Electric field dependence of intrasubband polar-optical-phonon scattering in a quantum well

D. Ahn and S. L. Chuang
Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 6 April 1987; revised manuscript received 3 August 1987)

The electric field dependences of intrasubband polar-optical-phonon scattering for electrons and holes in a semiconductor quantum well are studied theoretically using a simple infinite well model. It is found that the polar-optical-phonon scattering rates are enhanced with an applied electric field. They are higher for heavy holes for all ranges of electric fields, and the electric field dependence of the hole–polar-optical-phonon scattering is much stronger than that of the electron–polar-optical-phonon scattering. The higher subbands have, in general, weaker electron-phonon scattering rates and electric field dependences than those of the ground state. The tunneling effect in a finite well is also discussed. It is suggested that recent experimental results of the field-dependent line broadening of near band-edge optical absorption can be attributed, at least qualitatively, to the dominance of heavy-hole–polar-optical-phonon interaction and heavy-hole tunneling.

I. INTRODUCTION

There are many stimulating works on the semiconductor quantum-well structures because remarkable changes in electronic and optical properties are possible.[1,2] Since the polar-optical-phonon (POP) scattering is the major scattering mechanism in a polar semiconductor like GaAs over a considerable temperature range,[3] POP scattering in a two-dimensional structure has been the subject of study in some recent papers.[4–8] There has been discrepancy between some of the early calculations of POP scattering in two-dimensional layers[4,5] and the experimental results[9] which show a weakening of the electron-phonon interaction in a two-dimensional structure compared to that for a three-dimensional structure. The discrepancy is partially due to the crude approximations used in the early calculations, e.g., $q_z L \ll 1$ (strictly two-dimensional gas), where q_z is the transverse component of the phonon-wave vector and L is the quantum-well width.[4,5] Recently, Leburton[8] calculated the phonon scattering rate for two-dimensional electrons in the lowest subband of a quantum well without an applied electric field and showed that his results generally agree with the experimental data from Chiu *et al.*[9] and the model of Price.[7] In this paper we study the effects of an applied electric field perpendicular to the quantum well for both electrons and heavy holes in the ground level and the excited state. Previous calculations for POP scatterings show that the scattering rate is enhanced as the well width L is reduced.[6–8] When one applies an electric field perpendicular to the quantum well, electrons (or holes) are pushed to one side of the well, thus the effective well width is reduced.[10,11] This will result in the enhancement of the POP scattering rate. Our results show that the POP scattering rate is enhanced with increasing electric field. However, there are remarkable differences between the scattering rates for the electrons and heavy holes on the electric field dependence due to their different effective masses. Not only is the POP scattering rate dominated by heavy holes, but the electric field dependence of the POP scattering rate is also stronger for heavy holes. The former can be explained by Price's $\sqrt{m}$ law for the two-dimensional collision rate[7] and the latter can be accounted for by the electric field dependence of the wave-function variation. The wave-function variation of the heavy hole is more pronounced than that of the electron wave function at the same electric field.[10,11]

Let us consider an infinite quantum well with a width L [Fig. 1(a)]. When the quantum well has an applied electric field [Fig. 1(b)], it is found that (1) the energy level of the ground state lowers and the first excited state E_2 raises slightly, and (2) the wave functions are pushed to one side of the quantum well. Numerical results for the energy levels E_n in terms of the normalized energies $\tilde{E}_n$ and normalized electric field $\tilde{F}$ are shown in Fig. 2(a)

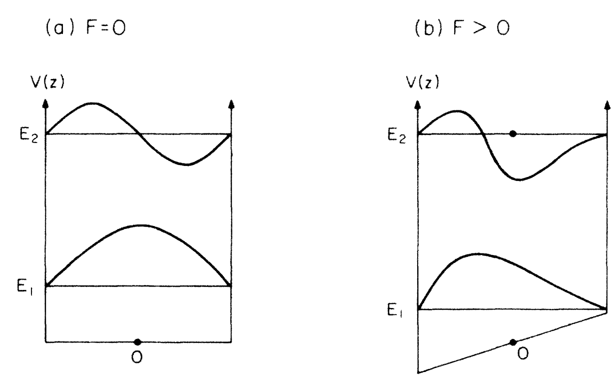

FIG. 1. Schematic diagrams for the energy levels and the wave functions of an infinite quantum well for (a) $F=0$ and (b) $F>0$.

