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High-field magnons in ferromagnets with random anisotropy axes

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We investigate magnons in ferromagnets with random anisotropy axes. It is assumed the applied field is large in comparison with the exchange and anisotropy fields. A coherent-anisotropy-field approximation is introduced, where the effect of the disorder is represented by a complex, energy-dependent anisotropy field, analogous to the coherent potential in the random-alloy problem. The real part of the anisotropy field represents the shift in the magnon energy, whereas the imaginary part gives the damping. The shift and damping of the uniform mode and the damping rate across the magnon band are calculated for both a uniform and a nonuniform distribution in the cosine of the angle between the anisotropy axis and the applied field.

I. INTRODUCTION

Since its introduction in 1973 by Harris, Plischke, and Zimmermann,[1] the ferromagnet with random anisotropy axes has served as a model of amorphous intermetallic compounds containing rare-earth atoms with nonzero orbital angular momentum such as a-$TbFe_2$.[2] The Hamiltonian for this model can be written

$$H=-\sum_{\langle i,j\rangle} J_{ij}\mathbf{S}_i\cdot\mathbf{S}_j-\mathcal{D}\sum_i(\hat{\mathbf{n}}_i\cdot\mathbf{S}_i)^2+g\mu H\sum_i S_z^i\ , \qquad (1.1)$$

where $\mathbf{S}$ is the spin, J_{ij} are the exchange constants, $\mathcal{D}$ is an anisotropy constant, H is the applied field and the $\hat{\mathbf{n}}_i$ are anisotropy axes which are assumed to vary randomly throughout the system.

Recently, Chudnovsky *et al.*[3] and Saslow[4] have investigated a continuum version of (1.1) with particular emphasis on the effect of a uniform applied field. When the anisotropy field is small in comparison with the exchange field, three regimes can be distinguished, depending on the relative magnitudes of the dimensionless ratios H/H_{ex} and H_a/H_{ex}, where H_{ex} and H_a denote the exchange and (random) anisotropy fields, respectively. When $H/H_{ex} \ll (H_a/H_{ex})^4$ the system is in the correlated spin-glass regime; for $(H_a/H_{ex})^4 \ll H/H_{ex} \ll 1$ it is in the ferromagnet with wandering axis (FWA) regime in which the spins are locally aligned with the axis varying slowly from region to region. For $H/H_{ex} \gtrsim 1$ the system is in the high-field regime where there is a slight misalignment of the spin with respect to the applied field, which varies randomly from site to site. As emphasized in Ref. 3, this regime occurs whenever $H \gg H_a, H_{ex}$, independent of the relative magnitudes of H_a and H_{ex}.

The focus in this paper is on the high-field regime. We assume that the temperature is sufficiently low that the fundamental excitations in the system are (quasi)harmonic magnons. Our interest is in the effect of the disorder on the magnon spectrum. The approach we are following leads to a coherent-anisotropy-field approximation (CAFA) which is the analogy of the coherent-potential approximation (CPA) that has found wide applicability in the characterization of the electronic states in disordered alloys.[5] A self-consistent equation is derived for the coherent anisotropy field, the real and imaginary parts of the field giving the shift and broadening of the magnon modes, respectively. As will be discussed in greater detail below, the effect of the random anisotropy can, in principle, be observed in ferromagnetic resonance and inelastic neutron scattering experiments carried out in high magnetic fields.

II. THEORY

Since the alignment of the spins is nearly complete in the high-field regime, at low temperatures we can use the Holstein-Primakoff transformation to express the spin operators in terms of boson annihilation and creation operators. Initially, we will assume that there is a periodic array of spins; later, the analysis will be generalized to amorphous systems. With a periodic array the harmonic magnons associated with the exchange and Zeeman terms in (1.1) have energies given by

$$E(\mathbf{k})=g\mu H+S[J(0)-J(\mathbf{k})]\ , \qquad (2.1)$$

where S is the spin and $J(\mathbf{k})$ denotes the Fourier transform of the exchange interaction

$$J(\mathbf{k})=\sum_j J_{ij}\exp[i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\ . \qquad (2.2)$$

