PHYSICAL REVIEW B VOLUME 37, NUMBER 7 1 MARCH 1988

Yang-Lee edge singularity in systems with correlated disorder

B. Tadić and R. Pirc
Jožef Stefan Institute, E. Kardelj University of Ljubljana, 61111 Ljubljana, Yugoslavia
(Received 25 June 1987)

We apply renormalization-group methods to study the density of Yang-Lee zeros at the edges of the gap in Ising systems with correlated random-T_c impurities. The impurity correlations are assumed to fall off at large distances as $\sim R^{-(d-\theta)}$ ($\theta > 0$). Both short- and long-range spin interactions decaying as $\sim R^{-(d+\sigma)}$ ($0<\sigma<2$) are considered. We find that the edge singularities are described by a new fixed point in the underlying ϕ^3-field model with imaginary coupling and imaginary correlated random fields. We obtain the edge exponents to leading order in the $\tilde{\epsilon}$ expansion, where $\tilde{\epsilon}=d_c-d$ with $d_c=8+\theta$ for short- and $d_c=4\sigma+\theta$ for long-range interactions which do not obey any apparent dimensional reduction rule.

I. INTRODUCTION

It has been recognized for some time that the analytic behavior of the magnetization $M(H,T)$ as a function of the magnetic field H can be related to the asymptotic distribution of the Yang-Lee zeros[1] (i.e., zeros of the partition function in the thermodynamic limit) in the complex magnetic-field plane. For a purely imaginary field the zeros occur along the field axis forming a gap for $|H| < H_c$, $H_c = H_c(T)$, within which the magnetization is an analytic function of T and H. The singular behavior of the density of Yang-Lee zeros at the edges of the gap, which appear to be branch points for the magnetization, is crucial to understanding the observable behavior of magnetization as a function of the real field and temperature. This has been first pointed out by Kortman and Griffiths[2] and studied in detail by Fisher[3] who termed these branch points the Yang-Lee edge singularities. Fisher[3] has also shown that the Yang-Lee edge singularity is essentially a critical point, where the behavior of the magnetization, which is determined by the density of zeros, can be described by a new exponent $\hat{\sigma}$, i.e.,

$$m \sim |H-H_c|^{\hat{\sigma}} , \tag{1}$$

where $m = M - M(H_c,T)$ and $\hat{\sigma}$ is related to the critical exponent δ of a ϕ^3 theory with imaginary coupling, whereas the critical behavior at $T=T_c$, $H=0$ is described by a ϕ^4-field model.

Recently Cardy[4] and Cardy and McKane[5] have applied field-theoretical methods to study the distribution of Yang-Lee zeros in a dilute Ising model. A unique feature of random systems is the existence of tails in the distribution of Yang-Lee zeros above the critical temperature of the random system $T_c(p)$, where $p<1$ is a measure of randomness. These tails are responsible for the nonanalytic behavior of the magnetization as a function of the field at the point $H=0$ in the temperature range $T_c(p) < T < T_c(1)$, where $T_c(1)$ is the critical temperature of the pure system. Such nonanalytic behavior of the thermodynamic functions is known as the Griffiths singularities.[6] Cardy[4] and Cardy and McKane[5] have applied the instanton technique for some range of parameters to describe the form that the Griffiths singularities take in these systems. The main point of their approach lies in the fact that the underlying ϕ^3 theory with imaginary coupling and imaginary random fields, which describes the Yang-Lee edge singularities in this case, is equivalent to a $(d-2)$-dimensional normal ϕ^3 theory with real coupling, which has real instantons.[7]

The equivalence of a ϕ^4- (or ϕ^3-) field theory with Gaussian random fields and a normal theory in $d-2$ dimensions has been proven to be the consequence of supersymmetry.[8,9] Similarly, in the perturbation expansion for the critical exponents a dimensional reduction rule applies to all orders. In the case of non-Gaussian distributions of random fields, however, and in particular if the random fields are spatially correlated such that their correlations decay with distance as $\sim R^{-(d-\theta)}$, $\theta>0$, the breaking of the supersymmetry and the absence of any apparent dimensional reduction rule has been demonstrated both for Ising[10] and Potts[11] models.