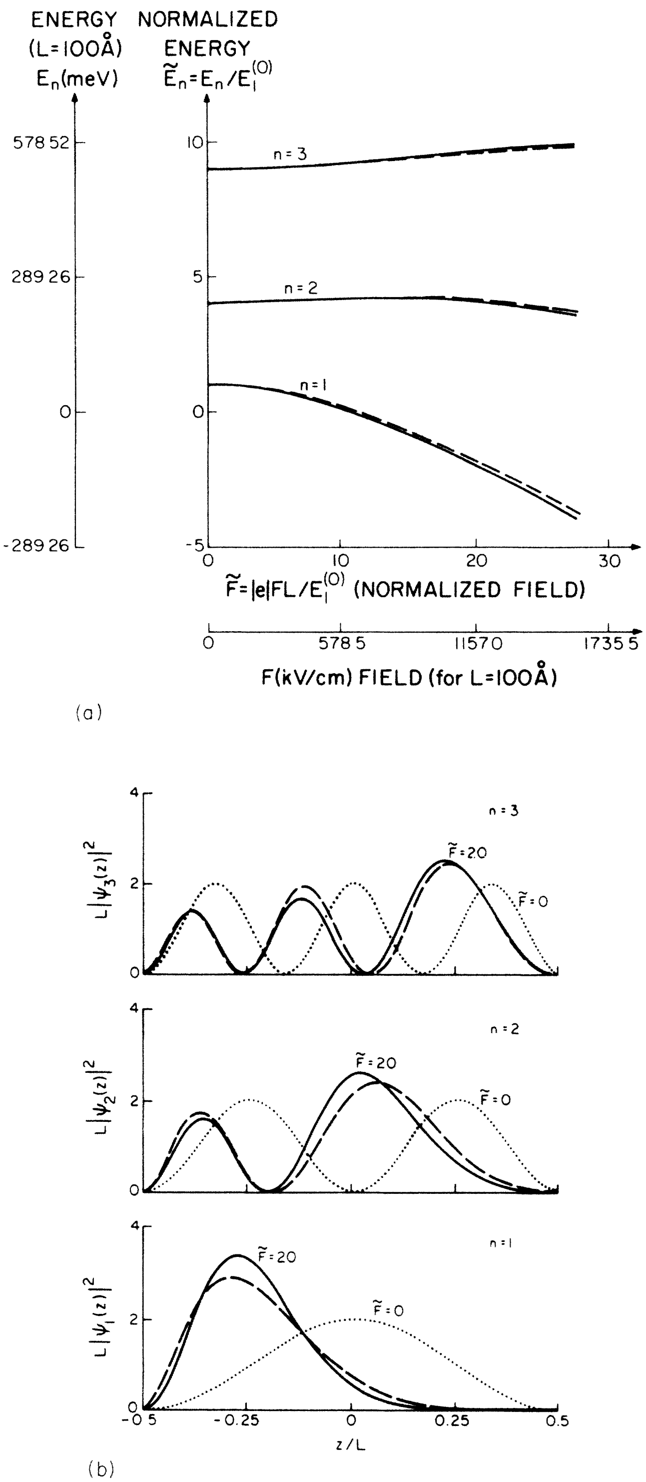

FIG. 2. (a) The normalized subband energies ($\tilde{E}_n=E_n/E_1^{(0)}$) are plotted vs the normalized electric field ($\tilde{F}=|e|FL/E_1^{(0)}$). (1) The exact solution (solid lines), (2) the variational method with the Gram-Schmidt orthogonalization procedure (dashed lines). (b) The magnitudes of the wave functions, $L|\psi_n(z)|^2$, are plotted vs the normalized distance z/L for the first three subbands. The dotted lines are for the zero electric field. The solid lines are the exact solutions and the dashed lines are those for the variational method with the Gram-Schmidt orthogonalization procedure for a quantuam well with an applied electric field ($\tilde{F}=20$) (from Ref. 11).