Using the Holstein-Primakoff transformation we can write the anisotropy terms in the form

$$-\mathcal{D}(\hat{\mathbf{n}}_i\cdot\mathbf{S}_i)^2=-\mathcal{D}[(-S+a_i^\dagger a_i)n_i^z+(S/2)^{1/2}(n_i^x+in_i^y)a_i+(S/2)^{1/2}(n_i^x-in_i^y)a_i^\dagger+\cdots]^2\ . \qquad (2.3)$$

The terms not shown explicitly in (2.3), when written in normal-ordering form, lead to the renormalization of the anisotropy constant and magnon-magnon scattering. In addition, there are terms which introduce admixtures of the vacuum state or multimagnon states into the one-magnon states. Since we have assumed $H \gg H_a, H_{ex}$, the admixed states will be on the order of $2\mathcal{D}S/(g\mu H) \ll 1$, and thus can be neglected.

Apart from a constant, the one-magnon terms in (2.3) take the form

$$-[\mathcal{D}(\hat{\mathbf{n}}\cdot\mathbf{S})^2]_{\text{one-magnon}} = 3DS(\cos^2\theta_i - \tfrac{1}{3})a_i^\dagger a_i \ , \tag{2.4}$$

where θ_i is the angle between $\hat{\mathbf{n}}_i$ and the applied field and $D = \mathcal{D}[1-1/(2S)]$ is the renormalized anisotropy constant.

The right-hand side of (2.4) is the analog of a potential fluctuation in the random-alloy problem. With this identification, one can adopt the CPA formalism. In the CAFA the ensemble average of the propagator takes the form

$$\langle G \rangle[\mathbf{k}, E - g\mu H_a^c(E)] = [E - g\mu H_a^c(E) - E(\mathbf{k})]^{-1} \ , \tag{2.5}$$

where the coherent anisotropy field, $H_a^c(E)$, is given by the nonlinear equation

$$\int_{-1}^{1} d(\cos\theta) \frac{P(\cos\theta)[3DS(\cos^2\theta - \frac{1}{3}) - g\mu H_a^c(E)]}{1 - [3DS(\cos^2\theta - \frac{1}{3}) - g\mu H_a^c(E)]\langle G_0 \rangle [E - g\mu H_a^c(E)]} = 0 \ , \tag{2.6}$$

in which

$$\langle G_0 \rangle (E) = N^{-1} \sum_{\mathbf{k}} \langle G \rangle (\mathbf{k}, E) \ , \tag{2.7}$$

N is the number of spins, and $P(\cos\theta)$ is the probability distribution for the cosine of the angle between the anisotropy axis and the applied field.

Equations (2.5) and (2.6) are the principal results of this section. The effect of the random anisotropy on the magnon spectrum is represented by a complex, energy-dependent coherent anisotropy field which is obtained self-consistently from (2.6). In analogy with the random alloy, both the real part of $H_a^c(E)$, which can be interpreted as a shift in the magnon energy, and the imaginary part of $H_a^c(E)$, which gives rise to a damping or dephasing, can be looked upon as arising from the scattering of the magnons off the fluctuations in the anisotropy field.

III. CAFA

In this section we consider the solution to Eq. (2.6). In the limit of weak anisotropy one can expand the coherent anisotropy field to order D^2 thus obtaining

$$g\mu H_a^c(E) \approx 3DS \int_{-1}^{1} d(\cos\theta) P(\cos\theta)(\cos^2\theta - \tfrac{1}{3}) + (3DS)^2 \langle G_0 \rangle (E) \int_{-1}^{1} d(\cos\theta) P(\cos\theta)[(\cos^2\theta - \tfrac{1}{3}) - \langle \cos^2\theta - \tfrac{1}{3} \rangle]^2 \ , \tag{3.1}$$

where $\langle \ \rangle$ refers to an average over $P(\cos\theta)$. In systems where there are no preferred directions for $\hat{\mathbf{n}}$, $P(\cos\theta)$ is equal to $\frac{1}{2}$ and $g\mu H_a^c$ vanishes to first order in D. In these cases the first nonvanishing term in H_a^c is of order D^2, viz.,

$$g\mu H_a^c(E) = \tfrac{4}{5} D^2 S^2 \langle G_0 \rangle (E) \ , \tag{3.2}$$

and arises from the fluctuations in $\cos^2\theta$ relative to the mean value of $\frac{1}{3}$.