In the present work we consider an Ising model with correlated random-T_c impurities, where the correlations fall off at large distances. Both the cases of short- and long-range spin interactions will be discussed. To describe the Yang-Lee edge singularities in the presence of correlated disorder, we map the model onto a corresponding ϕ^3-field model, which has an imaginary coupling and imaginary correlated random fields. Applying the ϵ expansion to leading order, we find that the edge exponents describing the singular behavior of the density of Yang-Lee zeros belong to a new universality class.

Another type of correlation occurs in the case of "line" defects,[12–14] where the impurities are perfectly correlated in the ϵ_d-dimensional subspace and random in the remaining spatial dimensions. Recently Ma, Halperin, and Lee[14] have argued that such a model describes the critical properties of strongly disordered superconductors and the superfluid ^{4}He on Vycor glass. (In this case the ϵ_d-dimensional subspace represents the "time" coordinates for the classical analogue of the original quantum model, and ϵ_d is eventually set to 1.) We argue that the problem of Yang-Lee edge singularities for this type of correlated disorder reduces—with the appropriate dimensional shift—to the problem with uncorrelated disorder studied earlier.[4,5]

Kaufman and Kardar[15] have recently shown that the Ginzburg criterion, governing the crossover from classical to nonclassical critical behavior, indicates that the critical region is expanded in the presence of correlated or uncorrelated random fields. A similar analysis applied to the ϕ^3 model suggests that the nonclassical regime in the Yang-Lee edge-singularity problem is broadened by the imaginary correlated random fields.

It should be noted that in random-field systems the ϵ expansion, which assumes the dimensionality d close to the upper critical dimensionality d_c, breaks down before d reaches the lower critical dimensionality d_l (e.g., $d_l=2$ for the Ising model with Gaussian random fields). A possible explanation[15,16] is that in addition to the random fluctuations the usual thermal fluctuations become important when $d \leq 4$. Thermal fluctuations, which are neglected in the renormalization group and supersymmetry formalisms, may thus invalidate conclusions derived by these methods. Specifically, the dimensional reduction by 2 breaks down in Ising systems with uncorrelated random fields in $d=3$. Another reason[16] for the failure of the dimensional reduction rule may be the existence of Griffiths singularities[6] in random-field Ising systems.[17]

In Sec. II we derive the model appropriate to the Yang-Lee edge-singularity problem, starting from the Ising model with correlated random-T_c impurities. The renormalization-group analysis for the case of short-range interactions is given in Sec. III, while the similar steps for the case of systems with predominantly long-range interactions are presented in Sec. IV. Finally, in Sec. V we summarize the results and discuss the case of extended defects.

II. THE MODEL

We consider an Ising spin system with random-T_c impurities correlated over large distances such that the correlation behaves as

$$\Delta(x-y) \sim |x-y|^{-(d-\theta)}, \quad \theta > 0 . \tag{2}$$

The model Hamiltonan is written in replica space $\alpha,\beta=1,\ldots,n$ in the form

$$\mathcal{H} = \int d^dx \sum_{\alpha} \{ \tfrac{1}{2}[r_0 S_\alpha^2 + (\nabla S_\alpha)^2] + \tfrac{1}{4} u S_\alpha^4 - iHS_\alpha \} - \tfrac{1}{2} \int d^dx \int d^dy \, \Delta(x-y) \sum_{\alpha,\beta} S_\alpha^2(x) S_\beta^2(y) . \tag{3}$$

To study the Yang-Lee singularities in this model we have added a purely imaginary field iH, and we fix the parameter $r_0 \propto T - T_c$ at a positive value.