where $\tilde{E}_n=E_n/E_1^{(0)}$, $E_1^{(0)}=\hbar^2\pi^2/(2m^*L^2)$, and $\tilde{F}=|e|FL/E_1^{(0)}$ where m^* is the effective mass. The amplitudes of the wave functions are shown in Fig. 2(b). Since the effective mass of the heavy holes ($m^*=0.45m_0$, m_0 is the free-electron mass) is heavier than that of the electrons ($m^*=0.0665m_0$), $E_1^{(0)}$ for the heavy hole is lower than that of the electron. As a result, the normalized $\tilde{F}$ for a heavy hole is larger than that for an electron at the same electric field strength F. This causes the stronger electric field dependence of the POP scattering in a two-dimensional gas for heavy holes. Our calculations show that the POP scattering rate for the electron is enhanced by about 10% when the electric field F is 200 kV/cm for $L=126.5$ Å compared with that for the zero-field case, while the POP scattering rate for the heavy hole is almost doubled when $F=200$ kV/cm compared with the zero-field results. This, together with tunneling of the heavy hole, may explain qualitatively the recent experimental results where the exciton linewidth is almost doubled when $F=220$ kV/cm compared with those for the zero-field case.[12] We also found that, in general, the POP scattering rates and their electric field dependences for higher subbands are smaller than those for the ground state. Many-body effects, however, are not considered here and will be a subject of future study. The electric field dependence of the carrier-POP scattering should also be important in the calculation of the intersubband optical absorption[13] in a quantum well.

II. ELECTRIC FIELD DEPENDENCE OF POLAR-OPTICAL-PHONON SCATTERING OF TWO-DIMENSIONAL CARRIERS IN A QUANTUM WELL

The Hamiltonian of the system (a single quantum well) subject to a uniform electric field perpendicular to the quantum well (the z direction) with carrier–polar-optical-phonon interaction is written as

$$H=H_0+H' , \tag{1}$$

where H_0 is the unperturbed Hamiltonian for an electron (or hole) in the quantum well in the presence of a perpendicular electric field. For simplicity, we assume isotropic and parabolic bands, and consider only one type of hole (heavy hole).

The Fröhilich Hamiltonian H' in SI units is[14]

$$H'=\sum_{\mathbf{q}}\frac{C}{q}(a_{\mathbf{q}}e^{i\mathbf{q}\cdot\mathbf{r}}-a_{\mathbf{q}}^{\dagger}e^{-i\mathbf{q}\cdot\mathbf{r}}) , \tag{2}$$

where $a_{\mathbf{q}}^{\dagger}$ and $a_{\mathbf{q}}$ are the second quantized creation and annihilation operators of the phonon with the wave vector $\mathbf{q}$, respectively, and

$$C=i\left[\frac{e^2\hbar\omega_q}{2\epsilon_0 V}\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_s}\right]\right]^{1/2} . \tag{3}$$

Here, V is the volume of the system, ϵ_0 is the permittivity of free space, ϵ_∞ and ϵ_s are the optical and static dielectric constants, respectively, and ω_q is the phonon angular frequency, $\hbar\omega_q = 36.202$ meV. Here we consider the intrasubband transition only, i.e., the energy separation of subbands is sufficiently larger than $\hbar\omega_q$. If we neglect the interaction between the electrons (or holes) in the well, the wave functions for the initial state $\psi_i^\alpha(\mathbf{r})$ and the final state $\psi_f^\alpha(\mathbf{r})$ can be written as

$$\psi_i^\alpha(\mathbf{r}) = \frac{1}{\sqrt{A}} e^{i\mathbf{k}_t^\alpha \cdot \mathbf{r}_t} \Phi_n^\alpha(z), \quad |z| \le \frac{L}{2} \tag{4a}$$

$$\psi_f^\alpha(\mathbf{r}) = \frac{1}{\sqrt{A}} e^{i\mathbf{k}_t'^\alpha \cdot \mathbf{r}_t} \Phi_n^\alpha(z), \quad |z| \le \frac{L}{2}, \tag{4b}$$

where A is the cross-sectional area of the well, L is the width of the well, α denotes either electrons or holes, $\mathbf{k}_t^\alpha$ and $\mathbf{k}_t'^\alpha$ are the wave vectors of electrons (or holes) in the x-y plane for the initial and final states, respectively, and $\mathbf{r}_t$ is the position vector in the x-y plane. We assumed in Eqs. (4) that the perturbations vary slowly so that the cell periodic function part will be integrated to one in the matrix element to be evaluated later. Thus the cell periodic function is not included here. The envelope function $\Phi_n^\alpha(z)$ for the nth subband satisfies the following Schrödinger equation in the effective-mass approximation:

$$-\frac{\hbar^2}{2m_\alpha^*}\frac{d^2}{dz^2}\Phi_n^\alpha(z) - eFz\Phi_n^\alpha(z) = E_n^\alpha \Phi_n^\alpha(z), \tag{5}$$

