Correction to acoustic transmission through a rough solid-superfluid ^{4}He interface (due to coupling between pressure and temperature waves in the liquid)

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(Received 6 August 1987)

We have calculated the reflection and transmission coefficients of high-frequency sound waves at the solid-superfluid ^{4}He interface. We have taken into account the coupling between pressure and temperature waves in the liquid due to thermal expansion and irreversible processes. This calculation is compared with recent experimental results of Moelter, Manning, and Elbaum and their interpretation.

The solid-liquid helium interface possesses very fascinating properties. In the case of a rough interface, ac melting of the interface under the influence of an incoming pressure wave occurs; the corresponding latent heat is removed in the liquid by a second sound wave and in the solid by diffusive processes. Considering first and second sound modes in the liquid as pure pressure or temperature waves, Castaing and Nozières[1] calculated the amplitudes r and t of the reflected and transmitted pressure waves. They found

$$t=\frac{2z_C}{z_L+z_C+z_Cz_L\xi}\ , \tag{1}$$

where

$$\xi=\left[\frac{\rho_C-\rho_L}{\rho_C\rho_L}\right]^2\left[\frac{1}{k}+\frac{1}{T}\frac{R_KZ_C(TS_C-\lambda)^2+R_KZ_{2L}(TS_L-\lambda)^2+Z_CZ_{2L}L^2}{(R_K+Z_C+Z_{2L})}\right]^{-1}\ , \tag{2}$$

and r is deduced from the relation $r=t-1$ reflecting the pressure conservation at the interface between the incident reflected and transmitted first sound modes; z_i is the acoustic impedance of phase i ($i=L$ for liquid, C for crystal), ρ_i the mass density, S_i the entropy, L the latent heat, Z_i the thermal impedance. k, R_K, and λ are the three Onsager coefficients describing the heat and mass transport at the interface (see, for example, Refs. 2 and 3); k is the growth coefficient, R_K the Kapitza resistance; and λ the thermal sharing coefficient.

Recently Moelter, Manning, and Elbaum[4] (MME) have measured the temperature dependence of the reflection and transmission of high-frequency sound wave at the ^{4}He superfluid-solid interface. They observe that their data are not consistent with the relation $r=t-1$. In order to account for the "missing" pressure amplitude they add a contribution to the pressure field in the liquid coming from the second sound wave: the thermal expansion coefficient introduces such a coupling effectively. But at the same time MME consider the first sound as a pure pressure mode. These two hypotheses are not mutually consistent. Indeed the ratio between the pressure and the temperature amplitudes in each eigenmode is determined only by the equation of state of the liquid or solid bulk phases. Actually the equations of continuity at the interface cannot be verified if, as in the simplifying MME assumption, the mixed character is given only to second sound mode. Moreover, we do not agree with the MME formula of the pressure associated to second sound.

On the other hand, pressure and temperature can also be coupled by the irreversible processes in liquid helium.

In order to solve accurately and coherently the problem of the acoustic reflection-transmission at the helium interface we have performed a complete calculation taking into account all these couplings (however, we have neglected the thermal expansion of solid helium for both modes).

Let i, r, t be respectively the emitted, reflected, and transmitted sound waves, r' the reflected second sound wave, and t' the transmitted diffusive temperature wave. In the liquid each wave is characterized by a normal fluid velocity $\mathbf{v}_n$, a superfluid velocity $\mathbf{v}_s$, a temperature fluctuation δT, and a pressure fluctuation δp, proportional to one another. Lifshitz[5] calculated the parameters a, b, c, defined by $v_n=av_s$, $\delta p=bv_s$, $\delta T=cv_s$ when the coupling between modes is due to thermal expansion and Dingle[6,7] did it for the irreversible processes in liquid helium (viscosity); in the hydrodynamical regime these processes can be described by two viscosity coefficients η and ζ_2. Using Lifshitz notation, we obtain

$$\delta T_{i,r}=\frac{c_1}{b_1}\delta p_{i,r}=\frac{T}{C_L\rho_L}\left[\beta+\sigma\frac{U_{2L}^2}{U_{1L}^2}\right]\delta p_{i,r}=\frac{T}{C_L\rho_L}\beta_1\delta p_{i,r}\ , \tag{3}$$

$$\delta p_{r'} = \frac{b_2}{c_2}\delta T_{r'} = -\rho_L U_{2L}^2 \left[\beta + \sigma \frac{U_{1L}^2}{U_{2L}^2}\right]\delta T_{r'} = -\rho_L U_{2L}^2 \beta_2 \delta T_{r'} , \tag{4}$$

$$\frac{1}{\rho_L}(\rho_n v_n + \rho_s v_s)_{i,r} = \frac{1}{\rho_L}\frac{a_1\rho_n + \rho_s}{b_1}\delta p_{i,r} = \frac{\delta p_{i,r}}{\rho_L U_{1L}}\left[1 + \frac{S_L T \sigma}{C_L}\right] , \tag{5}$$

$$\frac{1}{\rho_L}(\rho_n v_n + \rho_s v_s)_{r'} = \frac{1}{\rho_L}\frac{a_2\rho_n + \rho_s}{c_2}\delta T_{r'} = -U_{2L}(\beta+\sigma)\delta T_{r'} = -U_{2L}\beta_3 \delta T_{r'} , \tag{6}$$

where β is the thermal expansion coefficient, U_{1L} and U_{2L} the first and second sound velocities, and C_L the specific heat of the liquid. We put

$$\sigma = \frac{i\omega}{\rho_L U_{1L}^2}\left(\tfrac{4}{3}\eta + \zeta_2\right)\frac{C_L}{S_L T} .$$

