Tunneling through a spin-polarizing barrier: Boltzmann-equation study

M. J. DeWeert* and S. M. Girvin†
Surface Science Division, National Bureau of Standards, Gaithersburg, Maryland 20899
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We investigate the nonequilibrium distribution functions for electrons in the electrodes of a metal-insulator-metal junction where the insulator is a ferromagnet with its domains aligned parallel to the interfaces. In this geometry, the tunneling barrier for spin-up electrons differs from that for spin-down electrons, so that the two spins tunnel at different rates. If the junction is biased so that the Fermi levels on the right and left are shifted to $(\mu+eV/2)$ and $(\mu-eV/2)$, respectively, we find that the electrons are spin polarized *in the steady state.* Electrons in the right electrode are polarized in the direction opposite those on the left. The spin polarization increases with voltage and tunneling conductance and decreases as the spin relaxation time falls.

I. INTRODUCTION

There has recently been renewed interest[1] in metal-insulator-metal tunneling in the case where the insulator is a ferromagnet. If the barrier is thin enough, its magnetization **M** is parallel to the interfaces, so that it produces a negligibly small field in the tunneling electrodes. Nevertheless, the tunneling barrier will have different heights for spin-up and spin-down electrons (quantization axis in the direction of **M**), as shown in metal-insulator-vacuum tunneling experiments,[2] where the tunneled electrons are up to 85% spin polarized. This is due primarily to the exchange interaction $J\mathbf{S}\cdot\mathbf{s}$ in the barrier, which can produce differences in barrier height on the order of a few tenths of an electron volt for the two spins.[3] In a Wentzl-Kramers-Brillouin (WKB) approximation, this difference then appears in exponential factors in the tunneling matrix elements.[4]

Our hypothesis is that the polarization of the tunneled electrons leads to a steady-state polarization of the electrons in the electrodes. This polarization should be limited by the amount of spin-flip scattering in the bulk of the electrodes. We demonstrate this for normal metals with a Boltzmann equation calculation in the limit in which the electrodes are thin enough that the distribution functions throughout each are homogeneous. If the electrodes are not so thin, we expect our results to hold within some homogenization depth determined by the properties of the electrodes and the amount of scattering. Recent work by van Son *et al.*[5] on ferromagnet-metal interfaces shows this depth to be approximated by $\Lambda=(v_F^2\tau_0\tau_{SF}/3)^{1/2}$, where v_F is the Fermi velocity. τ_0 is the scattering lifetime, and τ_{SF} is the spin-relaxation time. If we take $v_f=10^{16}$ Å/s, $\tau_0=10^{-14}$ s, and $\tau_{SF}=10^{-10}$ s, then $\Lambda\approx 6000$ Å. Electrodes much thinner than this can easily be fabricated, so our approximation is reasonable.

II. SOLUTION

We take the energy to depend only on the magnitude of the momentum and the electric potential in each electrode

$$\xi_{R\sigma}(k)=(\hbar^2k^2/2m)-\mu-eV/2\ , \tag{1a}$$

$$\xi_{L\sigma}(k)=(\hbar^2k^2/2m)-\mu+eV/2\ . \tag{1b}$$

The chemical potentials μ relative to the band bottom of the two electrodes are taken to be equal. In equilibrium, the distribution functions $g_{R,L}(E)$ on the two sides (R for right, L for left) would be independent of spin, and would reduce to Fermi functions $g^0_{R,L}(E)$ with the energies given in (1).

When tunneling is allowed, the system is out of equilibrium unless the voltage is zero. The tunneling and scattering processes can be cast into scattering and tunneling lifetimes by assuming that g is independent of position within a given electrode and using Fermi's golden rule, so that only states of equal energy are connected. This allows us to write Boltzmann equations for the tunnel junction[6]

$$\dot{g}_{\sigma R}(E,t)=-(1/\tau_{SR})(g_{\sigma R}-g_{-\sigma R})-(1/\tau_{TR\sigma})(g_{\sigma R}-g_{\sigma L})\ , \tag{2a}$$

$$\dot{g}_{\sigma L}(E,t)=-(1/\tau_{SL})(g_{\sigma L}-g_{-\sigma L})-(1/\tau_{TL\sigma})(g_{\sigma L}-g_{\sigma R})\ , \tag{2b}$$