where F is the electric field, the subscript n refers to the nth subband with the energy E_n^α, m_α^* is the effective mass for specie α. Note that $e = -|e|$ for electrons ($\alpha = e$) and $e = +|e|$ for holes ($\alpha = h$). The solutions of Eq. (7) are well known and are given by a linear combination of the two independent Airy functions $\mathrm{Ai}(\eta)$ and $\mathrm{Bi}(\eta)$:[13,15]

$$\Phi_n^\alpha(z) = a_n^\alpha \mathrm{Ai}(\eta_n^\alpha) + b_n^\alpha \mathrm{Bi}(\eta_n^\alpha), \quad |z| \le \frac{L}{2}, \tag{6}$$

with the boundary conditions

$$\Phi_n^\alpha\left[\pm\frac{L}{2}\right] = 0, \tag{7}$$

where η_n^α is defined by

$$\eta_n^\alpha = -\left[\frac{2m_\alpha^*}{(e\hbar F)^2}\right]^{1/3}(E_n^\alpha + eFz). \tag{8}$$

The two constants a_n^α and b_n^α and the energy E_n^α can be determined from Eqs. (8)–(10) and the normalization of the wave function. From now on, we will omit the superscript α for convenience.

The transition rate for POP scattering is expressed in the Born approximation by Fermi's "golden rule:"

$$W_{fi}^\pm = \frac{2\pi}{\hbar}|M_{fi}^\pm|^2 \delta(E_f - E_i \pm \hbar\omega_q) \tag{9}$$

where E_i and E_f are the total initial and final energies of the carrier, respectively, and W_{fi}^+ and W_{fi}^- denote the transition rate for phonon emission and absorption, respectively. The matrix elements $M_{fi}^\pm$ are written as

$$M_{fi}^+ = -\frac{C}{q}(n_q+1)^{1/2}\delta_{\mathbf{k}_t',\mathbf{k}_t-\mathbf{q}_t}\int_{-L/2}^{L/2} dz\, e^{-iq_z z}|\Phi_n(z)|^2 \tag{10}$$

and

$$M_{fi}^- = \frac{C}{q} n_q^{1/2}\delta_{\mathbf{k}_t',\mathbf{k}_t+\mathbf{q}_t}\int_{-L/2}^{L/2} dz\, e^{+iq_z z}|\Phi_n(z)|^2. \tag{11}$$

The initial and the final energies E_i and E_f are defined in the parabolic band approximation, respectively,

$$E_i = \frac{\hbar^2 k_t^2}{2m^*} + E_n \tag{12a}$$

and

$$E_f = \frac{\hbar^2 k_t'^2}{2m^*} + E_n. \tag{12b}$$

After some mathematical manipulation, we obtain the total transition rates $W_n^+(\mathbf{k}_t)$ and $W_n^-(\mathbf{k}_t)$ (see the Appendix):

$$W_n^+(\mathbf{k}_t) \equiv \frac{2\pi}{\hbar}\sum_{\mathbf{k}_t'}\sum_{\mathbf{q}}|M_{fi}^+|^2\delta(E_f - E_i + \hbar\omega_q)$$
$$= \omega_q(n_q+1)\alpha_{e\text{-ph}} I_n^+(E_t, F, L), \tag{13}$$

with

$$I_n^+(E_t,F,L) = \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' \int_0^{\phi_{\max}} d\phi\, |\Phi_n(z)|^2 |\Phi_n(z')|^2 \left[\frac{E_t}{\hbar\omega_q}\cos^2\phi - 1\right]^{-1/2}$$
$$\times \left[\exp\left\{-|z-z'|\left[\frac{2m^* E_t}{\hbar^2}\right]^{1/2}\left[\cos\phi - \left[\cos^2\phi - \frac{\hbar\omega_q}{E_t}\right]^{1/2}\right]\right\} + \exp\left\{-|z-z'|\left[\frac{2m^* E_t}{\hbar^2}\right]^{1/2}\left[\cos\phi + \left[\cos^2\phi - \frac{\hbar\omega_q}{E_t}\right]^{1/2}\right]\right\}\right], \tag{14}$$

and

$$W_n^-(k_t) \equiv \frac{2\pi}{\hbar} \sum_{\mathbf{k}_t'} \sum_{\mathbf{q}} |M_{fi}^-|^2 \delta(E_f - E_i - \hbar\omega_q)$$
$$= \omega_q n_q \alpha_{e\text{-ph}} I_n^-(E_t, F, L) , \qquad (15)$$

with

$$I_n^-(E_t,F,L) = \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' \int_0^{\pi} d\phi \, |\Phi_n(z)|^2 |\Phi_n(z')|^2 \left[\frac{E_t}{\hbar\omega_q} \cos^2\phi + 1 \right]^{-1/2}$$
$$\times \exp\left\{ |z-z'| \left[\frac{2m^* E_t}{\hbar^2} \right]^{1/2} \left[\cos\phi - \left[\cos^2\phi + \frac{\hbar\omega_q}{E_t} \right]^{1/2} \right] \right\} . \qquad (16)$$