From the above analysis it is evident that the calculation of $H_a^c(E)$ to order $(DS)^2$ and higher requires knowledge of the Green's function for the unperturbed system. Using (2.7) we can write $\langle G_0 \rangle (E)$ in the form

$$\langle G_0 \rangle (E) = \int_0^\infty \frac{\rho(u)}{E - u} du \ , \tag{3.3}$$

where

$$\rho(u) = \frac{1}{N} \sum_{\mathbf{k}} \delta[u - E(\mathbf{k})] \ , \tag{3.4}$$

is the normalized magnon density of states in the absence of anisotropy. In our analysis we will measure energies relative to $g\mu H$ and take

$$\rho(E) = (8/\pi E_M)(E/E_M)^{1/2}(1 - E/E_M)^{1/2} \tag{3.5}$$

for $0 \le E/E_M \le 1$, and zero otherwise. For $E \ll E_M$, $\rho(E)$ varies as $E^{1/2}$, which is characteristic of a system where $E(\mathbf{k}) \sim k^2$ at long wavelengths. With $E(\mathbf{k}) = \hat{D}k^2$ we have

$$\rho(E) = v E^{1/2} / 4\pi^2 \hat{D}^{3/2} \ , \tag{3.6}$$

in which v is the volume per spin. Comparing (3.5) and (3.6) we can make the connection

$$E_M = (32\pi/v)^{2/3} \hat{D} \ . \tag{3.7}$$

The CAFA is developed under the assumption that the array of spins has translational symmetry. Although random anisotropy magnets are generally amorphous, the magnons have a quadratic dispersion relation at long wavelengths reflecting the (approximate) conservation of total spin. Since the $E^{1/2}$ behavior is present in both crystalline and amorphous ferromagnets, we expect our theory to be applicable to the latter for energies near the bottom of the magnon band, a point we explore in greater detail below.

With $\rho(E)$ given by (3.4), $\langle G_0 \rangle(E)$ assumes the form

$$\langle G_0 \rangle(E) = (8/E_M)\{E/E_M - \tfrac{1}{2} - [(E/E_M)^2 - (E/E_M)]^{1/2}\} \ . \quad (3.8)$$

The function $\langle G_0 \rangle(E)$ is analytic in the complex E plane with a cut along the real axis between 0 and E_M.

We have solved (2.6) with $\langle G_0 \rangle$ given by (3.8) for two cases of interest. In the first of these $\cos\theta$ is distributed uniformly between -1 and $+1$, viz.,

$$P(\cos\theta) = \tfrac{1}{2}, \quad 0 \le \theta \le \pi \ . \quad (3.9)$$

The resulting self-consistent equation takes the form

$$\tanh\{(3AG/2)^{1/2}[1+(h+A/2)G]^{1/2}\} = (3AG/2)^{1/2}/[1+(h+A/2)G]^{1/2} \ , \quad (3.10)$$

where

$$A = 2DS \ , \quad (3.11)$$

$$h = g\mu H_a^c(E) \ , \quad (3.12)$$

and

$$G = \langle G_0 \rangle[E - g\mu H_a^c(E)] \ . \quad (3.13)$$

In the second case we have

$$P(\cos\theta) = |\cos\theta| \ , \quad 0 \le \theta \le \pi \ . \quad (3.14)$$

The corresponding self-consistent equation takes the form

$$\ln\{[1+(h+A/2)G]/[1+(h-A)G]\} = 3AG/2 \ , \quad (3.15)$$

where the symbols have the same meaning as before.

Provided the magnon damping is relatively small, as is the case here, the magnon energies in the presence of the applied and anisotropy fields, $\tilde{E}_k$, are inferred from the energy at which the real part of the denominator of $\langle G \rangle[\mathbf{k}, E - g\mu H_a^c(E)]$ vanishes:

$$\tilde{E}_k - g\mu \,\mathrm{Re} H_a^c(\tilde{E}_k) - E(\mathbf{k}) = 0 \ , \quad (3.16)$$

where Re denotes real part. Likewise, the linewidths of these modes, Γ_k, are given by the imaginary part of the denominator evaluated at $\tilde{E}_k$:

$$\Gamma_k = g\mu \,\mathrm{Im} H_a^c(\tilde{E}_k) \ , \quad (3.17)$$

where Im denotes imaginary part.

