Exchange enhancement of the spin splitting in a GaAs-$Ga_xAl_{1-x}As$ heterojunction

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The spin splitting in a GaAs-$Ga_xAl_{1-x}As$ heterojunction has been studied by using the coincidence technique, where the Shubnikov–de Haas oscillations of the conductivity are measured in tilted magnetic fields, and by measurements of the activation energy associated with spin-split conductivity minima. The spin splitting is found to be very strongly enhanced by exchange interactions, and values for the effective g factor as high as 6.2 have been found. The coincidence measurements were made at 0.37 K, and required the use of tilt angles in the range 85°–89°. These show evidence of oscillatory spin splitting determined by the relative spin-population difference within the Landau levels. The activation energy was also studied as a function of tilt angle, and shows that the spin splitting is primarily determined by the perpendicular component of magnetic field for well-resolved levels. The dependence upon total field is sublinear and shows a saturation behavior at high tilt angles.

I. INTRODUCTION

An exchange enhancement of the electronic g factor in two-dimensional (2D) systems was first proposed by Janak[1] to explain measurements made by Fang and Stiles,[2] in which the g factor of silicon metal-oxide-semiconductor (MOS) transistors was found to be considerably larger than the bulk electron value of $g=2$. In the following years a number of authors[3–6] studied this very interesting consequence of many-body interactions in 2D systems, in particular Ando and Uemura,[4] who first pointed out that the g factor should be an oscillatory function of the filling of the Landau levels.

In practice is has not proved easy to make unequivocal measurements of the magnitude of the enhanced spin splitting, since electron-electron interactions should not influence optical transitions between the spin states.[7] ESR measurements in GaAs measure only the unenhanced, or "bare" g factor.[8,9] The most commonly employed method involves tilting of the sample relative to the magnetic field direction. The Landau splittings are only determined by the perpendicular component of magnetic field, whereas the bare spin splitting will be determined by the total field. At low fields the exchange enhancement will also be proportional to the bare spin splitting, and hence the total field, since the spin population difference will be proportional to the energy splitting for poorly resolved levels. At high fields the exchange enhancement should saturate once both spin states are completely separated. It is thus possible to produce level coincidences, or evenly spaced ladders of spin- and Landau-split levels, which can be detected by their influence on the form of the Shubnikov–de Haas oscillations as a function of either magnetic field or gate voltage. There are some problems associated with this method due to the fact that the g factor oscillates, and the level spacings change within the measurement. Nevertheless the measurements made on silicon inversion layers give very clear evidence for the exchange enhancement,[1,10–13] with some evidence also for oscillatory behavior.[14]

For III-V systems the coincidence method has been used to study both $Ga_xIn_{1-x}As$-InP (Ref. 15) and $Ga_xIn_{1-x}As$-$Al_yIn_{1-y}As$ (Ref. 16) heterojunctions, where considerable enhancement of the g factor was found, from a bare value of 3 to enhanced values of around 9. The enhancement was found to depend strongly upon the magnitude of the splitting, consistent with the theory of Ando and Uemura,[4] and some evidence was also found for oscillatory behavior from a study of the different coincidence conditions. Up to now however no coincidence-method measurements have been made on the GaAs-$Ga_xAl_{1-x}As$ system, due to the very low bare g factor of GaAs which is -0.44 (Ref. 17) at the band edge. There is however some experimental evidence that the g factor is enhanced in this system. Nicholas *et al.*[18] and Narita *et al.*[19] compared the spin splitting with the Landau-level broadening estimated from theory[20] or cyclotron resonance, and argued that the observation of spin splittings in Shubnikov–de Haas measurements implied an enhanced g factor of order 5. In a more detailed fit to the temperature-dependent conductivity, Englert *et al.*[21] deduced that there was a considerable exchange contribution to the spin splitting, which would lead to a g factor of 5 for completely resolved levels. In the present paper we present a study of the g factor in a GaAs-$Ga_xAl_{1-x}As$ heterojunction by the coincidence method, showing clear evidence for both enhancement and oscillatory behavior, and in addition present activation studies of the spin splitting, which demonstrate the two-dimensional nature of the exchange enhancement through its saturation at high fields.