where τ_{SR} and τ_{SL} are the spin-relaxation lifetimes in the right and left electrodes, and $\tau_{TR\sigma}$ and $\tau_{TL\sigma}$ are tunneling lifetimes for tunneling from right to left and left to right, respectively, for an electron of spin σ. The lifetimes $\tau_{TR\sigma}$ and $\tau_{TL\sigma}$ are not necessarily the same. If the electrodes are the same material, these lifetimes still depend on the thickness d, being longer for thicker electrodes so that

$$\tau_{TL\sigma}/\tau_{TR\sigma}=d_L/d_R\ . \tag{3}$$

This can be seen from conservation of current. If the tunneling current is thought of as coming from throughout the volume of one electrode and going to the entire volume of the other, then the density of tunneling events per volume must be lower for the thicker elec-

trode, so that, on average, electrons reside longer in a thicker film. Though Eqs. (2a), (2b), and (3) have fairly obvious forms, we have included a justification of them in Appendix A.

We have assumed that tunneling preserves the direction of spin, which will hold true for a ferromagnetic insulator far below its Curie temperature, since the spin directions in the insulator are held rigid. It should also be approximately true for a paramagnetic insulator in a high field if the net spin S of its atoms is much greater than that of an electron. Note that there is no non-spin-flip scattering term because we have assumed the distribution functions to be constant across each metal's thickness.

The four equations (2) can be expressed compactly in the form of a 4×4 matrix. Since we are really interested in number and current densities, we assume the τ's are independent of $E(k)$ and integrate over k vectors to obtain

$$\begin{bmatrix}\dot{n}_{R\uparrow}\\ \dot{n}_{R\downarrow}\\ \dot{n}_{L\uparrow}\\ \dot{n}_{L\downarrow}\end{bmatrix}=\begin{bmatrix}-(a_R+b_{R\uparrow}) & a_R & b_{R\uparrow} & 0\\ a_R & -(a_R+b_{R\downarrow}) & 0 & b_{R\downarrow}\\ b_{L\uparrow} & 0 & -(a_L+b_{L\uparrow}) & a_L\\ 0 & b_{L\downarrow} & a_L & -(a_L+b_{L\downarrow})\end{bmatrix}\begin{bmatrix}n_{R\uparrow}\\ n_{R\downarrow}\\ n_{L\uparrow}\\ n_{L\downarrow}\end{bmatrix}, \tag{4}$$

where

$$a_R=1/\tau_{SR}\ , \tag{5a}$$

$$a_L=1/\tau_{SL}\ , \tag{5b}$$

$$b_{R\sigma}=1/\tau_{TR\sigma}\ , \tag{5c}$$

$$b_{L\sigma}=1/\tau_{TL\sigma}\ . \tag{5d}$$

We will use the ratios

$$r_{R\sigma}=b_{R\sigma}/a_R\ , \tag{6a}$$

and

$$r_{L\sigma}=b_{L\sigma}/a_L\ . \tag{6b}$$

In the steady state, the n's *and* their time derivatives are taken to be constant in time. This is a familiar approach, used, for example, in a simplified derivation of the Josephson effect.[7] It is a way of taking into account the fact that there exist reservoirs of particles in contact with each electrode which are sufficient to keep their electron distributions constant despite the steady transfer of particles from one electrode to the other, and we do not concern ourselves with the exact mechanisms by which the contact is effected.

Thus, Eq. (4) really represents a system of four equations with eight unknowns, and we must impose constraints. The first constraint is

$$\dot{n}_{R\uparrow}=\dot{n}_{R\downarrow}\equiv\dot{n}_R\ , \tag{7a}$$

$$\dot{n}_{L\uparrow}=\dot{n}_{L\downarrow}\equiv\dot{n}_L\ . \tag{7b}$$

Equations (7a) and (7b) constrain the electrodes to have spin polarizations which are constant in time. The second constraint is that the total (spin up plus spin down) density of electrons in each metal is the same as if the up and down tunneling rates were the same:

$$n_{\uparrow R}+n_{\downarrow R}=2n_{0R}\ , \tag{8a}$$

$$n_{\uparrow L}+n_{\downarrow L}=2n_{0L}\ . \tag{8b}$$

That is, the effect of the difference in tunneling rates is to redistribute the density between up and down without changing the total density.