In Figs. 1 and 2 we display our results for the energy and width of the uniform ($\mathbf{k}=0$) mode, which is excited in ferromagnetic resonance, versus $2DS/E_M$ for the case of a uniform distribution for $\cos\theta$, Eq. (3.9). Energies are measured relative to $g\mu H$, and energies and linewidths are in units of E_M. In Fig. 3 we show the energy dependence of the linewidth in a plot of $g\mu \,\mathrm{Im} H_a^c(E)$ vs $(E - g\mu H)/E_M$ for $2DS/E_M = 0.2$ and 0.4, assuming a uniform distribution for $\cos\theta$. These data correspond to linewidths which would be observed in inelastic neutron-scattering experiments with energy transfer E. In Figs. 4–6 we show equivalent results for the linear distribution [Eq. (3.14)]. The data displayed in Figs. 1–3 are obtained from Eq. (3.10), whereas the data shown in Figs. 4–6 are found by solving (3.15). We postpone discussion of these results until Sec. IV.

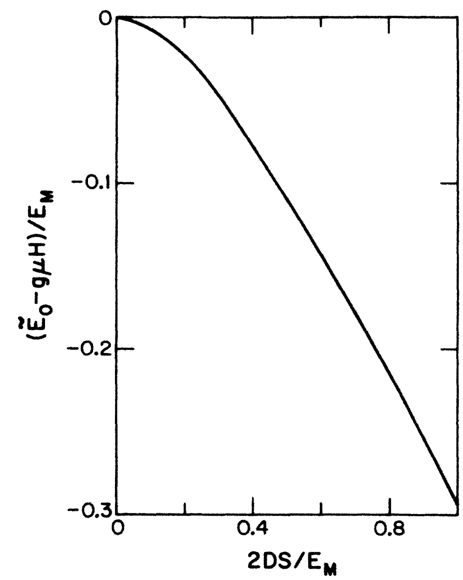

FIG. 1. Shift in energy of the uniform ($\mathbf{k}=0$) mode vs $2DS/E_M$. Energy in units of E_M. Uniform probability distribution, Eq. (3.9).

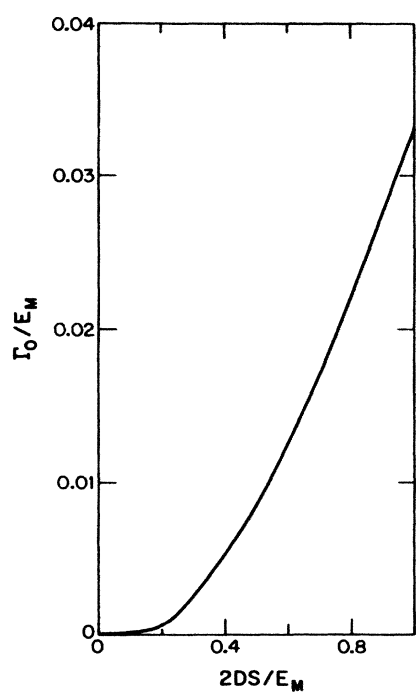

FIG. 2. Linewidth of the uniform ($\mathbf{k}=0$) mode vs $2DS/E_M$. Linewidth in units of E_M. Uniform probability distribution, Eq. (3.9).

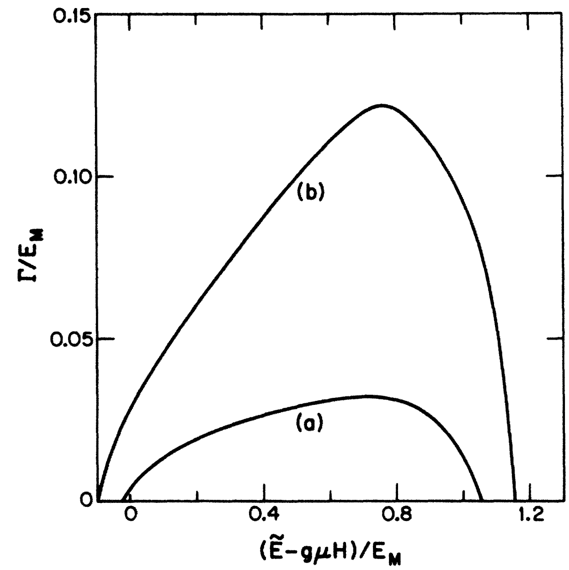

FIG. 3. Magnon linewidth vs $(\tilde{E}-g\mu H)/E_M$. (a) $2DS/E_M=0.2$; (b) $2DS/E_M=0.4$. Linewidth in units of E_M. Uniform probability distribution, Eq. (3.9).