II. EXPERIMENTAL METHOD

The samples studied were from a GaAs-$Ga_xAl_{1-x}As$ heterojunction, with a 750 Å spacer layer, with an electron concentration of 8.1×10^{10} cm^{-2} and a mobility of 450 000 $cm^2/V\,s$ at 4.2 K. Electrical measurements were performed on devices with both Hall bar and circular Corbino geometry (true circular enclosed contact geometry) in a rotation gear mounted in a ^3He immersion cryostat. The Corbino geometry allows a direct measurement of σ_{xx}, whereas ρ_{xx} can be directly determined in a Hall bar geometry. Although lithographically defined adjacent to each other, the carrier concentration of the Hall bar was approximately 4% lower than the Corbino sample. The tilt angle was determined from the period of the Shubnikov–de Haas oscillations, assuming that the electron concentration remained constant and the degeneracy of the Landau levels is proportional to the perpendicular component of magnetic field $B\cos\theta$. This assumption was verified by mounting the two samples at a slight angle ($\theta=2.2°$) to each other and studying both simultaneously. This showed that both gave consistent values for the tilt angle, which could be determined to an accuracy of 0.01° for $\theta>85°$. The coincidence measurements were made at a temperature $T=0.37$ K. In addition activation studies were made of the conductivity and resistivity minima in the range $0.37\text{ K}<T<4.2\text{ K}$.

It is also worth discussing the coincidence conditions used in some detail, since these can be quite varied for a system with an oscillating g factor. In a tilted field the energy levels are given by

$$E=(n+\tfrac{1}{2})\hbar\omega_c\pm\tfrac{1}{2}g^*\mu_B B$$
$$=(n+\tfrac{1}{2})\frac{\hbar eB}{m^*}\cos\theta\pm\tfrac{1}{2}g^*\mu_B B\ , \tag{1}$$

where g^* is the effective g factor. For a constant g factor this will lead to an evenly spaced ladder of levels whenever the condition

$$|g^*\mu_B B|=r\hbar\omega_c=r\frac{\hbar eB}{m^*}\cos\theta \tag{2}$$

is satisfied, where $r=\frac{1}{2},\frac{3}{2},\frac{5}{2}$ corresponds to a ladder of alternating spin levels, and $r=1,2,3,\ldots$ corresponds to a ladder with coincident spin-up and spin-down levels from different Landau levels. These conditions are shown schematically in Fig. 1, where the energy levels are shown for a constant perpendicular field component and increasing total field, and hence spin splitting. We will use "r" to denote the generic type of coincidence condition in the subsequent discussion.

Since the g-factor enhancement depends upon the spin-population difference within a Landau level,[4] the g factor will however depend upon the position of the Fermi level. Resistivity, or conductivity minima occuring when the Fermi level lies between two spin split states of the same Landau level will correspond to a maximum enhancement, leading to a larger g factor, usually known as g_S^*, while for a Fermi level between the spin states originating from different levels the population difference will be much smaller leading to a smaller g factor, usually known as g_L^*. For clarity we introduce a new notation for the g factors, $g^*(p)$, where p corresponds to the relative spin-population difference (assuming infinitely sharp levels) for the Landau level under consideration. Thus we have $g_S^*=g^*(1)$ and $g_L^*=g^*(0)$, which we will usually take to be the bare spin splitting, $g^*(0)=g_0$. Thus the $r=\frac{1}{2}$ condition, defined as the condition where the minima, corresponding to the Fermi level lying between Landau levels, and those, where it lies within one Landau level, are equally resolved, corresponds to

$$g^*(1)\mu_B B=\hbar\omega_c-g^*(0)\mu_B B\ . \tag{3}$$

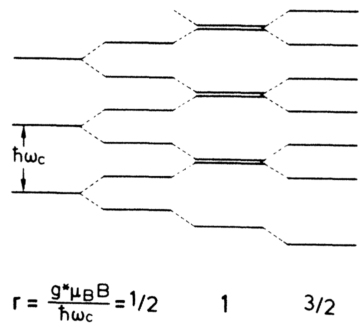

FIG. 1. A schematic view of the energy levels at a constant perpendicular field, for increasing total magnetic field, assuming a constant g factor.