Within the free-electron approximation at low temperature, the n_0's are

$$n_{0R(L)\sigma}=n_0[1+(-)eV/2\mu]^{3/2}\approx n_0[1+(-)3eV/4\mu]\ , \tag{9}$$

$$\delta n_0\equiv(n_{0R\uparrow}-n_{0L\uparrow})=(n_{0R\downarrow}-n_{0L\downarrow})\approx(3n_0/2\mu)eV\ . \tag{10}$$

We now can combine Eqs. (4), (7), and (8) into the final set

$$\begin{bmatrix}-2a_Rn_{0R}\\ 2a_Rn_{0R}+2b_{R\downarrow}\delta n_0\\ -2a_Ln_{0L}\\ 2a_Ln_{0L}-2b_{L\downarrow}\delta n_0\end{bmatrix}=\begin{bmatrix}-(2a_R+b_{R\uparrow}) & -1 & b_{R\uparrow} & 0\\ (2a_R+b_{R\downarrow}) & -1 & -b_{R\downarrow} & 0\\ b_{L\uparrow} & 0 & -(2a_L+b_{L\uparrow}) & -1\\ -b_{L\downarrow} & 0 & (2a_L+b_{L\downarrow}) & -1\end{bmatrix}\begin{bmatrix}n_{R\uparrow}\\ \dot{n}_R\\ n_{L\uparrow}\\ \dot{n}_L\end{bmatrix}. \tag{11}$$

The solution of this is straightforward, and together with (3), yields

$$n_{R\uparrow,\downarrow}=n_{OR}\pm\delta n_R\ , \tag{12a}$$

$$n_{L\uparrow,\downarrow}=n_{0L}\pm\delta n_L\ , \tag{12b}$$

$$\delta n_R=\delta n_0\left[\frac{r_{R\uparrow}-r_{R\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right] \tag{13a}$$

$$\delta n_L=-\delta n_0\left[\frac{r_{L\uparrow}-r_{L\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right], \tag{13b}$$

$$2\dot{n}_R = -2\dot{n}_L = -\delta n_0(1/\tau_{TR\uparrow}+1/\tau_{TR\downarrow}) - (1/\tau_{TR\uparrow}-1/\tau_{TR\downarrow})(\delta n_R - \delta n_L) , \tag{14}$$

In Eq. (14), the first term represents the current density which would flow if the up and down tunneling rates were the same, and the second term is a correction. Equations (12) for the electronic densities show a shift of the spin-up and spin-down densities in opposite directions away from their equilibrium values. This shift depends on the voltage through δn_0 and on the ratios of spin-relaxation lifetimes to tunneling lifetimes. If we assume that the system is close enough to equilibrium that the densities g can be modeled as Fermi functions with different chemical potentials $\mu_{R,L\sigma}$ for the two spin populations, then the chemical potential shifts

$$2\nu_{R,L} = (\mu_{R,L\uparrow} - \mu_{R,L\downarrow})$$

can be inferred from Eq. (12) within the approximation of Eq. (9):

$$n_{R\sigma} \approx n_0(1+3eV/4\mu+3\nu_{R\sigma}/2\mu) , \tag{15a}$$

$$n_{L\sigma} \approx n_0(1-3eV/4\mu+3\nu_{L\sigma}/2\mu) , \tag{15b}$$

$$\therefore \nu_{R\uparrow} = -\nu_{R\downarrow} \equiv \nu_R \approx eV\left[\frac{r_{R\uparrow}-r_{R\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right] , \tag{16a}$$

$$\nu_{L\uparrow} = -\nu_{L\downarrow} \equiv \nu_L \approx -eV\left[\frac{r_{L\uparrow}-r_{L\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right] , \tag{16b}$$

and

$$\nu_L/\nu_R = -d_R/d_L . \tag{17}$$

The maximum magnitude of the splitting occurs when one of the r's, say $r_{L\uparrow}$, is much greater than both unity and the other r's. Then $\nu_L \approx -$eV, and $\nu_R \approx 0$.

III. DISCUSSION

The effect of the different tunneling rates for the two spins resembles the Zeeman splitting which would occur in an applied magnetic field. The major differences are that the splitting has opposite signs and possibly different magnitudes on the two sides of the tunneling barrier. The field H which would yield an equivalent splitting can be estimated from

$$\nu = \mu_B H = (5.8\times10^{-3}\ \text{meV/kG})H . \tag{18}$$

In Appendix B, we estimate tunneling parameters. If we have $r_{L\uparrow} \approx r_{R\uparrow} \approx 0.15$, $r_{L\downarrow} r_{R\downarrow} \approx 0.1$, and $V = 10$ mV, we get $\nu_R \approx -\nu_L \approx 0.12$ meV. This would correspond to a field $H \approx 20$ kG, much greater than, for example, the 1.1-kG magnetization of an EuS insulator. In an experiment, this could mimic the effect of the barrier magnetization leaking into the electrodes.