Here $\alpha_{e\text{-ph}}$ is a dimensionless coupling constant defined as[16]

$$\alpha_{e\text{-ph}} = \frac{e^2}{4\pi\hbar\epsilon_0} \left[\frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_s} \right] \left[\frac{m^*}{2\hbar\omega_q} \right]^{1/2} , \qquad (17)$$

which gives Price's $(m^*)^{1/2}$ law[7] for the scattering rate when $q_z L_z \ll 1$. However, due to the size effect, $I_n^{\pm}$ also depends on the effective mass m^*, so the deviation from $(m^*)^{1/2}$ law for the scattering rate is expected. The numerical results of W_n^+ and W_n^- for several cases will be presented in the next section.

III. NUMERICAL RESULTS AND DISCUSSIONS

The total transition rates $W_n^+(E_t)$ and $W_n^-(E_t)$ of the nth subband with a two-dimensional wave vector $\mathbf{k}_t$ for phonon emission and absorption, respectively, have been calculated by numerically integrating I_n^+ and I_n^- in Eqs. (14) and (16) for the lowest two subbands with an applied electric field F. The parameters are as follows:

$$m^*(\text{electron}) = 0.0665 m_0, \quad m^*(\text{hole}) = 0.45 m_0 ,$$
$$T = 300 \text{ K}, \quad L = 126.5 \text{ Å}, \quad \epsilon_\infty = 10.92, \quad \epsilon_s = 12.90 ,$$
$$\hbar\omega_q = 0.036\,202 \text{ eV} ,$$

where m_0 is the free-electron mass. The phonon occupation number n_q is given by the Bose-Einstein distribution,

$$n_q = \{\exp[(\hbar\omega_q)/k_B T] - 1\}^{-1} , \qquad (18)$$

where k_B is the Boltzmann constant. We use an effective well width $L = 126.5$ Å chosen to give the same ground-state energy of a finite well with a width $L_f = 100$ Å and the barrier height $V_0 = 340$ meV in the GaAs-$Al_xGa_{1-x}As$ system.

In Fig. 3 we plot the scattering rates for the electrons and the heavy holes with an applied electric field (1) $F = 200$ kV/F (solid lines) and (2) $F = 0$ (dashed lines) for the first subband. Although noticeable, the effects of an applied electric field on electron-POP scattering rates are generally small (electron-POP scattering rates are increased by about 10% when $F = 200$ kV/cm with $L = 126.5$ Å compared with those for the zero-field case). However, as seen from the figure, the electric field changes the heavy-hole-POP scattering rates drastically by almost a factor of 2 when $F = 200$ kV/cm compared with those for the zero-field case. Higher scattering rates of the heavy hole for all electric fields can be attributed to its heavier effective mass than that of the electron. For a finite quantum well, however, tunneling of carriers is also important at high electric field regime compared with the zero-field case for which the polar-optical-phonon scattering is the dominant scattering mechanism for $T > 200$ K.[17,18] To compare the POP scattering rate with the tunneling rate, we have also calculated the tunneling rate at $F = 200$ kV/cm from the complex energy eigenvalues of quasibound state[19] for electron ($L = 100$ Å and potential well height $V_0 = 340$ meV) and heavy hole ($L = 100$ Å and $V_0 = 60$ meV). Tunneling rates for electron and hole in their ground states at $F = 200$ kV/cm are 5.37×10^9 sec^{-1} and 43.7×10^{12} sec^{-1}, respectively. Recently, a band offset ratio of 60:40 for $Al_xGa_{1-x}As$/GaAs (240 meV for electrons, 160 meV for holes) is more commonly accepted. In this case the tunneling rates for the electron and hole