Then Onsager relations and continuity equations are written with Nozières[3] notations:

$$\delta T_i + \delta T_r + \delta T_{r'} - \delta T_{t'} = R_K\left[\frac{\delta T_i}{Z_{1L}} - \frac{\delta T_r}{Z_{1L}} - \frac{\delta T_{r'}}{Z_{2L}} + J(TS_L - \lambda)\right] \tag{7}$$

$$= R_K\left[\frac{\delta T_{t'}}{Z_C} + J(TS_C - \lambda)\right] , \tag{8}$$

$$J = k\left[\frac{\rho_C - \rho_L}{\rho_C \rho_L}\delta p_t - \frac{1}{T}(TS_L - \lambda)(\delta T_i + \delta T_r + \delta T_{r'}) + \frac{1}{T}(TS_C - \lambda)\delta T_{t'}\right] , \tag{9}$$

$$\delta p_i + \delta p_r + \delta p_{r'} = \delta p_t , \tag{10}$$

$$J = \frac{\rho_C \rho_L}{\rho_C - \rho_L}\left[\frac{(\rho_n v_n + \rho_s v_s)_i}{\rho_L} - \frac{(\rho_n v_n + \rho_s v_s)_r}{\rho_L} - \frac{(\rho_n v_n + \rho_s v_s)_{r'}}{\rho_L} - \frac{\delta p_t}{z_C}\right] . \tag{11}$$

J is the mass current, $Z_{2L} = 1/\rho_L C_L U_{2L}$ the second sound thermal impedance for an infinite medium, $Z_C = 1/\sqrt{2\omega K_C C_C}$ the solid impedance for a diffusive thermal conduction. K_C is the solid thermal conductivity coefficient.

The liquid first sound thermal impedance Z_{1L} is calculated by writing the heat flux density $\mathbf{J}_Q = \rho_L T S_L \mathbf{v}_n$ but

$$v_n = \frac{a_1}{c_1}\delta T = \frac{C_L}{\beta T U_{1L}}\delta T + \frac{S_L}{U_{1L}}\frac{\rho_L}{\rho_n}\delta T .$$

The first term is the velocity of pure first sound wave whereas the second term is associated with the temperature fluctuation. Therefore

$$J_Q = \rho_L T S_L \frac{S_L}{U_{1L}}\frac{\rho_L}{\rho_n}\delta T = \frac{\rho_L C_L U_{2L}^2}{U_{1L}}\frac{\rho_L}{\rho_s}\delta T, \quad Z_{1L} = \frac{\delta T}{J_Q} = \frac{U_{1L}}{\rho_L C_L U_{2L}^2}\frac{\rho_s}{\rho_L} .$$

At the first order in β and in η and ζ_2 the calculations give

$$t = 2z_C \frac{1 + Q\beta_1 U_{1L}/C_L}{z_C + (z_L + z_C z_L \xi)[1 - \sigma(S_L T/C_L)] - z_C Q[\rho_L U_{2L} Z_{1L}(\beta_3 U_{1L} - \beta_2 U_{2L}) - (\beta_1 U_{1L}/C_L)(1 - Z_{1L}/Z_{2L})]} , \tag{12}$$

$$r = \frac{z_C - (z_L + z_C z_L \xi)[1 - \sigma(S_L T/C_L)] + z_C Q[\rho_L U_{2L} Z_{1L}(\beta_3 U_{1L} + \beta_2 U_{2L}) + (\beta_1 U_{1L}/C_L)(1 + Z_{1L}/Z_{2L})]}{z_C + z_L + z_C z_L \xi - z_C Q[\rho_L U_{2L} Z_{1L}(\beta_3 U_{1L} - \beta_2 U_{2L}) - (\beta_1 U_{1L}/C_L)(1 - Z_{1L}/Z_{2L})]} , \tag{13}$$

where

$$Q = \frac{\rho_C \rho_L}{\rho_C - \rho_L}\frac{Z_{2L}}{Z_{1L}}\frac{R_K(S_L T - \lambda) + Z_C L}{R_K + Z_C + Z_{2L}}\xi .$$

Figure 1 shows the results of the numerical calculation between 0.5 K and 1.5 K at 10 MHz and 25 bars. We have drawn the values of t, r, and $e = 1 - r^2 - t^2(z_L/z_C)$; the sound wave vector is supposed to be parallel to the c axis of the crystal. The values of $\frac{4}{3}\eta + \zeta_2$ have been deduced from the ultrasonic attenuation measurements.[8]

TABLE I. Comparison between experimental results and numerical calculation at 10 MHz, 25 bars, and 1 K.