IV. CASE OF FINITE APPLIED FIELD

Since we are interested in Zeeman-type splittings, we take the films to be thin enough that the effect of the **A** field on the canonical momentum can be neglected. Then the effect of an external field **H** is to shift the energies to

$$\xi_{R\sigma}(k) = (\hbar^2k^2/2m) - \mu - eV/2 + \mu_B\sigma\cdot\mathbf{H} , \tag{19a}$$

$$\xi_{L\sigma}(k) = (\hbar^2k^2/2m) - \mu + eV/2 + \mu_B\sigma\cdot\mathbf{H} , \tag{19b}$$

so that Eq. (9) becomes

$$n_{0R(L)\sigma} \approx n_0[1+3\mu_B\sigma H/\mu + (-)3eV/4\mu] \tag{20}$$

and δn_0 is unchanged. To account for the fact that, in the absence of tunneling, the g's would relax to Fermi functions with the energies given by (19), we make a relaxation-time approximation for the *polarizations* in each electrode

$$\dot{g}_{\sigma R}(E,t) = -(1/\tau_{SR})[(g_{\sigma R}-g_{-\sigma R})-(g^0_{\sigma R}-g^0_{-\sigma R})] - (1/\tau_{TR\sigma})(g_{\sigma R}-g_{\sigma L}) , \tag{21a}$$

$$\dot{g}_{\sigma L}(E,t) = -(1/\tau_{SL})[(g_{\sigma L}-g_{-\sigma L})-(g^0_{\sigma L}-g^0_{-\sigma L})] - (1/\tau_{TL\sigma})(g_{\sigma L}-g_{\sigma R}) . \tag{21b}$$

Constraint (8) is unchanged, but instead of (7) we assume that reservoirs supply the equilibrium polarization to the electrodes

$$\dot{n}_{R\uparrow} - \dot{n}_{R\downarrow} = p_0(\dot{n}_{R\uparrow}+\dot{n}_{R\downarrow}) , \tag{22a}$$

$$\dot{n}_{L\uparrow} - \dot{n}_{L\downarrow} = p_0(\dot{n}_{L\uparrow}+\dot{n}_{L\downarrow}) , \tag{22b}$$

where

$$p_0 = (n_{0\uparrow}-n_{0\downarrow})/(n_{0\uparrow}+n_{0\downarrow}) . \tag{23}$$

Defining

$$q = (1-p_0)/(1+p_0) , \tag{24}$$

(11) becomes

$$\begin{bmatrix} -2a_R n_{0R\uparrow} \\ 2a_R n_{0R\uparrow}+2b_{R\downarrow}\delta n_0 \\ -2a_L n_{0L\uparrow} \\ 2a_L n_{0L\uparrow}-2b_{L\downarrow}\delta n_0 \end{bmatrix} = \begin{bmatrix} -(2a_R+b_{R\uparrow}) & -1 & b_{R\uparrow} & 0 \\ (2a_R+b_{R\downarrow}) & -q & -b_{R\downarrow} & 0 \\ b_{L\uparrow} & 0 & -(2a_L+b_{L\uparrow}) & -1 \\ -b_{L\downarrow} & 0 & (2a_L+b_{L\downarrow}) & -q \end{bmatrix} \begin{bmatrix} n_{R\uparrow} \\ \dot{n}_R \\ n_{L\uparrow} \\ \dot{n}_L \end{bmatrix} . \tag{25}$$

The solution of this is again straightforward, yielding

$$v_{R\uparrow}=-v_{R\downarrow}\equiv v_R\approx\mu_B H+2eV\left[\frac{qr_{R\uparrow}-r_{R\downarrow}}{2(1+q)+q(r_{L\uparrow}+r_{R\uparrow})+r_{L\downarrow}+r_{R\downarrow}}\right] \tag{26a}$$

$$v_{L\uparrow}=-v_{L\downarrow}\equiv v_L\approx\mu_B H-2eV\left[\frac{qr_{L\uparrow}-r_{L\downarrow}}{2(1+q)+q(r_{L\uparrow}+r_{R\uparrow})+r_{L\downarrow}+r_{R\downarrow}}\right] . \tag{26b}$$

The main effect of the different tunneling rates is to add a correction to the Zeeman splitting. Since q is close to unity for all practical fields, this correction is not much different from Eqs. (16a) and (16b). It enhances the Zeeman effect on one side of the junction and suppresses it on the other. The discussion in Sec. III shows that, as the voltage increases, the correction becomes *larger* than the equilibrium splitting.

