)}$ and the self-energies $\Sigma_l^{(1)}$ separately. Let us denote the eigenfunctions and eigenvalues of H_0 for these two boundary conditions by $\{|\phi_l\rangle, \mathcal{E}_l^{(0)}\}$ and $\{|\phi_l(\theta)\rangle, \mathcal{E}_l^{(0)}(\theta)\}$, respectively. Since the routine used for eigenanalysis orders the eigenvalues (and the corresponding eigenfunctions) with $\mathcal{E}_l^{(0)}$ increasing with l, we are assured that $\{|\phi_l(\theta)\rangle, \mathcal{E}_l^{(0)}(\theta)\}$ corresponds to the state $\{|\phi_l\rangle, \mathcal{E}_l^{(0)}\}$ provided θ is taken to be small enough so that the change $\mathcal{E}_l^{(0)}(\theta) - \mathcal{E}_l^{(0)}$ is much less than the level spacing.

Now, for these two cases the quasiparticle energies to first order in U are

$$\mathcal{E}_l^{(1)} = \mathcal{E}_l^{(0)} + \Sigma_l^{(1)} , \tag{13}$$

$$\mathcal{E}_l^{(1)}(\theta) = \mathcal{E}_l^{(0)}(\theta) + \Sigma_l^{(1)}(\theta) . \tag{14}$$

The phase sensitivity energy of state l is

$$\begin{aligned}\Theta_l &= [\mathcal{E}_l^{(1)}(\theta) - \mathcal{E}_l^{(1)}]/\theta^2 \\ &= \{[\mathcal{E}_l^{(0)}(\theta) - \mathcal{E}_l^{(0)}] + [\Sigma_l^{(1)}(\theta) - \Sigma_l^{(1)}]\}/\theta^2 \\ &\equiv \Theta_l^0 + U\Theta_l^1 .\end{aligned} \tag{15}$$

The Θ_l^0 and Θ_l^1 are distributed with zero mean and have Lorentzian wings. Following McMillan[13] we define a Lorentzian width $\tilde{\Theta}$ which minimizes the mean-square error between the observed distribution and a Lorentzian of width $\tilde{\Theta}$. It is given by the condition

$$4\tilde{\Theta}^2 \left[\frac{\tilde{\Theta}^2 - \Theta_l^2}{(\tilde{\Theta}^2 + \Theta_l^2)^2} \right]_{\rm av} = 1 . \tag{16}$$

If we expand $\tilde{\Theta}$ in powers of U and substitute Eq. (13) into Eq. (14), then to first order in U we have

$$\tilde{\Theta} = \tilde{\Theta}^0 + U\tilde{\Theta}^1 , \tag{17}$$

where $\tilde{\Theta}^0$ and $\tilde{\Theta}^1$ are given by

$$4(\tilde{\Theta}^0)^2 \left[\frac{(\tilde{\Theta}^0)^2 - (\Theta_l^0)^2}{[(\tilde{\Theta}^0)^2 + (\Theta_l^0)^2]^2} \right]_{\rm av} = 1 , \tag{18}$$

$$\tilde{\Theta}^1 = \tilde{\Theta}^0 \frac{\{\Theta_l^0\Theta_l^1[3(\tilde{\Theta}^0)^2 - (\Theta_l^0)^2]/[(\tilde{\Theta}^0)^2 + (\Theta_l^0)^2]^2\}_{\rm av}}{\{(\Theta_l^0)^2[3(\tilde{\Theta}^0)^2 - (\Theta_l^0)^2]/[(\tilde{\Theta}^0)^2 + (\Theta_l^0)^2]^2\}_{\rm av}} . \tag{19}$$

The averaging is done as before, over one-fourth of the quantum states in the middle of the band, over three directions of applying the phase shift, and over several configurations of the random potential, typically 10 000 samples of Θ_l^0 and Θ_l^1 in all. $\tilde{\Theta}^0$ is a measure of free-particle localization. We interpret $\tilde{\Theta}^1$ as a measure of localization due to the interaction effect. The net contribution to it comes predominantly from situations where the the noninteracting energy and the self-energy both change with the same sign when the boundary condition is modified.

We define a dimensionless coupling constant, $G = N(0)\tilde{\Theta}$, which is a function only of the ratios $w = W/V$ and $u = U/V$. From Eqs. (5)–(7) we get a two-parameter recursion relation contained implicitly in

$$G_{n'}(w',u') = G_n(w,u) , \tag{20}$$

$$F_{n'}^a(w',u') = F_n^a(w,u) . \tag{21}$$

B. Results

We calculate G_n and F_n (for $n = 4$ and 6) for different values of w near the noninteracting fixed point value of 15. The data for each n are fitted appropriately to get the w dependence in an algebraic form. We can now study the (RG) equations contained implicitly in Eqs. (20) and (21) to examine flows in the parameter space formed by (w,u). The noninteracting fixed point is obtained (Fig. 3) at (14.8,0) as before,[13] and it represents the Anderson transition. The flow to its right is towards $(\infty,0)$, which represents the noninteracting insulator phase, whereas to its left it is towards (0,0), which corresponds to the noninteracting metallic phase. The Anderson fixed point is, however, unstable with respect to interaction and the flow out of it is towards the interacting fixed point which is, as we mentioned before, beyond the reach of perturbation theory.