The quadratic term in the expression (3) is eliminated by introducing a shifted field ϕ_α *via*

$$S_\alpha = i(r_0/3u)^{1/2} + \phi_\alpha . \tag{4}$$

After this transformation the Hamiltonian (3) becomes

$$\mathcal{H} = \int d^dx \left| \sum_\alpha [\tfrac{1}{2}(\nabla\phi_\alpha)^2 + iw\phi_\alpha^3 + i(H_c - H)\phi_\alpha] + \tfrac{2}{3}(r_0/u) \int d^dy \, \Delta(x-y) \sum_{\alpha,\beta} \phi_\alpha(x)\phi_\beta(y) + \cdots \right| , \tag{5}$$

where $H_c = 2u(r_0/3u)^{3/2}$ and $w=(r_0 u/3)^{1/2}$. We have neglected all constant terms as well as the terms which vanish in the $n \to 0$ limit. The ellipsis indicate the higher-order terms such as $\sum_\alpha \phi_\alpha^4(x)$ and the terms representing more complicated effects of disorder, e.g., $\sum_{\alpha,\beta}\phi_\alpha^2(x)\phi_\beta^2(y)$ and $\sum_{\alpha,\beta}\phi_\alpha(x)\phi_\beta^2(y)$. These terms can be neglected under the conditions which are discussed in Ref. 5.

The symmetry-breaking term $\sum_{\alpha,\beta}\phi_\alpha(x)\phi_\beta(y)$ in (5) can be thought of as arising from an imaginary random field in the unreplicated Hamiltonian with long-range correlations of the type (2), apart from a constant factor $2r_0/3u$. Thus the effective Hamiltonian written out in reciprocal space, before averaging over the random fields, is of the form

$$\mathcal{H} = \int \frac{d^dq}{(2\pi)^d} \left| \tfrac{1}{2}(r+cq^2)\phi(q)\phi(-q) + iw \int \frac{d^dp}{(2\pi)^d} \phi(q)\phi(p)\phi(-q-p) + i\xi(q)\phi(-q) \right| + i(H_c - H)\phi(0) . \tag{6}$$

Here $\xi(q)$ represents a random field with the distribution

$$[\xi(q)\xi(-q)]_d = h_0^2 q^{-\theta} , \tag{7}$$

the symbol $[\ ]_d$ denoting a random average.

The parameter r has been reinstated, since it will be generated by the renormalization-group transformations, as discussed below.

III. RENORMALIZATION-GROUP ANALYSIS

In this section we apply the renormalization-group transformations to the Hamiltonian (6). We will adopt a renormalization scheme analogous to the one used earlier for the q-state Potts model,[11] i.e., we will impose the condition $\delta H' = \delta H = 0$, where $\delta H \equiv H_c - H$, and allow r in (6) to vary. An alternative procedure, which leads to the same results, is to allow δH to vary; however, a shift in the field $\phi(q)$ is then needed at each step to keep $r' = r = 0$.[3]

The critical exponent $\hat{\sigma}$ which characterizes the Yang-Lee edge singularity in pure systems has been calculated within the ϵ expansion about the upper critical dimensionality $d_c = 6$ by Fisher[3] for the case of short-range interactions. Theumann and Gusmão[18] have similarly considered the case of long-range interactions decaying with distance as $\sim R^{-(d+\sigma)}$ ($d_c = 3\sigma$ in this case). It has been argued[3] that the exponent $\hat{\sigma}$ can be expressed in terms of the critical exponents δ or η *via* the hyperscaling relation

$$\hat{\sigma} = 1/\delta = \frac{d-2+\eta}{d+2-\eta} , \tag{8}$$

where δ and η are the critical exponents for the model (6) *without* random fields.

In the presence of random fields with correlations of the type (7) we find that the upper critical dimensionality, which corresponds to the combination $w^2 h_0^2$ as an expansion parameter, is $d_c = 8+\theta$. Thus for small $\tilde{\epsilon} = 8+\theta-d$ and taking only the most divergent diagrams (see Fig. 1), we derive the recursion relations for the relevant parameters in differential form:

$$\frac{dr}{dl} = (2-\eta)r - K_d 3^2 2^2 (wh_0)^2 (1+r)^{-3} , \tag{9}$$

$$\frac{dh_0^2}{dl} = (2-\eta+\theta)h_0^2 + K_d 3^2 2 (wh_0)^2 h_0^2 , \tag{10}$$

$$\frac{dw}{dl} = \tfrac{1}{2}(6-d-3\eta)w + K_d 3^3 2^2 (wh_0)^2 w , \tag{11}$$

where

$$\eta = K_d 3^2 2^2 (wh_0)^2 \frac{4+\theta}{8+\theta} \tag{12}$$

and $K_d^{-1} = 2^{d-1}\pi^{d/2}\Gamma(d/2)$.