V. CONCLUSIONS

We have constructed a Boltzmann-equation model of single-electron tunneling through a ferromagnetic insulator which predicts that spin-polarized tunneling leads to a steady-state spin polarization of the electrons in the metals of a metal-insulator-metal tunnel junction. This polarization increases with voltage. At an applied voltage of 1 mV, this effect causes a splitting equal to that produced by a 1-kG field when the tunneling rate is on the order of a few percent of the spin-relaxation rate in the electrodes. Because the tunneling lifetimes increase with the thickness of the electrode, the splitting decreases with electrode thickness, or with the homogenization depth, if it is less than the thickness.

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APPENDIX A: DERIVATION OF BOLTZMANN EQUATION

The purpose of this section is to justify Eqs. (2a), (2b), and (3). Our starting point is the semiclassical Boltzmann equation for the distribution function for electrons of spin σ and momentum $\mathbf{k}$ in each electrode[8]

$$\dot{g}_{\sigma R}(\mathbf{r},\mathbf{k},t)=-\mathbf{v}\cdot\nabla g_{\sigma R}-\mathbf{F}\cdot\frac{1}{\hbar}\frac{\partial g_{\sigma P}}{\partial\mathbf{k}}+\left[\frac{\partial g_{\sigma R}}{\partial t}\right]_{\text{scat}}+\left[\frac{\partial g_{\sigma R}}{\partial t}\right]_{\text{tunn}} . \tag{A1}$$

The subscript R denotes the right electrode. An analogous equation applies in the left electrode. Our first approximation is to assume that $g_{\sigma R,L}$ is independent of position within a given electrode. Then the gradient term in (A1) vanishes. We also assume that negligible electric field can be supported by the electrodes, so $F\cdot\partial/\partial\mathbf{k}$ also vanishes, leaving only the scattering and tunneling terms:

$$[\dot{g}_{\sigma R}(\mathbf{r},\mathbf{k},t)]_{\text{scat}}=-\int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}\{W_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')g_{\sigma R}(\mathbf{r},\mathbf{k},t)[1-g_{\sigma' R}(\mathbf{r}',\mathbf{k}',t)] - W_{\sigma'\sigma}(\mathbf{r}',\mathbf{k}',\mathbf{r},\mathbf{k})g_{\sigma R}(\mathbf{r}',\mathbf{k}',t)[1-g_{\sigma R}(\mathbf{r},\mathbf{k},t)]\} , \tag{A2a}$$

$$[g_{\sigma R}(\mathbf{r},\mathbf{k},t)]_{\text{tunn}}=-\int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}\{T_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')g_{\sigma R}(\mathbf{r},\mathbf{k},t)[1-g_{\sigma' L}(\mathbf{r}',\mathbf{k}',t)] - T_{\sigma'\sigma}(\mathbf{r}',\mathbf{k}',\mathbf{r},\mathbf{k})g_{\sigma L}(\mathbf{r}',\mathbf{k}',t)[1-g_{\sigma R}(\mathbf{r},\mathbf{k},t)]\} . \tag{A2b}$$

If we assume that the scattering and tunneling matrices are invariant when the indices $(\mathbf{r},\mathbf{k},\sigma)$ and $(\mathbf{r}',\mathbf{k}',\sigma')$ are interchanged, then

$$\dot{g}_{\sigma R}(\mathbf{r},\mathbf{k},t)=-\int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}W_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma R}(\mathbf{r},\mathbf{k},t)-g_{\sigma' R}(\mathbf{r}',\mathbf{k}',t)] - \int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}T_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma R}(\mathbf{r},\mathbf{k},t)-g_{\sigma' L}(\mathbf{r}',\mathbf{k}',t)] , \tag{A3a}$$

$$\dot{g}_{\sigma L}(\mathbf{r},\mathbf{k},t)=-\int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}W_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma L}(\mathbf{r},\mathbf{k},t)-g_{\sigma' L}(\mathbf{r}',\mathbf{k}',t)] - \int d\mathbf{r}'\int d\mathbf{k}'\sum_{\sigma'}T_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma L}(\mathbf{r},\mathbf{k},t)-g_{\sigma' R}(\mathbf{r}',\mathbf{k}',t)] . \tag{A3b}$$