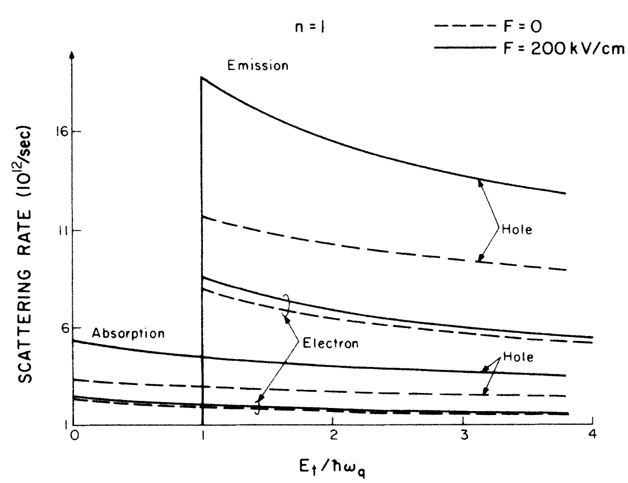

FIG. 3. Plots of the polar-optical-phonon scattering rates of electrons and holes for an applied electric field (1) $F = 200$ kV/cm (solid lines) and (2) $F = 0$ (dashed lines) for the ground state with a well width $L = 126.5$ Å at $T = 300$ K.

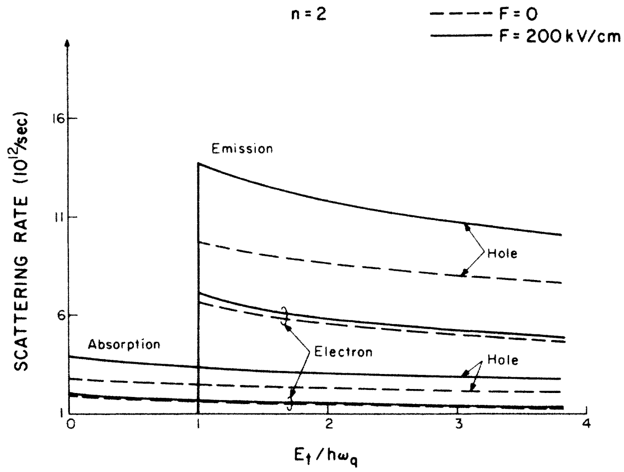

FIG. 4. Plots of the polar-optical-phonon scattering rates of the electrons and the holes for an applied electric field (1) $F=200$ kV/cm (solid lines) and (2) $F=0$ (dashed lines) for the second subband with a well width $L=126.5$ Å at $T=300$ K.

in their ground states at $F=200$ kV/cm are 9.33×10^{11} $\sec^{-1}$ and 2.04×10^{12} $\sec^{-1}$, respectively. So heavy-hole-POP scattering and heavy-hole tunneling have comparable transition rates. From these results we can explain, at least qualitatively, the recent experimental data[12] of field-dependent exciton linewidth broadening in a two-dimensional structure by heavy-hole-POP interaction and heavy-hole tunneling. We do not consider, however, other linewidth broadening mechanisms such as the interface roughness,[20] the field inhomogeneity, and the carrier-carrier interaction. In Fig. 4 the scattering rates for electrons and heavy holes with an applied electric field (1) $F=200$ kV/cm (solid lines) and (2) $F=0$ (dashed lines) for the second subband are plotted. In general, the scattering rates and their electric field dependences of the second subband are smaller than those of the ground level. Smaller electric field dependences can be explained by the fact that the wavefunction change due to the applied electric field of higher subbands is smaller than that of the ground state.[11]

IV. CONCLUSIONS

The electric field dependence of the polar-optical-phonon scattering rates for both the electrons and the holes in a quantum well are studied. For all electric field ranges, the scattering rates for the holes are higher than those of the electrons due to the effective-mass difference. When the electric field $F=200$ kV/cm, the scattering rates for the holes are almost doubled, while the scattering rates for the electrons are increased by about 10% only, compared with those for the zero-field case. It is found that higher subbands have smaller scattering rates and weaker electric field dependences, for both the electrons and the holes, than those of ground state. In our opinion, the field-dependent linewidth broadening of interband edge optical transitions could be dominated by heavy-hole–polar-optical-phonon interaction and heavy-hole tunneling. Other mechanisms, however, such as the interface roughness scatterings, may also contribute to the total linewidth.

ACKNOWLEDGMENTS

The authors would like to thank Professor J. P. Leburton for supplying Ref. 8 and Professor K. Hess for his encouragement in this research. This work was partially supported by the U.S. Air Force, Contract No. F33615-84-K-1557. One of the authors (D.A.) also acknowledges support by General Telephone and Electronics Laboratories Inc. (GTE).