Combining (10) and (11) we have

$$\frac{d(wh_0)^2}{dl} = (\tilde{\epsilon}-4\eta)(wh_0)^2 + K_d 3^2 2 6 (wh_0)^4 , \tag{13}$$

whence by using (12) we obtain the fixed-point value

$$(wh_0)^* = i\frac{1}{3}\left[\frac{1}{2}K_d^{-1}\tilde{\epsilon}\frac{8+\theta}{72+5\theta}\right]^{1/2} . \tag{14}$$

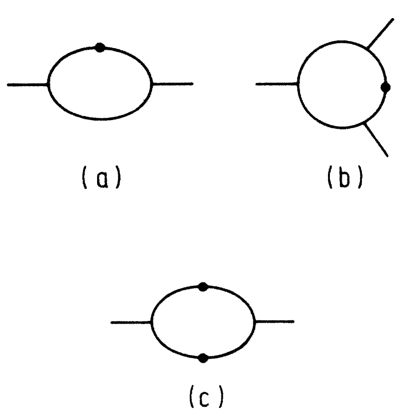

FIG. 1. One-loop diagrams contributing to the recursion relation for the parameters (a) r, (b) w, and (c) h_0^2. A heavy dot on an internal line carrying the momentum k represents the factor $h_0^2 k^{-\theta}$.

Inserting this back into Eq. (12) we find

$$\eta = -2\tilde{\epsilon}\frac{4+\theta}{72+5\theta} . \tag{15}$$

Notice that the expression (15) for $\theta=0$ reduces to $\eta = -\tilde{\epsilon}/9$, i.e., the value of the critical exponent η obtained for the pure model[3] but with a replacement of $\epsilon = 6-d$ by $\tilde{\epsilon} = 8-d$. This dimensional reduction by 2 is known to be due to the supersymmetry of the field-theoretic model with Gaussian random fields.[4,5] The presence of long-range correlations, however, breaks the supersymmetry. Consequently, there is no apparent relation between the critical exponents of a system with correlated random fields on the one hand, and the pure system in $d-2-\theta$ dimensions on the other. However, the effective spatial dimensionality which enters the hyperscaling relations is reduced, i.e.,

$$d \to d' = d + \lambda_w , \tag{16}$$

where $\lambda_w < 0$ is the scaling exponent of the field w^2, which enters the scaling part of the free energy in a singular manner[11] and becomes irrelevant in dimensions d above 6. Going back to Eq. (11), where wh_0 is now held fixed at the fixed-point value (14), we find $w(l) = w(0)\exp(\frac{1}{2}\lambda_w l)$ with

$$\lambda_w = -(2+\theta) - \tilde{\epsilon}\frac{\theta}{72+5\theta} . \tag{17}$$

Notice that the hyperscaling relation (8) is modified by the presence of random fields in the sense that the dimensionality d is replaced by d' defined by (16). Thus, using Eqs. (8), (15), (16), and (17) we find the critical exponent $\hat{\sigma}$,

$$\hat{\sigma} = \frac{1}{2} - \frac{3}{4}\tilde{\epsilon}\frac{4+\theta}{72+5\theta} , \tag{18}$$

which, as expected, reduces to the value found by Fisher[3] in $d-2$ dimensions when the correlations of the random fields vanish ($\theta=0$).