We are now ready to make the final approximations. The scattering is taken to be local and independent of the momenta, so that

$$W_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')=\delta(\mathbf{r}-\mathbf{r}')W_{\sigma\sigma'} .$$

The distributions g are assumed to depend on momentum only through the energies $E(\xi_{R,L}(\mathbf{k}),\sigma)$, where

$$\xi_{R,L}(\mathbf{k})=(\hbar^2k^2/2m-\mu_{R,L}) .$$

By neglecting the dependence on the direction of $\mathbf{k}$, we are assuming that scattering very rapidly redistributes any nonisotropic distribution of $\mathbf{k}$ directions. Finally, Fermi's golden rule allows only states of equal E to be connected by scattering or tunneling events. Thus, the scattering term becomes

$$\int d\mathbf{r}' \int d\mathbf{k}' \sum_{\sigma'} W_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma R}(\mathbf{r},\mathbf{k},t)-g_{\sigma' R}(\mathbf{r}',\mathbf{k}',t)] \approx N_0 W_{\sigma\sigma'}[g_{\sigma R}(E)-g_{\sigma' R}(E)] , \tag{A4}$$

where N_0 is the density of states at the Fermi surface. If $\sigma=\sigma'$, (A4) vanishes. Thus, only the spin-flip scattering remains, and we characterize it with a lifetime

$$\tau_{SR,L}=1/(N_0 W_{\sigma\sigma'})_{R,L} .$$

The tunneling is somewhat different from the scattering in that is manifestly nonlocal. In fact, we will take it to be so nonlocal that $T(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')$ will be independent of the coordinates as well as the momenta. Assuming T to be independent of $\mathbf{k}$ and $\mathbf{k}'$ requires that the temperature and the voltage across the insulator be very small compared to the barrier height. Then the tunneling term reduces to

$$\int d\mathbf{r}' \int d\mathbf{k}' \sum_{\sigma'} T_{\sigma\sigma'}(\mathbf{r},\mathbf{k},\mathbf{r}',\mathbf{k}')[g_{\sigma R}(\mathbf{r},\mathbf{k},t)-g_{\sigma' L}(\mathbf{r}',\mathbf{k}',t)] \approx V_L N_0 \sum_{\sigma'} T_{\sigma\sigma'}[g_{\sigma R}(E)-g_{\sigma' L}(E)] , \tag{A5}$$

where $V_{L(R)}=Ad_{L(R)}$ is the volume of the left (right) electrode, and A is its area. If tunneling preserves spin, then $T_{\sigma\sigma'}=T\delta_{\sigma\sigma'}$. We define the tunneling lifetime from right to left (left to right) for spin σ as

$$\tau_{TR(L)\sigma}=1/(d_{L(R)}AN_0T) .$$

Thus, we arrive at the desired result

$$\dot{g}_{\sigma R}(E,t)=-(1/\tau_{SR})(g_{\sigma R}-g_{-\sigma R}) -(1/\tau_{TR\sigma})(g_{\sigma R}-g_{\sigma L}) , \tag{A6a}$$

$$\dot{g}_{\sigma L}(E,t)=-(1/\tau_{SL})(g_{\sigma L}-g_{-\sigma L}) -(1/\tau_{TL\sigma})(g_{\sigma L}-g_{\sigma R}) , \tag{A6b}$$

$$\tau_{TL\sigma}/\tau_{TR\sigma}=d_L/d_R . \tag{A7}$$

In summary, the approximations we are using are the semiclassical Boltzmann equation and Fermi's golden rule, and the assumptions required to obtain the (standard) forms (2a) and (2b) are: (1) g depends only on $|\mathbf{k}|$, not on direction; (2) within a given electrode, g is independent of position; (3) the scattering and tunneling matrix elements are symmetric in $(\mathbf{k},\mathbf{r},\sigma)$ and $(\mathbf{k}',\mathbf{r}',\sigma')$; (4) tunneling preserves spin direction; (5) tunneling electrons can be considered as coming from and going to the entire volume of each electrode; and (6) eV and k_BT are much less than the tunneling barrier. The disappearance of the non-spin-flip scattering term in (A3) does *not* mean that non-spin-flip scattering is unimportant. In fact, scattering justifies assumptions (1), (2), and (5). Put another way, we have assumed non-spin-flip scattering to be so important that these assumptions completely describe its effects.

