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Flux distribution and penetration depth measured by muon spin rotation in high-T_c superconductors

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The determination of the magnetic penetration depth λ in type-II superconductors from the field distribution $n(B)$ measured by muon spin rotation (μSR) is discussed. Particular stress is put on high-T_c oxide superconductors (coherence length $\xi \ll \lambda$; upper critical field much greater than the applied field; anisotropy and pinning). Fitting of the highly asymmetric $n(B)$ by a Gaussian or Lorentzian, as done in existing μSR experiments, yields for a perfect vortex lattice λ values which are too large, and for a strongly distorted (due to pinning) lattice λ values which are too small. An improved evaluation of μSR data is suggested.

Since the discovery of high-T_c superconductors[1] several papers on transverse muon spin rotation (μSR) in these promising materials have appeared.[2-5] The main aim of these experiments was the determination of the magnetic penetration depth λ from the relaxation of the (complex) polarization $P^+ = P_x + iP_y$ of the muon spin. The positive muons, created by pion decay, are implanted into the specimen one at a time and rotate their spin in the local magnetic field $B(\mathbf{r})$ with precession frequency $\omega = \gamma_\mu B$ where $\gamma_\mu = 8.513$ rad/sT is the gyromagnetic ratio of the muons. In transverse μSR the orientation of the spin component perpendicular to the applied field B_{appl} is sensored by detection of the positron emitted predominantly along the spin direction when the muon decays (lifetime $\tau_\mu = 2.2$ μs). When the muons see different values of B the amplitude of the ensemble-averaged rotating $P^+(t)$ relaxes due to destructive interference. Here t is the time which elapses between the implantation and the decay of the muon. In this way the distribution of the magnetic field inside the specimen may be determined.

In the present paper it will be shown how $P^+(t)$ looks in a perfect type-II superconductor, how anisotropy and inhomogeneity of the material (vortex pinning) may influence the μSR signal, and how the evaluation of experiments can be improved.

In type-II superconductors $B(\mathbf{r})$ varies spatially since in the range $B_{c1} < B_{\text{appl}} < B_{c2}$ magnetic flux penetrates in form of a (more or less regular) lattice of flux lines (tiny current vortices) each carrying one quantum of flux $\phi_0 = 2.07\times10^{-15}$ Tm^2. These vortices have a core of radius $\approx \xi$ (coherence length) within which the superconducting order parameter goes to zero, and they carry a flux tube of radius λ (penetration depth for weak magnetic fields).

The new high-T_c oxide superconductors are of extreme type II: From their very large (extrapolated) upper critical fields $B_{c2} = \phi_0/2\pi\xi^2$ (>50 T at low temperatures[6]) and very small lower critical fields $B_{c1} \approx \phi_0 \ln\kappa + \text{const.})/4\pi\lambda^2$ [<0.02 T (Refs. 6–8)] follows a large Ginsburg-Landau parameter $\kappa \equiv \lambda/\xi \gg 1$. One useful consequence of this fact is that there exists a large range of applied fields $B_{\text{appl}} < B_{c2}/4$ where the simple London picture applies. This means (i) the vortex cores are well separated (they do not overlap and do not interact since their separation is $d > 5.4\xi$ in a triangular lattice) and therefore (ii) the vortex fields superimpose linearly such that (iii) the energy and elasticity of the vortex lattice can immediately be written down as the magnetic interaction of all vortex pairs.[9] The theory of the vortex lattice is thus much simpler and more transparent than in the Ginsburg-Landau case ($\bar{B} > B_{c2}/4$, $T \approx T_c$).

Another consequence of $\lambda \gg \xi$ is that for $B_{\text{appl}} > 2B_{c1}$ (the condition $2B_{c1} < B_{\text{appl}} < B_{c2}/4$ is satisfied in most experiments) one has $(2\pi\lambda)^2 \gg d^2$ (d = vortex spacing) and therefore the vortex fields overlap strongly. This has three consequences which all are important for the interpretation of μSR experiments. (i) The field variation $(\overline{\Delta B^2})^{1/2}$ in a perfect vortex lattice is much smaller than the average internal field $\bar{B} \approx B_{\text{appl}}$ and is (ii) independent of B_{appl} (this is nicely confirmed by the experiments of Gygax *et al.*[5]). A further, less obvious consequence is that (iii) the elastic response of the vortex lattice to pinning forces is highly nonlocal. This means that the elastic energy required to compress or tilt the vortex lattice inhomogeneously is reduced (with respect to homogeneous strain) by a factor $1/(1+k^2\lambda^2)$ when a Fourier component of wavelength $2\pi/k$ is considered[9,10] (see below).