APPENDIX: DERIVATION OF EQS. (13)–(16)

The total transition rates $W_n^+(\mathbf{k}_t)$ and $W_n^-(\mathbf{k}_t)$ for the carriers in the nth subband with a two-dimensional wave vector $\mathbf{k}_t$ are

$$\begin{aligned} W_n^+(\mathbf{k}_t) &= \frac{2\pi}{\hbar}\sum_{\mathbf{k}_t'}\sum_{\mathbf{q}} |M_{fi}^+|^2\delta(E_f-E_i+\hbar\omega_q) \\ &= \frac{2\pi}{\hbar}|C|^2(n_q+1)\sum_{\mathbf{k}_t'}\sum_{\mathbf{q}}\delta_{\mathbf{k}_t',\mathbf{k}_t-\mathbf{q}_t}\frac{1}{q^2}\left|\int_{-L/2}^{L/2}dz\, e^{-iq_z z}|\Phi_n(z)|^2\right|^2\delta\left[\frac{\hbar^2}{2m^*}(\mathbf{k}_t-\mathbf{q}_t)^2-\frac{\hbar^2}{2m^*}\mathbf{k}_t^2+\hbar\omega_q\right] \\ &= \frac{V|C|^2(n_q+1)}{4\pi^2\hbar}\int d^2q_t G_n(q_t,L,F)\delta\left[\frac{\hbar^2}{2m^*}(q_t^2-2k_tq_t\cos\phi)+\hbar\omega_q\right], \end{aligned} \tag{A1}$$

and

$$\begin{aligned} W_n^-(\mathbf{k}_t) &= \frac{2\pi}{\hbar}\sum_{\mathbf{k}_t'}\sum_{\mathbf{q}} |M_{fi}^-|^2\delta(E_f-E_i-\hbar\omega_q) \\ &= \frac{2\pi}{\hbar}|C|^2 n_q\sum_{\mathbf{k}_t'}\sum_{\mathbf{q}}\delta_{\mathbf{k}_t',\mathbf{k}_t+\mathbf{q}_t}\frac{1}{q^2}\left|\int_{-L/2}^{L/2}dz\, e^{iq_z z}|\Phi_n(z)|^2\right|^2\delta\left[\frac{\hbar^2}{2m^*}(\mathbf{k}_t+\mathbf{q}_t)^2-\frac{\hbar^2}{2m^*}\mathbf{k}_t^2-\hbar\omega_q\right] \\ &= \frac{V|C|^2 n_q}{4\pi^2\hbar}\int d^2q_t G_n(q_t,L,F)\delta\left[\frac{\hbar^2}{2m^*}(q_t^2+2k_tq_t\cos\phi)-\hbar\omega_q\right], \end{aligned} \tag{A2}$$

where ϕ is the azimuthal angle between $\mathbf{k}_t$ and $\mathbf{q}_t$, $-\pi \le \phi \le \pi$. Here, $G_n(q_t,L,F)$ is defined by

$$G_n(q_t,L,F)=\int_{-\infty}^{\infty} dq_z \frac{1}{q_z^2+q_t^2}\left| \int_{-L/2}^{L/2} dz\, e^{\pm i q_z z} |\Phi_n(z)|^2 \right|^2 , \tag{A3}$$

and can be evaluated by contour integration. The result is

$$\begin{aligned} G_n(q_t,L,F) &= \int_{-\infty}^{\infty} dq_z \frac{1}{q_z^2+q_t^2}\left| \int_{-L/2}^{L/2} dz\, e^{\pm i q_z z} |\Phi_n(z)|^2 \right|^2 \\ &= \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' |\Phi_n(z)|^2 |\Phi_n(z')|^2 \int_{-\infty}^{\infty} dq_z \frac{e^{iq_z(z-z')}}{q_z^2+q_t^2} \\ &= \frac{\pi}{q_t} \int_{-L/2}^{L/2} dz \int_{-L/2}^{L/2} dz' |\Phi_n(z)|^2 |\Phi_n(z')|^2 e^{-q_t|z-z'|} . \end{aligned} \tag{A4}$$