There is an additional hyperscaling relation between the critical exponents of a scalar ϕ^3 theory, namely,

$$1/\nu = \tfrac{1}{2}(d-2+\eta) , \tag{19}$$

which appears to be the consequence of the structure of Feynman diagrams contributing to the two-point vertex function with ϕ^2 insertions and the three-point vertex function, as discussed by Theumann and Gusmão.[18] Replacing d by $d' = d + \lambda_w$ in (19) and using the expressions (15) and (17), we obtain

$$1/\nu = 2 - 4\tilde{\epsilon}\frac{10+\theta}{72+5\theta} . \tag{20}$$

On the other hand, by linearizing the recursion relation (9) we find

$$1/\nu = 2 + 3^2 2^3 K_d [(wh_0)^*]^2 \frac{10+\theta}{8+\theta} , \tag{21}$$

and upon inserting the fixed-point value $(wh_0)^*$ from (14) we indeed obtain the same expression as (20). This guarantees that the modified relation (19) in the presence of correlated random fields holds to linear order in the $\tilde{\epsilon}$ expansion.

In the Yang-Lee edge-singularity problem one can define another exponent ν_c, which measures the divergence of the correlation length ξ as the magnetic field approaches its critical value H_c at fixed temperature, i.e.,

$$\xi \approx |H-H_c|^{-\nu_c} . \tag{22}$$

According to Fisher,[3] the following hyperscaling relation holds:

$$1/\nu_c = \tfrac{1}{2}(d+2-\eta) . \tag{23}$$

Here again one has to replace d by $d+\lambda_w$. Thus, with the help of (15) and (17) we have

$$\nu_c = \frac{1}{4} + \frac{1}{8}\tilde{\epsilon}\frac{16+\theta}{72+5\theta} , \tag{24}$$

which for $\theta=0$ obeys the dimensional reduction rule $d \to d-2$. It should be noted that, due to the relation (19) [as well as its modified counterpart $1/\nu = (d+\lambda_w-2+\eta)/2$] and the scaling arguments,[3] the Yang-Lee exponent $\hat{\sigma}$ can be expressed as

$$\hat{\sigma} = \nu_c/\nu . \tag{25}$$

IV. THE CASE OF LONG-RANGE INTERACTIONS

We now extend the study of the Yang-Lee edge singularities in random systems to the case of long-range interactions between the spins which decay with distance as $\sim R^{-(d+\sigma)}$, $\sigma<2$. Therefore, the quadratic term in the Hamiltonian (6) is rewritten as

$$\mathcal{H}_0 = \int \frac{d^dq}{(2\pi)^d}\frac{1}{2}(r+cq^2+aq^\sigma)\phi(q)\phi(-q) . \tag{26}$$

The effects of long-range interactions on the phase transitions in various ϕ^3-field models without random fields have been discussed earlier in the literature.[18–20] It has been recognized that if, for $\sigma<2$, the correlation function exponent $\eta=\eta_{\rm SR}$ evaluated at the fixed point controlled by the short-range interactions is negative (i.e., $\eta_{\rm SR}<0$), then the term aq^σ in (26) controls the expansion for the critical exponents. In particular, the value for η at the long-range fixed point retains its classical value $\eta_{\rm LR}=2-\sigma$, and the expansion becomes singular in the limit $\sigma\to 2$.[21] In the presence of random fields and short-range interactions, as described in Sec. III, the critical behavior is controlled by the fixed point (14) on the (wh_0) axis. The value of the critical exponent $\eta_{\rm SR}$ at this fixed point is negative [cf. Eq. (15)]. Thus we expect that in some region of parameter space the critical behavior will be dominated by the long-range part of the interaction, as discussed in greater detail at the end of this section. To describe the critical behavior in this case, we apply the renormalization-group analysis to the Hamiltonian (6) in which the quadratic term has been replaced by (26). By fixing $c=0$ and $a=1$ we see that to leading order $\eta_{\rm LR}=2-\sigma$. Thus, assuming a random-field distribution of the type (7), we find the upper critical dimensionality $\bar{d}_c=4\sigma+\theta$, as opposed to $d_c=8+\theta$ in the short-range case. Considering the one-loop contribution to the pair-correlation function we have formally

$$\eta_{\rm LR} = 2-\sigma-3^2 2^2 (wh_0)^2 \frac{\epsilon'}{\sigma-2}\frac{\sigma}{2}\frac{2\sigma+\theta}{3\sigma+\theta+2} , \tag{27}$$

where $\epsilon'=4\sigma+\theta-d$ and $(wh_0)^2$ at the long-range fixed point is of the order ϵ', as shown below. Thus there are no corrections to the classical value $\eta_{\rm LR}=2-\sigma$ to linear order in ϵ'. In the limit $\sigma\to 2$, the expression (27) reduces to the result found in the short-range case [cf. Eq. (12)] provided that $\epsilon'/(\sigma-2)\to -1$.[21]

