PHYSICAL REVIEW B VOLUME 37, NUMBER 7 1 MARCH 1988

Computer study of the electrical noise in high-dimensional percolating systems

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(Received 5 October 1987)

We report the first numerical simulation of the relative resistance noise in percolating networks of dimensions higher than two. The results confirm the scaling estimates of the corresponding noise exponents up to six dimensions. Since hypercontinuum systems with unit resistors are used, this confirmation indicates the universality of the derived exponents.

In recent papers, Rammal, Tremblay, and co-workers have analyzed the critical behavior of the resistance fluctuations noise in percolating networks.[1-3] Their work was followed by intensive experimental[4-9] and theoretical[10-12] activity. The experiments, however, yielded results different than those expected from their simple model, which is based on a percolating network of unit resistors. While a proper explanation for this discrepancy seems to be given,[10] there is still no confirmation of their universal predictions. In particular, there is no experimental or computational proof for the fulfillment of their scaling argument[1] or the scaling argument presented by Wright *et al.*[11] for the dimensional dependence of the noise exponent κ. The only result which can be checked against the scaling arguments was given in the pioneering paper on the subject[1] by a numerical simulation on a two-dimensional lattice. Hence, an all-dimension confirmation of their theory and the scaling relation was called for.[2] Since experimental systems are limited to two- and three-dimensional networks and since these systems usually contain a distribution in the values of the resistors[10] it appears that the only way to test the above-mentioned predictions is by computer simulations.

Following these considerations we have carried out computer simulations on continuum systems composed of intersectable hyperspheres such that each intersection has a unit resistor attached to it. This work on systems of two to six dimensions enabled us to directly determine the noise-critical exponents. Other quantities that can be expressed in terms of current moments (see below) were also studied. In particular, the critical behavior of the backbone (a geometrical quantity) and the critical behavior of the electrical conductivity (a physical quantity) were investigated, and the corresponding critical exponents β' and t were determined. For example, for three-dimensional systems we found $\beta' = 0.9 \pm 0.1$ and $t = 1.8 \pm 0.1$, and for six-dimensional systems we found $\beta' = 1.7 \pm 0.2$ and $t = 2.9 \pm 0.1$. The fact that these values are in agreement with the results established in the literature[13] is then a confirmation of the validity of our computational procedure.

The fact that this work is carried out on a continuum geometry makes it also a test of universality. This is a consequence of the fact that a confirmation of the theoretical predictions will indicate that (as in the case of the electrical conductance[14]) the critical noise exponents are the same for lattice and continuum geometries. This means that the continuum *connectivity* as such does not affect the critical behavior of the electrical resistance noise. Continuum nonuniversality could arise however from a "bond strength" distribution[10] which is not considered in this paper.

The generation of the samples for the present work was similar to that used for the study of the cluster statistics and the resistivity of three-dimensional systems.[14] Hyperspheres are randomly implanted in the continuum of a unit hypercube. Correspondingly, the diameter of the sphere d is given below in units of the hypercube edge. Two hyperspheres are considered connected if there is some overlap between them. The onset of percolation is associated with the formation of a continuous path of connected hyperspheres between two opposite faces of the hypercube. Hence, the percolation threshold is determined by the critical concentration of hyperspheres N_c which is required for this onset. Applying the computational procedure which we have used for low dimensions,[14] one could study the geometrical-statistical properties and the transport properties as a function of the hypersphere concentration N. However, separate procedures were needed for each group of properties. A considerable simplification, using current moments, has been shown recently by de Arcangelis, Redner, and Coniglio[15] to enable a single procedure for the determination of some geometrical and statistical properties as well as the transport properties for $N \geq N_c$. Such a study[15-17] has to be carried out, however, for a network in which a unit resistor is associated with each bond of the percolating system, and in which a unit current is set between the opposite plane electrodes. For the property of interest in the present work, the relative resistance noise S_R, it has been shown by Rammal and co-workers[1,2] that it can be given in terms of the fourth and second current moments, i.e., that[10]

$$S_R = A\sum i^4 \Big/ \left(\sum i^2\right)^2 . \qquad (1)$$

Here A is a constant[1,2] (that is taken in this work to be 1), and the sums are over all the resistors in the network. One notes of course that the denominator is simply the square of the sample's resistance and that contributions to the sums come only from the backbone elements (see below). Following Eq. (1) and Ref. 15 we have associated a unit resistor with each intersection of two conducting hyperspheres and we have "set" a unit current between the two opposite hyperplanes which provide the "electrodes" to the cube.