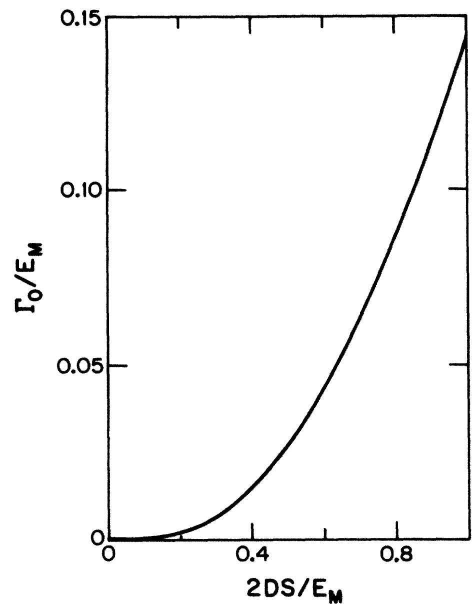

FIG. 5. Linewidth of the uniform ($\mathbf{k}=0$) mode vs $2DS/E_M$. Linewidth in units of E_M. Linear probability distribution, Eq. (3.14).

IV. DISCUSSION

In the preceding sections we have outlined a theory for the influence of random anisotropy on the magnon modes in a ferromagnet. The theory applies to the high-field regime where the applied field is large in comparison with the exchange and anisotropy fields. A coherent anisotropy field approximation is introduced, and a self-consistent equation, (2.6), is obtained for $H_a^c(E)$. The real and imaginary parts of $H_a^c(E)$ determine the shift and broadening of the magnon lines, which come from the fluctuations in the anisotropy.

The results displayed in Figs. 1–6 can be understood with reference to Eqs. (3.1) and (3.8). In Figs. 1–3 it is assumed that $P(\cos\theta)$ is constant. As a consequence the first term in (3.1) vanishes and $H_a^c(E)$ is given by (3.2) for $2DS/E_M \ll 1$. Thus both $\mathrm{Re}H_a^c$ and $\mathrm{Im}H_a^c$ vary quadratically for small D. The decrease in $\tilde{E}_0$ with increasing D shown in Fig. 1 is a result of $\mathrm{Re}\langle G_0\rangle(E)$ being negative for $E<\frac{1}{2}$. The near-semicircular shape for Γ shown in

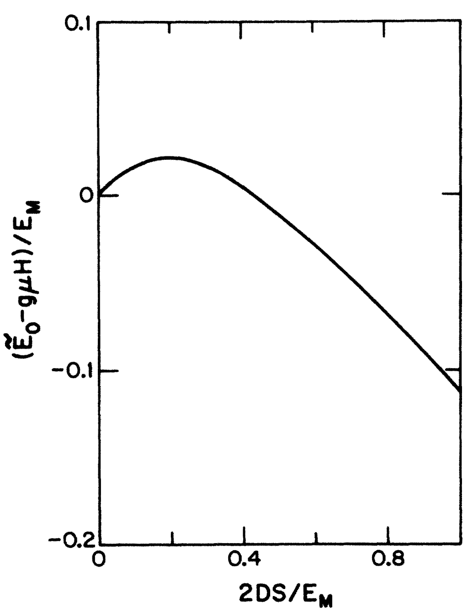

FIG. 4. Shift in energy of the uniform ($\mathbf{k}=0$) mode vs $2DS/E_M$. Energy in units of E_M. Linear probability distribution, Eq. (3.14).