APPENDIX B: ESTIMATES OF PARAMETERS

For the spin polarization to be significant, two conflicting conditions must be met: the tunneling resistances must be small enough for $\tau_{TR,L\sigma}$ to be not much longer than τ_{SF}, and the resistances $R_\uparrow$ and $R_\downarrow$ must be large enough to differ significantly. Of course, raising the voltage will increase the polarization, according to Eq. (16), but high voltages invalidate our assumption that tunneling lifetimes are independent of electron energy.

The tunneling lifetimes $\tau_{TR\sigma}$ and $\tau_{TL\sigma}$ can be estimated in terms of the tunneling resistance for each spin and the thickness of the metals. Separating (14) into spin-up and spin-down currents, we have

$$I_\uparrow=-V(3n_0e^2V_R/4\mu\tau_{TR\uparrow}) \times\left[1+\left[\frac{r_{R\uparrow}+r_{R\downarrow}-r_{L\uparrow}+r_{L\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right]\right] , \tag{B1a}$$

$$I_\downarrow=-V(3n_0e^2V_R/4\mu\tau_{TR\downarrow}) \times\left[1+\left[\frac{r_{R\uparrow}+r_{R\downarrow}-r_{L\uparrow}-r_{L\downarrow}}{4+r_{L\uparrow}+r_{L\downarrow}+r_{R\uparrow}+r_{R\downarrow}}\right]\right] , \tag{B1b}$$

where $V_R=d_RA$ is the volume of the right electrode and A is the area. If we take the r's to be much less than one, we have the resistances R_σ:

$$R_\sigma\approx(4\mu\epsilon_0/3n_0e^2C_0td_R)\tau_{TR\sigma} , \tag{B2}$$

where the area has been eliminated in favor of a capacitance $C_0=(\epsilon_0A/t)$, where t is the thickness of the barrier and ϵ_0 is the vacuum dielectric constant. This is solely a convenience, since the actual dielectric constant appears nowhere in Eqs. (B1a) or (B1b), allowing us to eliminate conversions from mks units to electrostatic units. For $\mu=10$ eV, $n_0=0.2$ Å^{-3}, and $d_R=50$ Å, we have

$$\tau_{R\sigma}\approx 0.69\ R_\sigma C_0[t/(1\text{ Å})] .$$

Conduction-electron spin-resonance data for aluminum[9] yields an intrinsic spin-relaxation time $\approx 10^{-10}$ s, and more recent data from superconducting tunneling[10] gives

10^{-9} s. To get $\tau_{R\sigma} \approx 10^{-9}$ s for $t = 5$ Å, $R_\sigma C_0$ must be $\approx 3 \times 10^{-10}$ s. For $R = 1$ KΩ, the area of the junction would then be $\approx 1.7 \times 10^{-7}$ cm^2.

We can also estimate the effect of the barrier-height difference on the tunneling lifetimes. Using a WKB approximation,[4] we have

$$R_\uparrow \propto \exp[(\phi_\uparrow / E_0)^{1/2}] , \tag{B3a}$$

and

$$R_\downarrow \propto \exp[(\phi_\downarrow / E_0)^{1/2}] , \tag{B3b}$$

where φ_σ is the barrier height for spin σ and $E_0 = (\hbar^2 / 4mt^2)$. For a 5-Å barrier, $E_0 \approx 7.7 \times 10^{-2}$ eV. For EuS, $\phi_\uparrow \approx 1.32$ eV and $\phi_\downarrow \approx 1.08$ eV. This leads to $R_\uparrow / R_\downarrow = \tau_{TR\uparrow} / \tau_{TR\downarrow} \approx 1.5$.

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*Present address: Condensed Matter and Radiation Sciences Division, Code 4680, Naval Research Labs, Washington, D.C. 20375.

†Present address: Department of Physics, Swain Hall West 117, Indiana University, Bloomington, IN 47405.

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