The recursion relations to linear order in ϵ' are

$$\frac{dr}{dl} = \sigma r - K_d 3^2 2^2 (wh_0)^2(1+r)^{-3} , \tag{28}$$

$$\frac{dh_0^2}{dl} = (\sigma+\theta)h_0^2 + K_d 3^2 2 (wh_0)^2 h_0^2 , \tag{29}$$

$$\frac{dw}{dl} = \tfrac{1}{2}(3\sigma-d)w + K_d 3^3 2^2 (wh_0)^2 w . \tag{30}$$

From (29) and (30) we find the fixed-point value for the relevant expansion parameter $(wh_0)^2$, i.e.,

$$(wh_0)^* = i\tfrac{1}{3}(\epsilon'/26)^{1/2} . \tag{31}$$

Linearizing Eq. (28) near this fixed point we obtain

$$1/\nu = \sigma - \tfrac{6}{13}\epsilon' . \tag{32}$$

In analogy to the case of short-range interactions discussed in Sec. III, one can show that the exponent ν may be expressed *via* the modified hyperscaling relation $1/\nu=(d'-2+\eta)/2$. Here $d'=d+\bar{\lambda}_w$ is the effective dimensionality, where according to (30) and (31) we find

$$\bar{\lambda}_w = -(\sigma+\theta)+\tfrac{1}{13}\epsilon' . \tag{33}$$

This can also be written as

$$\bar{\lambda}_w = -(9\sigma+12\theta+d)/13 < 0 .$$

It should be stressed that for vanishing θ (i.e., Gaussian random fields) the dimensional reduction $d\to d+\bar{\lambda}_w$ in the case of long-range interactions does not imply a simple dimensional reduction rule for the critical exponents, in contrast to the case of short-range interactions [cf. Eq. (17)]. A similar feature has been found in ϕ^4-field models with long-range interactions and uncorrelated random fields.[22]

Using the expression (33) to replace d by $d+\bar{\lambda}_w$ in the hyperscaling relations (8) and (23), we find the critical exponents $\hat{\sigma}$ and ν_c characterizing the behavior of the magnetization and the correlation length, respectively:

$$\hat{\sigma} = \tfrac{1}{2}(1-\tfrac{3}{13}\epsilon'/\sigma) , \tag{34}$$

$$\nu_c = \frac{1}{2\sigma}(1+\tfrac{3}{13}\epsilon'/\sigma) . \tag{35}$$

It is surprising that—to given order—the results for the critical exponents do not depend explicitly on the range of impurity correlations. It is worthwhile to compare the results of this section with the corresponding results obtained for pure systems with long-range interactions.[18] In particular, based on the observation that the result $\eta_{\rm LR}=2-\sigma$ may be correct to all orders in perturbation expansion and using the hyperscaling relations (8) and (19), Theumann and Gusmão[18] concluded that the expressions for the other critical exponents, i.e., $\nu=(d-\sigma)/2$ and $\hat{\sigma}=(d-\sigma)/(d+\sigma)$, may also be exact results. In the presence of random fields, however, though the relation $\eta_{\rm LR}=2-\sigma$ may still be exact (we have proven it to leading order only), the hyperscaling relations are modified by lowering the dimensionality by an approximate amount, Eq. (33). Consequently, the results for the critical exponents (32), (34), and (35) cannot be matched to the corresponding expressions in the pure case.[18] This can again be regarded as a manifestation of the breaking of supersymmetry by the long-range interactions.