The validity of the simple London picture (for impure superconductors with $\lambda \gg \xi$ at $B_{\text{appl}} < B_{c2}/4$) is independent of the detailed mechanism which causes superconductivity but follows from the mere presence of a Meissner effect.

In Refs. 2, 4, and 5 the penetration depth λ is extracted from the μSR data as follows; for improved evaluation methods see Ref. 3 and below. First, the field variation $\overline{\Delta B^2}$ for a perfect vortex lattice is given. For the triangular lattice [reciprocal lattice vectors $K^2 = K_{mn}^2 = (16\pi^2/3d^2)(m^2+mn+n^2)$; m,n integer; $d^2 = 2\phi_0/\sqrt{3}\bar{B}$] the London model yields

$$\overline{\Delta B^2} = \sum_{K\neq0} B_K^2 = \phi_0^2\lambda^{-4}(9/32\pi^4) \times(1+3^{-2}+4^{-2}+2\times7^{-2}+\cdots)\ . \quad (1)$$

This correct result $\overline{\Delta B^2}=0.00371\phi_0^2\lambda^{-4}$ is 2.13 times larger than the crude approximation (sum replaced by integral)[11] used in Refs. 2, 4, and 5. In Eq. (1) $B_K=\bar{B}/(1+K^2\lambda^2)$ are the Fourier coefficients of $B(\mathbf{r})$. Note that $K^2\lambda^2>72$ for $B_{\text{appl}}>2B_{c1}$ and that $(\overline{\Delta B^2})^{1/2}\approx 0.76B_{c1}/\ln\kappa\approx B_{c1}/6$. If desired, a cutoff at $K\approx 1.4/\xi$ is provided by the numerical solution of the (isotropic) Ginsburg-Landau theory for $\kappa\gg 1$.[12] This gives at reduced fields $b\equiv\bar{B}/B_{c2}<0.25$, $B_K\approx\bar{B}(1+K^2\lambda^2)^{-1}\exp(-K^2\xi^2/2)$, and at $b>0.7$,

$$\overline{\Delta B^2}=7.52\times 10^{-4}(1-b)^2\phi_0^2\lambda^{-4}\ . \tag{2}$$

A useful approximate expression valid at arbitrary field $0<b<1$ is

$$\overline{\Delta B^2}\approx 7.5\times 10^{-4}(1-b)^2[1+3.9(1-b)^2]\phi_0^2\lambda^{-4}\ . \tag{3}$$

Now consider the field distribution

$$n(B')=\langle\delta[B'-B(\mathbf{r})]\rangle\ , \tag{4}$$

where $\langle\ldots\rangle$ denotes spatial averaging and δ the one-dimensional delta function. $n(B)$ is the probability of finding a field B at an arbitrarily chosen point $\mathbf{r}$ inside the specimen. The integral from $B=-\infty$ to $B=\infty$ over $n(B)$ thus yields unity, and the integral over $B^2n(B)$ yields $\overline{\Delta B^2}$. When the implanted muons are immobile $n(B)$ is the (real) Fourier transform of the (complex) muon spin polarization,[13]

$$P^+(t)=P^+(0)\int_{-\infty}^{\infty}n(B)\exp(i\gamma_\mu Bt)dB\ . \tag{5}$$

From Eq. (5) follows that if $n(B)$ were a Gaussian its relaxation would also be Gaussian, $|P^+(t)|\sim\exp(-t^2/T_2^2)$, where

$$T_2=(2/\gamma_\mu^2\overline{\Delta B^2})^{1/2}\ . \tag{6}$$

In order to estimate λ the measured relaxation was therefore fitted by a Gaussian[2-4] or, somewhat inconsistently, by the exponential $\exp(-t/T_2)$.[5] [This gave a better fit but means a Lorentzian $n(B)$ which has a diverging $\overline{\Delta B^2}$.] Combination of Eqs. (1) and (6) then yielded $\lambda=(0.043\gamma_\mu\phi_0T_2)^{1/2}$.