	$\beta,\eta,\zeta_2 = 0$	$\beta \neq 0$ [a] $\eta,\zeta_2 = 0$	$\beta \neq 0$ [a] $\eta,\zeta_2 \neq 0$ [b]	MME
t	0.617	0.616	0.581	0.2
r	−0.383	−0.384	−0.303	−0.7
e	0.619	0.619	0.700	0.48
$t-1-r$	0	0	−0.116	

[a]Reference 12. [b]Reference 8.

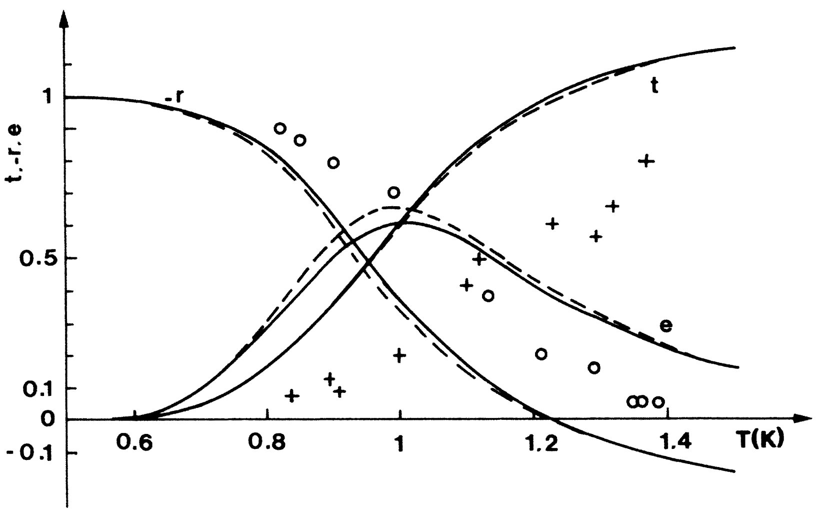

FIG. 1. Comparison between MME experimental results (dots and crosses are respectively t and r measured values) and calculation. The solid lines are the theoretical curves of t, r, and e vs T from Eqs. (12) and (13), calculated for 10 MHz, 25 bars, between 5 K and 1.5 K, without any correction. Thermal expansion effect is negligible. The dashed lines are obtained taking viscosity into account. The numerical values are $\rho_L = 0.1729$ g/cm^3, $\rho_C = 0.1908$ g/cm^3, $U_{1L} = 36\,620$ cm s^{-1}, $U_{1C} = 54\,400$ cm s^{-1}. First and second viscosities are deduced from the ultrasonic attenuation measurements of Roach, Ketterson, and Kuchnir (Ref. 8). The liquid and solid entropies are taken from Donnelly and Roberts (Ref. 9) and Gardner *et al.* (Ref. 10), respectively; thermal conductivity of the solid from Golub (Ref. 11), thermal expansion coefficient from Grilly (Ref. 12). We have taken $\rho_C/k = 0.1 + 1.5T^4 + (5.3\times10^5)e^{-7.8/T}$ cm s^{-1} (Ref. 13) and $R_K = (2\times10^{-9})T^{-5} + (1\times10^{-8})T^{-3}$ cm^{2}K erg^{-1}s^{-1} (Ref. 14).

Actually, for $T < 1.2$ K, it is no longer justified to speak of hydrodynamical regime in liquid helium, and the viscosity looses a clear meaning: the ultrasonic attenuation arises instead from three phonon events. Then at low temperatures we use $2\rho C_{1L}^3 \alpha_1/\omega$ in the place of $\omega(\frac{4}{3}\eta + \zeta_2)$ where α_1 is the sound attenuation coefficient. From experimental values of α_1 and theory α_1/ω varies very smoothly with ω between 10 and 150 MHz.

Figure 1 shows that the thermal expansion produces negligible effect on the interface transmission and reflection properties of incident sound waves. On the other hand, viscosity effect is measurable especially on reflection coefficient. At low temperatures ($T < 0.5$ K), the interface melts and crystallizes very fast and becomes a pressure mode, therefore r tends towards 1 and t towards 0. At high temperatures ($T > 1.5$ K), the melting is slowed down by liquid excitations like phonons and rotons, and the interface tends towards the usual acoustic mismatch behavior with $t = 2z_C/(z_L + z_C) = 1.238$ and $r = 0.238$ for the considered orientation. The effect is maximum at intermediate temperatures. Table I shows a comparison between experimental results and numerical calculation at 10 MHz, 25 bars, and 1 K. These numerical values do not coincide with those of MME; the discrepancy is as high as 50% and consequently the correct explanation relative to their experiment is still to come.

We acknowledge helpful discussions with Dr. A. Levelut who verified some of our calculations. Laboratoire d'Ultrasons and Laboratoire de Physique des Solides are associated with the Centre National de la Recherche Scientifique.

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