The evaluation of the δ function is straightforward and is given by

$$\delta\left[\frac{\hbar^2}{2m^*}(q_t^2-2k_tq_t\cos\phi)+\hbar\omega_q\right]=\frac{m^*}{\hbar^2\left[k_t^2\cos^2\phi-\frac{2m^*}{\hbar}\omega_q\right]^{1/2}}\left[\delta\left[q_t-k_t\cos\phi-\left[k_t^2\cos^2\phi-\frac{2m^*}{\hbar}\omega_q\right]^{1/2}\right]\right.$$
$$\left.+\delta\left[q_t-k_t\cos\phi+\left[k_t^2\cos^2\phi-\frac{2m^*}{\hbar}\omega_q\right]^{1/2}\right]\right] , \tag{A5}$$

and

$$\delta\left[\frac{\hbar^2}{2m^*}(q_t^2+2k_tq_t\cos\phi)-\hbar\omega_q\right]=\frac{m^*}{\hbar^2\left[k_t^2\cos^2\phi+\frac{2m^*}{\hbar}\omega_q\right]^{1/2}}\delta\left[q_t+k_t\cos\phi-\left[k_t^2\cos^2\phi+\frac{2m^*}{\hbar}\omega_q\right]^{1/2}\right] . \tag{A6}$$

In Eq. (A6) the condition $q_t \ge 0$ makes the other δ function with an argument

$$q_t+k_t\cos\phi+\left[k_t^2\cos^2\phi+\frac{2m^*}{\hbar}\omega_q\right]^{1/2}$$

vanish.

For the emission process [Eq. (A5)], the condition $q_t \ge 0$, $k_t^2\cos^2\phi-(2m^*/\hbar)\omega_q \ge 0$, and $\cos^2\phi \le 1$ give

$$-\phi_{\max} < \phi < \phi_{\max} \tag{A7}$$

and

$$E_t/\hbar\omega_q > 1 \tag{A8}$$

with

$$\phi_{\max}=\cos^{-1}\left[\frac{\hbar\omega_q}{E_t}\right]^{1/2}$$

and

$$E_t=\frac{\hbar^2k_t^2}{2m^*} .$$

The condition $E_t/\hbar\omega_q > 1$ gives the two-dimensional emission rate a steplike behavior. For the absorption process [Eq. (A6)], all values of ϕ between $-\pi$ and π are allowed. Using Eqs. (A1)–(A8), we obtain the total transition rates $W_n^+(\mathbf{k}_t)$ and $W_n^-(\mathbf{k}_t)$:

$$W_n^+(\mathbf{k}_t)=\omega_q(n_q+1)\alpha_{e\text{-ph}}I_n^+(E_t,F,L) , \tag{A9}$$

with

$$I_n^+(E_t,F,L)=\int_{-L/2}^{L/2}dz\int_{-L/2}^{L/2}\int_0^{\phi_{\max}}d\phi\,|\Phi_n(z)|^2|\Phi_n(z')|^2\frac{1}{\left[\frac{E_t}{\hbar\omega_q}\cos^2\phi-1\right]^{1/2}}$$
$$\times\left[\exp\left\{-|z-z'|\left[\frac{2m^*E_t}{\hbar^2}\right]^{1/2}\left[\cos\phi-\left[\cos^2\phi-\frac{\hbar\omega_q}{E_t}\right]^{1/2}\right]\right\}\right.$$
$$\left.+\exp\left\{-|z-z'|\left[\frac{2m^*E_t}{\hbar^2}\right]^{1/2}\left[\cos\phi+\left[\cos^2\phi-\frac{\hbar\omega_q}{E_t}\right]^{1/2}\right]\right\}\right], \qquad \text{(A10)}$$

where we have reduced the ϕ integration to $0<\phi<\phi_{\max}$ with a factor of 2 added since $\cos\phi$ is an even function of ϕ, and

$$W_n^-(\mathbf{k}_t)=\omega_q n_q \alpha_{e\text{-ph}} I_n^-(E_t,F,L) \qquad \text{(A11)}$$

with

$$I_n^-(E_t,F,L)=\int_{-L/2}^{L/2}dz\int_{-L/2}^{L/2}dz'\int_0^{\pi}d\phi\,|\Phi_n(z)|^2|\Phi_n(z')|^2\frac{1}{\left[\frac{E_t}{\hbar\omega_q}\cos^2\phi+1\right]^{1/2}}$$
$$\times\exp\left\{|z-z'|\left[\frac{2m^*E_t}{\hbar^2}\right]^{1/2}\left[\cos\phi-\left[\cos^2\phi+\frac{\hbar\omega_q}{E_t}\right]^{1/2}\right]\right\}, \qquad \text{(A12)}$$

and $\alpha_{e\text{-ph}}$ is defined in Eq. (17).

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