How good are the λ values obtained in this way? First we note that for a perfect vortex lattice the field distribution $n(B)$ is far from being a Gaussian or a Lorentzian as fitted in these first μSR experiments.[2-5] The correct $n(B)$ is highly asymmetric (in particular at $b\ll 1$) and exhibits van Hove singularities, namely, jumps at the maximum ($B_{\max}$) and minimum ($B_{\min}$) values of the two-dimensionally periodic $B(\mathbf{r})$, and a logarithmic pole at the saddle-point value B_{sad}.[13] This can be seen in Fig. 1 where $n(B)$ is depicted for $b=1$, 0.2, and 0.05 with $B(\mathbf{r})$ taken from the Ginsburg-Landau solution.[12] For $b<0.05$ $n(B)$ looks similar to $b=0.05$ but has a longer tail since $B_{\max}$ is larger (see the field profiles inserted in Fig. 1).

The corresponding spin polarization $P^+(t)$ (Ref. 5) is shown in Fig. 2 for $b=0.2$ and $b=0.05$ [units $P^+(0)=1$, $\overline{\Delta B^2}=1$]. For $b>0.2$ ($b<0.05$) the curves look similar as those for $b=0.2$ ($b=0.05$). Note that neither $|P^+(t)|$ nor $p_x(t)\equiv\text{Re}\{P^+(t)\exp(-i\gamma_\mu\bar{B}t)\}$ (Re$\{\ldots\}$=real part) are well fitted by the curves

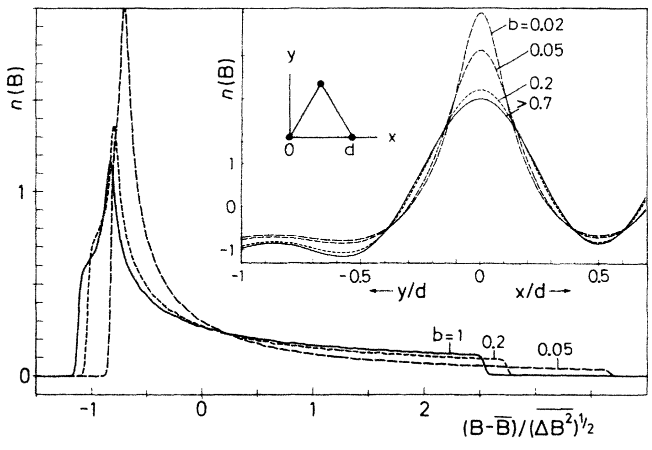

FIG. 1. The field distribution $n(B)$, Eq. (4), for the triangular vortex lattice at various reduced fields $b\equiv\bar{B}/B_{c2}$. Shown are histograms normalized to $\overline{\Delta B^2}=1$ and slightly smoothened by convolution with a Gaussian of width 0.03. The inset shows profiles of the field $B(x,y)$ along two different directions, $B(0,0)=B_{\max}$ (vortex centers), $B(0,\sqrt{3}d/6)=B_{\min}$, and $B(d/2,0)=B_{\text{sad}}$.

$\exp(-t^2/T_2^2)$ or $\exp(-t/T_2)$ when T_2 from Eq. (6) is used (dashed curves). Both fits yield too-large values for T_2 and λ.

The reason for this poor fit is the long tail in $|P^+(t)|$ caused by the sharp peak of $n(B)$ at $B=B_{\text{sad}}$. This situation changes little when anisotropy is allowed for in a monocrystal. Anisotropy will stretch the triangular lattice and split the peak in $n(B)$ since there are now $\simeq 3$ types of saddle points. However, these peaks and the jump at $B_{\min}$ nearly coincide since $B(\mathbf{r})$ is quite flat between the vortices when $b\ll 1$. Another effect of anisotropy seems more important. In a granular superconductor anisotropy of the randomly oriented grains pins the vortices since their energy varies from grain to grain. In high-T_c superconductors further pinning may be caused by grain boundaries, twin boundaries inside the grains, or even by the large atomic lattice cell (intrinsic pinning due to the small radius of the vortex core which may lead to a Peierls potential of the vortices). One therefore has to consider the effect of pinning on $n(B)$.