An interesting feature of the phase diagram is the manner in which the flows emerge from the vicinity of the noninteracting fixed point—they are tilted to the right. The line which emerges from the noninteracting fixed point corresponds to the phase boundary which separates the metallic phase from the (gapless) insulator phase. Thus, the metallic phase is stabilized by short-ranged interaction and for small but finite U more disorder is needed to make the system insulating. This behavior can be understood within a self-consistent Hartree picture in which interaction is seen to screen disor-

der. A spin-σ electron moves in an effective potential which is a sum of the bare impurity potential and a repulsive potential due to spin $(-\sigma)$ electrons:

$$\epsilon_{i\sigma}^{\text{eff}}=\epsilon_i+U\langle n_i^{-\sigma}\rangle \ .$$

The amount of disorder seen by the spin-σ electron is therefore reduced by the presence of spin $(-\sigma)$ electrons because their density is more on sites with deeper bare potential. This behavior is contrary to the generally believed additional localizing influence of Coulomb interaction. However, for long-range interaction it is the exchange (Fock) term which is more important in a disordered system. The Hartree term gets averaged out and just leads to a shift in the chemical potential.

C. Higher orders in interaction strength

In this subsection we discuss some possible consequences of going to higher orders in interaction. When disorder is treated in the rainbow approximation to get the impurity-averaged Green's function for the parabolic-band case, e.g., the Hartree term leads to simply to a chemical shift. However, in the exact-eigenstates analysis the self-consistent treatment of the Hubbard interaction is nontrivial and, as we shall show now, is important in determining the critical behavior of the system. In the following we shall evaluate approximate expressions for the Landau parameter and self-energy in the bubble approximation and study how the MRG equations are modified qualitatively.

Using the result in Ref. 12 we can write an approximate expression for the antisymmetric Landau parameter in the bubble approximation:

$$F^a=-N(0)\sum_i\langle(\phi_l^i)^4\rangle_{\text{au}}\frac{U}{1-U\bar{I}(\pi)} \ , \tag{22}$$

where I_{ij} represents the expression for the polarization propagator for the noninteracting system and is given by the same expression as the impurity susceptibility [Eq. (2)]. Here also, performing the averaging separately amounts to ignoring the vertex corrections and crossed impurity and interaction lines. From the discussion in Sec. II we know that the divergence in F^a when $1-U\bar{I}(\pi)=0$ corresponds to the transition from a disordered paramagnetic phase to a disordered antiferromagnetic phase. From the behavior of $\bar{I}(\pi)$ with disorder [Fig. 3 (insert)] this phase boundary is given roughly by $U/W\approx 1$. The self-energy in the bubble approximation is given by[12]

$$\Sigma_l=\sum_{\epsilon_m\,(<\epsilon_F)}\sum_{ij}(\phi_l^i)^2(\phi_m^j)^2\left[\frac{U}{\underline{1}+U\underline{I}}\right]_{ij} \ . \tag{23}$$

Using the fact that matrix $\underline{I}$ is almost diagonal in the region of interest ($W/V\approx 15$) and performing the averaging separately we get

$$\langle\Sigma_l\rangle_{\text{av}}=\sum_{\epsilon_m\,(<\epsilon_F)}\sum_i\langle(\phi_l^i)^2(\phi_m^i)^2\rangle_{\text{av}}\left[\frac{U}{1+U\bar{I}_{ii}}\right] \ . \tag{24}$$

Comparison with Eqs. (9) and (10) suggests that as far as the density of states is concerned, U is to be replaced by $U/(1+U\bar{I}_{ii})$, which is approximately $U/2$ near the phase boundary. Since $\underline{I}_{ij}$ is extremely local and hence we expect it not to be much susceptible to changes in the boundary conditions, the same argument carries through for the phase sensitivity energy in Eq. (15).

Thus in going from first order in interaction to the bubble approximation we observe that the effective interaction gets changed from U to $U/2$. This is actually a consequence of a partial cancellation between self-energy terms with different signs coming from different numbers of bubbles. However, the bubble diagrams for self-energy in the bubble approximation only form a subset of all diagrams contributing to the self-consistent Hartree level. Since these additional diagrams also contain bubbles we expect this cancellation to be even more effective leading to a further reduction in the effective interaction strength, U_{eff}. This reduction in U_{eff} implies that the contribution to localization due to the interaction effect (which stabilizes the metallic phase) is actually much reduced at the self-consistent level.

IV. CONCLUSION

We have studied the static magnetic susceptibility of the disordered Hubbard model with on-site disorder for the half-filled-band case. The impurity-averaged susceptibility $\bar{\chi}^0(\mathbf{q})$ has been evaluated using the exact eigenstates of the noninteracting system in 1D and 3D. The susceptibility for the interacting system is obtained using $\bar{\chi}^0$ within an RPA-type approximation which neglects certain vertex corrections. The system exhibits a magnetic instability which is of a disordered-antiferromagnetic nature.

The Stoner criterion is found to be valid for the onset of this instability. The density of states per spin goes roughly as $1/W$ (for $W/V\gg 1$) and the critical interaction strength U^* at which a transition between the paramagnetic and disordered-antiferromagnetic phases occurs, is given by $U^*/W=1$. In their decimation study of the disordered Hubbard model, Shimizu *et al.*[16] find that for large W/V the critical line approaches the $U=W$ line. The RPA-type approximation used here therefore gives an asymptotically correct description of the magnetic phase boundary.

We expect that nonmagnetic impurities do not qualitatively affect the critical behavior. Spin-dependent impurity scatterings lead to a competition between different types of orderings and may cause frustration and lead to a spin-glass-like phase. We have applied the macroscopic renormalization group method to study the critical behavior of the system for weak interaction near the noninteracting fixed point which describes the Anderson transition. We have chosen to preserve the density of states, the antisymmetric Landau parameter, and the width of the phase-sensitivity-energy distribution as the macroscopic length scale is varied. A small Hubbard term is actually seen to stabilize the metallic phase. Within the bubble approximation we find a reduction of the effective interaction and hence of this stabilization. This indicates the necessity of treating strong interaction within a self-consistent scheme—especially within the exact-eigenstates approach.

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