Preliminary simulations, in which the sums given by Eq. (1) were computed, have shown that our inverse matrix method (used previously[14] for two- and three-dimensional resistor network simulations) can handle samples which are too small to get the proper configuration averages[10] required for the study of S_R. In particular, the $N \leq 1000$ limitation associated with the method was found to be quite severe. This is a result of the fact that such a sample size is insufficient for the simulations of samples of higher dimensions for which much better statistics are required. Hence, we have turned to a preconditioned conjugate gradient algorithm which is designed for the solution of large systems of linear equations. The basic difference between this method and our previous inverse matrix method[14] is in the technique used for the solution of the node-voltage vector ($\mathbf{v}$) from the Kirchhoff equation

$$G(\mathbf{v}) - (\mathbf{I}) = (\mathbf{0}) . \qquad (2)$$

Here G is the conductance matrix,[14] ($\mathbf{I}$) is the net current (through-a-node) vector which is given by $(\mathbf{I}) = (1,0,0,\ldots,0,0,-1)$ and ($\mathbf{0}$) is the zero vector. On the other hand, the current through an existing bond ij between hypersphere-node i and hypersphere-node j is simply given by $(v_i - v_j)$. In the inverse matrix method one inverts G and then solves ($\mathbf{v}$) while in the present method one looks for a solution for ($\mathbf{v}$) such that the left-hand side of Eq. (2) is minimized by consecutive iterations. Details of this method will be presented in a subsequent paper. For the present work it is important that this method enabled the use of much larger samples than those which were available to us previously (the only limitations were computer-time costs). We have typically used this new procedure for samples as large as $N = 50000$. One notes that since ($\mathbf{v}$) is solved for the entire sample there is no need to single out the percolating cluster prior to the currents determination.

In this paper we present the data obtained for three-dimensional and six-dimensional systems and report the κ values derived from these data as well as from the data obtained for the other dimensions. These values are compared with the scaling predictions.[1,11] We have chosen to show here the results for the largest samples (largest N_c or smallest d) used in our simulations in order to exhibit the behavior when finite-size effects are not important. In smaller samples the behavior was much the same except for deviations expected[18] from such effects. In Fig. 1 we present the dependence of the relative resistance noise on the proximity to the percolation threshold. The data points, down to $N/N_c - 1 = 0.1$, are shown to exhibit a power-law dependence. The deviation from the power-law behavior below this point is well understood in terms of the smearing of the transition in finite-size samples.[18] Applying our least-squares-fit procedure[14] we found that the data yield an exponent value of $\kappa = 1.57 \pm 0.08$. This value is in excellent agreement with the scaling estimates of Wright, Bergman, and Kantor[11] who predicted that κ will be between 1.53 and 1.60. In Fig. 2 we show the results obtained in the six-dimensional hypercontinuum. Here too, the power-law behavior of the data is apparent and the least-squares fit yields a value which is very close to the scaling prediction[1,11] of $\kappa = 2.0$. These results suggest then an agreement of our numerical data with the scaling estimates.[1,11]

The scaling argument of Wright, Bergman, and Kantor[11] predicts that

$$D\nu + 1 - 2\zeta_R \leq \kappa \leq D\nu - \zeta_R , \qquad (3)$$

where D is the dimensionality of the system, ν is the correlation length exponent, and ζ_R is the "two-terminal" resistance exponent.[17,19,20] For $D \geq 6$ one expects[1,11] that $\kappa = 2.0$ for all dimensions. The comparison made in Fig. 3 between the present numerical results and the limits given by Eq. (3) leaves very little doubt that the present values for the "noise exponent" κ are indeed within the scaling limits. The values of ν and ζ_R used for drawing the scaling predicted estimates are taken from Ref. 19. One should note however that the values of the above ex-