Weak random pinning slightly distorts the vortex lattice. As a consequence the extrema $B(\mathbf{r})$ fluctuate spatially, very likely with Gaussian distribution. Calculation of these fluctuations is in preparation. The van Hove singularities in $n(B)$ will thus be smeared, even as the Bragg reflections in neutron scattering from the perturbed vortex lattice.[14] For intermediate pinning this smearing becomes so strong that $n(B)$ resembles a Gaussian or, due to the long tail in Fig. 1, an asymmetric Lorentzian-like curve. Smearing of $n(B)$ results in the multiplication of $P^+(t)$ by a Gaussian which cuts off the tails in Fig. 2 even before $\overline{\Delta B^2}$ is increased markedly above its perfect-lattice value [Eq. (1)]. This might explain why the experiments[2-5] (in which the vortex lattice was admittedly far from perfect) allow a fit by a Gaussian or exponential relaxation and why the resulting values for λ (1050 to 2500

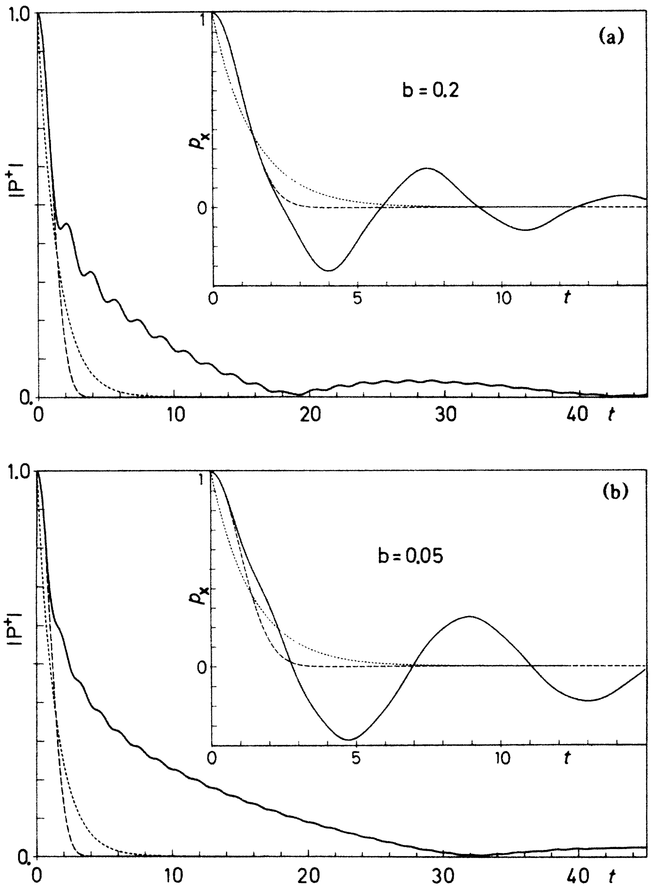

FIG. 2. The complex muon spin polarization $P^+(t)$ [Eq. (5)]. Depicted are the modulus $|P^+|$ and the real part in a frame rotating with the average frequency $\gamma_\mu \bar{B}$, p_x, for (a) $b=0.2$, and (b) $b=0.05$, cf. Fig. 1. Time unit is $(\gamma_\mu^2 \overline{\Delta B^2})^{-1/2}$, and $P^+(0)=1$ was put. The dashed curves show $\exp(-t/T_2)$ and $\exp(-t^2/T_2^2)$ which in μSR experiments were hoped to fit the relaxation of P^+ with T_2 from Eq. (6). Larger T_2 values give better fit but yield too-large values for $\lambda \sim T_2^{1/2}$ The long tail in $|P^+|$ comes from the peak in $n(B)$ at $B_{\rm sad}$, and the rapid oscillations from the jump at $B_{\rm max}$.

Å) are in reasonable agreement with λ values obtained from (difficult to measure) B_{c1} values. But note also that a misinterpretation of $\overline{\Delta B^2}$ by a factor of 16 would change the resulting λ only by a factor of 2, cf. Eqs. (1) to (3).

For strong pinning the global average $\overline{\Delta B^2}$ exceeds the perfect lattice value [Eq. (1)] by far. This results in an enhanced relaxation rate which indeed was observed in increasing field in Ref. 5. Thus, if pinning is strong throughout the specimen then the above fitting method yields too small values for λ. If, however, strong fluctuation of $B(\mathbf{r})$ occurs only in restricted regions, e.g., near grain boundaries or near the surface, but a sufficient part of the specimen volume exhibits almost perfect vortex lattice, then the contribution of this perfect lattice can be separated in the μSR signal: Its van Hove singularities will not be smeared but are superimposed on a broad background which is caused by the strong pinning regions in $n(B)$. In this case the correct λ value can be determined even with the strong pinning by appropriate Fourier transform of the μSR signal (see below).