In deriving the above results we have assumed that the short-range term cq^2 in Eq. (26) can be ignored compared to the long-term part aq^σ. The validity of the long-range expansion for $\sigma<2$ can be justified by retaining both terms in the recursion relations.[18,19] Then at the fixed point one has

$$(wh_0)^*=i\tfrac{1}{3}[\epsilon'(c^*+a^*)/26]^{1/2}\ . \qquad (36)$$

The right-hand side reduces to the fixed-point value (31) if one sets $c^*=0$ and $a^*=1$, as chosen above. Moreover, the irrelevance of the parameter c follows from its recursion relation, namely,

$$\frac{dc}{dl}=(\sigma-2)c+3^2 2^2(wh_0)^2(c+a)^{-3}\ . \qquad (37)$$

Solving this equation near the fixed point (36) we derive the scaling exponent which governs c under the renormalization

$$\lambda_c=\sigma-2-\tfrac{2}{13}\epsilon'\ . \qquad (38)$$

The negativity of λ_c for $\sigma<2$ confirms the validity of the long-range expansion employed in the above analysis.

V. DISCUSSION AND CONCLUSIONS

We have considered the problem of Yang-Lee edge singularities in Ising systems with long-range correlated disorder of the random-T_c type. The correlations are assumed to decay at large distances as $\sim R^{-(d-\theta)}$, $\theta>0$. Both short- and long-range spin interactions have been considered. By mapping the problem onto a scalar ϕ^3-field model with imaginary coupling and imaginary correlated random fields, we have calculated the critical exponents to leading order in the ϵ expansion about the corresponding upper critical dimensionality. We find that in both cases the critical behavior at the Yang-Lee edge in the presence of correlated disorder forms a new universality class. In particular, in the case of short-range interactions the critical exponents characterizing the behavior of the magnetization and the correlation length at the Yang-Lee edge explicitly depend on the range of impurity correlations. For a vanishingly small correlation range, however, our results for the critical exponents can be mapped onto the ones obtained earlier for pure systems[3] by a simple dimensional reduction $d\rightarrow d-2$. This feature of the critical behavior under Gaussian random fields has been well understood recently in terms of the supersymmetry of the underlying field theory.[4,5,8,9] For nonzero θ, as well as in the case of long-range spin interactions, the effective dimensionality of the system is still reduced due to random fields; however, our results for the critical exponents exhibit no apparent dimensionality reduction rule. This can be understood as a consequence of the breaking of supersymmetry due to the momentum dependent correlations and/or long-range interactions. Similar results have been reported earlier for the critical behavior under correlated random fields in Ising[10] and Potts[11] models, and also in the case of long-range interactions in n-vector models.[22]

The existence of supersymmetry in the case of uncorrelated disorder and short-range interactions has enabled the study of Griffiths singularities by means of the instanton technique.[4,5] For disordered systems with long-range correlations and/or long-range spin interactions, however, our perturbation expansion to leading order suggests that the supersymmetry is broken. Thus the possibility of mapping the Yang-Lee edge-singularity problem onto a lower-dimensional normal ϕ^3 theory—which has instantons—is ruled out. We conclude that alternative nonperturbative methods should be considered in order to determine the actual form of the Griffiths singularities in the present model.

The situation is, however, different in the case of extended defects[12,13] and some disordered quantum systems.[14] In this case the random-T_c type impurities are perfectly correlated in a ϵ_d-dimensional subspace of a $(d+\epsilon_d)$-dimensional lattice and uncorrelated in the remaining d dimensions. In the spirit of the results presented in Sec. II, the same kind of correlations characterize the imaginary random fields in the corresponding scalar ϕ^3 model which describes the Yang-Lee edge singularity in these systems. A straightforward analysis, analogous to Sec. III, leads to the conclusion that the edge singularity in this case belongs to the same universality class as that of a nonrandom classical system in $(d+\epsilon_d+\hat{\lambda}_w)$ dimensions, where for $\hat{\lambda}_w$ we find $\hat{\lambda}_w=-(2+\epsilon_d)+O(\hat{\epsilon}^2)$, $\hat{\epsilon}=8-d$. Therefore, the universality class is specified by a $(d-2)$-dimensional classical system or, equivalently, a $(d-\epsilon_d-2)$-dimensional quantum one.[23] Thus the edge singularity is governed by the exponents obtained in Sec. III with $\theta=0$ or, as already discussed, by the Yang-Lee exponents derived by Fisher[3] for pure classical systems with d replaced by $d-2$. This implies that, in contrast to the case of isotropic impurity correlations considered in Sec. III ($\theta\neq0$), the supersymmetry is preserved in the case of extended defects. Consequently, the form of Griffiths singularities found by Cardy and McKane[4,5] also applies to this class of disordered systems. It should be mentioned that in quantum systems the role of temperature is played by some other quantity[14] which drives the transition at zero temperature.