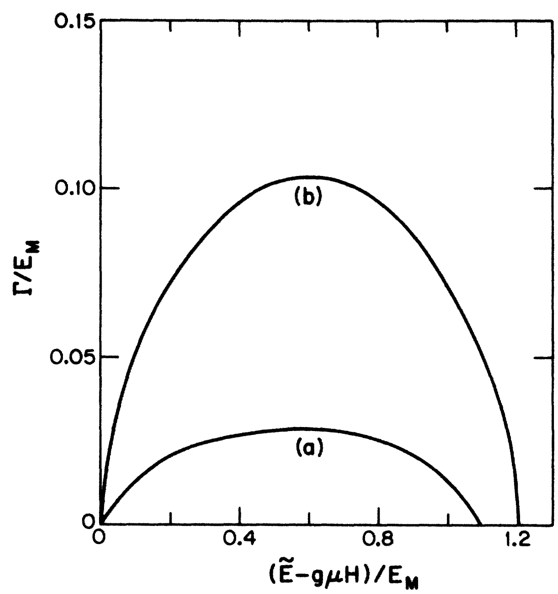

FIG. 6. Magnon linewidth vs $(\tilde{E}-g\mu H)/E_M$. (a) $2DS/E_M=0.2$; (b) $2DS/E_M=0.4$. Linewidth in units of E_M. Linear probability distribution, Eq. (3.14).

Fig. 3 reflects the fact that for small D the damping is determined by $\mathrm{Im}\langle G_0\rangle(E)$ with E real, which is the same as the density of unperturbed magnon modes apart from a factor of π [cf. Eq. (3.5)]. With increasing D the semicircular form becomes more and more distorted.

In contrast, in Figs. 4–6, the first term in (3.1) does not vanish. The shift in the energy of the uniform mode varies linearly with D for small D, i.e., $g\mu H_a^c \approx DS/2$, with the rolloff at larger D coming from the quadratic term. The similarity in the behavior of Γ shown in Figs. 2 and 3 and Figs. 5 and 6 reflects the fact that in both cases it is $\mathrm{Im}\langle G_0\rangle$ which determines the damping for small D. It is somewhat surprising, however, that the damping obtained with the linear distribution (Fig. 3) is symmetric about the point $E-g\mu H=E_M/2+DS/2$, whereas the damping calculated with the linear distribution is asymmetric.

We also note that the shift in the energy of the $\mathbf{k}=0$ mode for a uniform probability distribution,

$$\tilde{E}_0-g\mu H \approx -\tfrac{16}{5}D^2S^2/E_M \ ,$$

agrees in order of magnitude but has the opposite sign from the result obtained by Saslow[4,6] for the wandering axis regime

$$\tilde{E}_0-g\mu H \approx g\mu H_a^2/H_{ex} \ . \tag{4.1}$$

It is also worth pointing out that the shift in the energy of the uniform mode is the analog in the shift in the band edge in the random-alloy problem.

We mentioned earlier that even though the CAFA was derived assuming a periodic array of spins, we expect our findings to have applicability to amorphous systems for energies near the bottom of the magnon band where in the absence of anisotropy the magnon energy varies as k^2, the linewidth as k^4 (according to hydrodynamic theory), and $\rho(E)\sim E^{1/2}$. In the limit of weak anisotropy we can obtain general expressions for two quantities of direct physical interest. When $2DS/E_M \ll 1$ we can identify the shift in the energy of the uniform mode with $g\mu H_a^c(0)$. We have, from (3.1) and (3.3),

$$\begin{aligned} g\mu H_a^c(0) = {} & 3DS\int_{-1}^{1} d(\cos\theta)P(\cos\theta)(\cos^2\theta-\tfrac{1}{3}) \\ & -(3DS)^2\int_0^\infty du\,\rho(u)u^{-1}\int_{-1}^{1} d(\cos\theta)P(\cos\theta)[(\cos^2\theta-\tfrac{1}{3})-\langle\cos^2\theta-\tfrac{1}{3}\rangle]^2 \ , \end{aligned} \tag{4.2}$$

whereas to order D^2 the linewidth is given by

$$\Gamma(E)=(3DS)^2\pi\rho(E)\int_{-1}^{1} d(\cos\theta)P(\cos\theta)[(\cos^2\theta-\tfrac{1}{3})-\langle\cos^2\theta-\tfrac{1}{3}\rangle]^2 \ . \tag{4.3}$$

In both cases $\rho(E)$ is interpreted as the density of states in the amorphous material in the absence of anisotropy. Equation (4.3) gives a picture of the damping across the magnon band. However, since $\rho(0)$ is zero, it cannot be used to predict the linewidth of the uniform mode. This can only be obtained from (3.16) and (3.17).

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