How large are the pinning-caused fluctuations of $B(\mathbf{r})$ in superconductors with large κ? From the theory of elasticity one might argue that pinning only shears the vortex lattice but does not much compress it since it is caused mainly by local forces on the vortex cores and since the compressional modulus of the vortex lattice, $c_{11}=\bar{B}^2/\mu_0$, is much larger than its shear modulus, $c_{66}\approx c_{11}(1-b)^2/8b\kappa^2$.[10] But only compression will alter B and thus ΔB^2 markedly. A randomly sheared, or even amorphous, vortex arrangement exhibits an $n(B)$ which is some (smeared) average between that of the triangular (Fig. 1) and that of the square (Fig. 6 of Ref. 13) vortex lattice; the latter is more symmetric and less sharply peaked as can be understood from the smaller number of minima and saddle points per maximum in the $B(\mathbf{r})$ of the square lattice. Shearing thus does not change $\overline{\Delta B^2}$ considerably.

There are two nontrivial effects which cause random compression and thus enhance $\overline{\Delta B^2}$. First, the collective action of many pins may create a gradient in B. Calculation of the maximum gradient (the critical current density j_c) is the task of complicated statistical theories which sum random pinning forces acting on a deformable vortex lattice.[15-19]

Second, the pronounced nonlocality of the elastic response mentioned above facilitates compression with wavelength $<2\pi\lambda$. We show this for a pinning-caused periodic compression $\varepsilon(\mathbf{r})$ with wavelength $2\pi/k$. This strain leads to an elastic energy density[10]

$$U_k=\tfrac{1}{2}\langle\varepsilon^2\rangle(\bar{B}^2/\mu_0)/(1+k^2\lambda^2)\ , \tag{7}$$

and to a periodic field variation [Eq. (50) of Ref. 20][21]

$$B_k(\mathbf{r})=\bar{B}\varepsilon(\mathbf{r})/(1+k^2\lambda^2)\ . \tag{8}$$

Eliminating $\langle\varepsilon^2\rangle$ we find that the field fluctuation

$$\langle B_k^2\rangle=2\mu_0 U_k/(1+k^2\lambda^2)\ , \tag{9}$$

for the same elastic (or pinning) energy U_k, is larger at short wavelengths. Between $k=0$ (homogeneous compression) and $k=\pi/d$ (shortest wavelength) $\langle B_k^2\rangle$ increases by a factor $\approx b\lambda^2/\xi^2 \gg 1$. Therefore, in large κ materials even weak pinning forces may increase ΔB^2 above its ideal lattice value (1). Though the dependence of U_k on $\bar{B}$ and k is not known at present, it is conceivable that the pinning-caused $\overline{\Delta B^2}$ is independent of $\bar{B}\ll B_{c2}$, even as is the ideal ΔB^2 of Eq. (1), and as was observed in decreasing field in Ref. 5. The fitted λ value would then be too small.

These uncertainties can be overcome by a more transparent evaluation of μSR experiments in type-II superconductors. As suggested in Ref. 13, the real Fourier transform of the measured muon spin polarization should be inspected; this is just the field distribution with stochastic noise. Taking into account that $P^+(t)$ is defined only for positive times one obtains from Eq. (5)[13]

$$n(B)=\mathrm{Re}\{[\gamma_\mu/\pi P^+(0)]\int_0^\infty P^+(t)\exp(-i\gamma_\mu Bt)\}dt\ . \tag{10}$$

The real, positive, and slowly varying function $n(B)$ (Fig. 1) is more readily interpreted than the rapidly oscillating complex function $P^+(t)$ (Fig. 2). After appropriate noise reduction (filtering and smoothening), the inspection of $n(B)$ directly reveals whether there is regions of relatively perfect vortex lattice. After this, either $P^+(t)$ or (preferably) its Fourier transform (10) may be fitted to an appropriate theory which then yields information both on vortex pinning and on the penetration depth λ. Computer simulations show that, e.g., from 10^6 counted events the position of the van Hove jump at $B=B_{\max}$ in $n(B)$ can be determined with high accuracy (reproducibility), $\delta B_{\max}/\bar{B} < 10^{-4}$, even when its pinning-caused smearing has a width equal to the width ΔB^2 (1) of the pin-free distribution.

In conclusion, with the present knowledge it cannot be excluded that the penetration depths λ obtained from the relaxation time of the muon spin[2-5] are close to the correct λ values, but more likely these are too small since vortex pinning increases the field variation $\overline{\Delta B^2}$ far above its perfect lattice value. An improved evaluation of μSR data by means of Eq. (10) can clarify this point and will also yield detailed information on vortex pinning.

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