Finally, it should be noted that throughout this paper we have restricted our discussion to the random-T_c Ising systems. The results obtained for the Yang-Lee edge singularities are, however, valid for a larger class of systems; namely, the model (6) also applies to spin systems of continuous symmetry with a finite number of spin components n,[3] provided that the impurities do not affect the invariance of the Hamiltonian under spin rotations.

ACKNOWLEDGMENT

This work has been supported by the Research Council of Slovenia (Yugoslavia) under Contracts Nos. C1-0502-106-87 and C1-0104-106-87.

[1] C. N. Yang and T. D. Lee, Phys. Rev. **87**, 404 (1952); T. D. Lee and C. N. Yang, *ibid.* **87**, 410 (1952).
[2] P. J. Kortman and R. B. Griffiths, Phys. Rev. Lett. **27**, 1439 (1971).
[3] M. E. Fisher, Phys. Rev. Lett. **40**, 1610 (1978).
[4] J. L. Cardy, Physica (Utrecht) **15D**, 123 (1985).
[5] J. L. Cardy and A. J. McKane, Nucl. Phys. **B257** [FS14], 383 (1985).
[6] R. B. Griffiths, Phys. Rev. Lett. **23**, 17 (1969).
[7] A. J. McKane, Nucl. Phys. **B152**, 166 (1979).
[8] G. Parisi and N. Sourlas, Phys. Rev. Lett. **46**, 871 (1981).
[9] J. L. Cardy, Phys. Lett. **125B**, 470 (1983).
[10] M. Kardar, B. McClain, and C. Taylor, Phys. Rev. B **27**, 5875 (1983).
[11] B. Tadić and R. Pirc, Phys. Rev. B **32**, 350 (1985).
[12] S. N. Dorogovtsev, Phys. Lett. **76A**, 169 (1980); Zh. Eksp. Teor. Fiz. **83**, 1851 (1982) [Sov. Phys.—JETP **56**, 1070 (1982)].
[13] D. Boyanovsky and J. L. Cardy, Phys. Rev. B **26**, 154 (1982); **27**, 6971(E) (1982).
[14] M. Ma, B. I. Halperin, and P. A. Lee, Phys. Rev. B **34**, 3136 (1986).
[15] M. Kaufman and M. Kardar, Phys. Rev. B **31**, 2913 (1985).
[16] M. Kaufman, Superlattices Microstructures **1**, 511 (1985).
[17] M. Schwartz, Y. Shapir, and A. Aharony, Phys. Lett. **106A**, 191 (1984).
[18] W. K. Theumann and M. A. Gusmão, Phys. Rev. B **31**, 379 (1985).
[19] M. Chang and J. Sak, Phys. Rev. B **29**, 2652 (1984).
[20] G. Kotliar, P. W. Anderson, and D. L. Stein, Phys. Rev. B **27**, 602 (1983).
[21] See, e.g., M. E. Fisher, S.-k. Ma, and B. G. Nickel, Phys. Rev. Lett. **29**, 917 (1972), where a similar analysis has been carried out for the pure ϕ^4 model.
[22] A. P. Young, J. Phys. C **10**, L257 (1977).
[23] This is analogous to the destruction of the quantum to classical crossover occurring in quantum spin systems under real random fields. See, e.g., A. Aharony, Y. Gefen, and Y. Shapir, J. Phys. C **15**, 673 (1982); D. Boyanovksy and J. L. Cardy, Phys. Rev. B **27**, 5557 (1983); R. Micnas and K. A. Chao, Phys. Rev. B **30**, 6785 (1984).