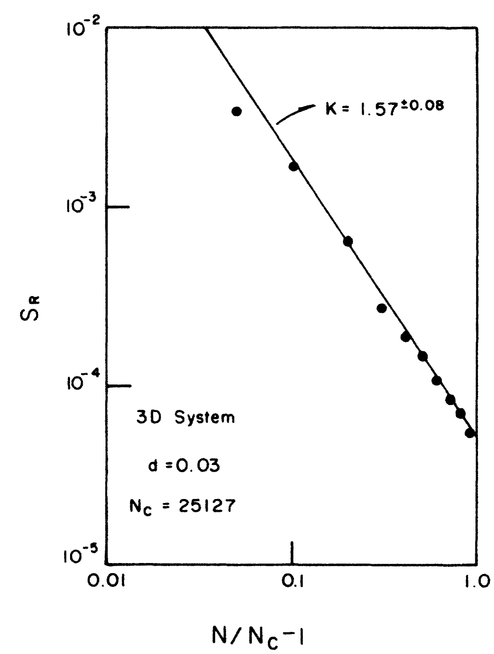

FIG. 1. The dependence of the relative resistance noise on the proximity to the percolation threshold in a system of intersectable spheres.

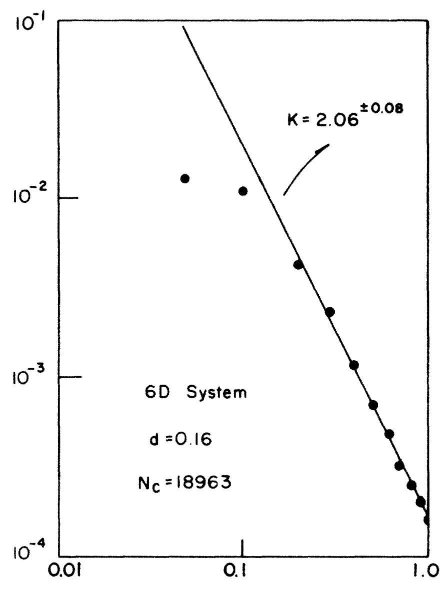

FIG. 2. The dependence of the relative resistance noise on the proximity to the percolation threshold in a six-dimensional system of intersectable hyperspheres.

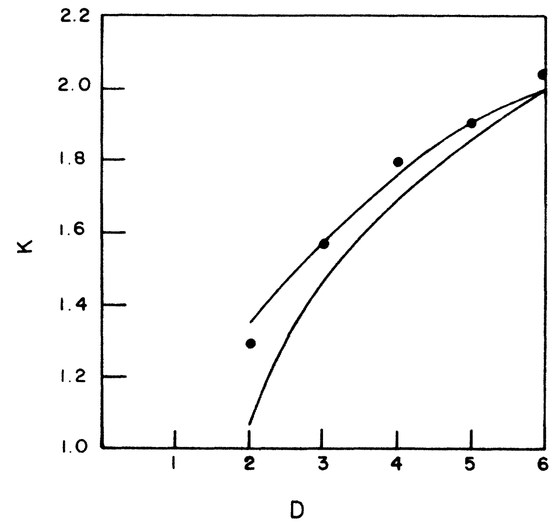

FIG. 3. A summary of the noise-exponent values derived in the present work. Also shown are the low and high scaling estimates for these values [Eq. (3)].

ponents are accurately established only for $D=2$, $D=3$, and $D=6$. For $D=4$ and $D=5$ different authors have reported somewhat different values for the exponents.[19-21] These values vary from one report to another by up to 20%. Hence, the true limits may deviate somewhat from those drawn in Fig. 3. This however cannot change the above conclusion that out simulations confirm the scaling predictions of Eq. (3) for the hypercontinuum.

In conclusion, we have shown that within our computational accuracy and the accuracy of available values for ν and ζ_R the scaling predictions are fulfilled for the noise exponents. This result and the fact that the simulations were carried out on a continuum geometry further show that our numerically derived exponents are universal.

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