The mean value of $\bar{g}$ deduced from Eq. (2) is thus $[g^*(1)+g^*(0)]/2$. When the g factor is a very strongly oscillating function, however, there may be some error in the definition of the condition corresponding to Eq. (3), since the relative overlap of the different levels may be considerably different with the Fermi level in the two different conditions. To overcome this difficulty we have chosen to analyze the strength of features occurring at a fixed position of the Fermi level, i.e., at a fixed value of perpendicular field, or occupancy ν ($\nu=nh/eB$).

If we consider the case of the Fermi level lying between two spin-split states of the same Landau level, as shown in Fig. 2(a), then the absolute values of the minima in σ_{xx} and ρ_{xx} will be lowest when the splitting is

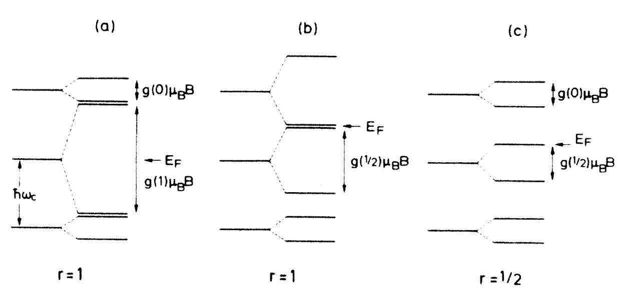

FIG. 2. A schematic view of the energy levels for different coincidence conditions, with an oscillatory g factor. (a) corresponds to Eq. (4), (b) to Eq. (2), and (c) to Eq. (6). (a) and (b) differ only in the position of the Fermi energy E_F.

largest. This occurs when the spin-up and spin-down states from one Landau level just begin to cross the levels from adjacent Landau levels, giving the conditions

$$\bar{g}\mu_B B=\frac{g^*(1)\mu_B B+g^*(0)\mu_B B}{2}=\hbar\omega_c \ . \tag{4}$$

For a constant g factor this is equivalent to the $r=1$ coincidence condition [Eq. (2)]. This measurement should give the largest values of the exchange-enhanced spin splitting, since it corresponds to the maximum spin population difference. It will however usually be assumed that inter-Landau-level exchange is negligible, and hence $g^*(0)=g_0$ for the adjacent, unoccupied Landau level.

The other conditions used involve a study of the absolute values of the conductivity and resistivity maxima. The simplest of these also corresponds to the coincidence condition $r=1$ in Eq. (2), as illustrated in Fig. 2(b). In this case a maximum value will occur for the resistivity and conductivity maxima at even integer filling factors when the spin-up and spin-down levels from adjacent Landau levels are coincident, giving a maximum in the joint density of states. The g factor in this case will be a partially enhanced value for both half-filled spin levels which we denote as $g^*(\frac{1}{2})$.

Finally we can also deduce a value for g close to the conventional $r=\frac{1}{2}$ condition [Eqs. (2) and (3)] from the values of the conductivity and resistivity peaks when all splittings are resolved. Any overlap of adjacent levels, due to level broadening, will cause an increase in the conductivity, and so we can equate a minimum value of the conductivity peak with minimum overlap. This will correspond to the situation shown in Fig. 2(c), where the Fermi level lies in one spin state which is equidistant from its neighbors. In this case we find the condition

$$g^*(\tfrac{1}{2})\mu_B B=\hbar\omega_c-\tfrac{1}{2}[g^*(\tfrac{1}{2})\mu_B B+g^*(0)\mu_B B] \tag{5}$$

or

$$\bar{g}\mu_B B=\frac{3g^*(\frac{1}{2})\mu_B B+g^*(0)\mu_B B}{4}=\tfrac{1}{2}\hbar\omega_c \ , \tag{6}$$

where $g^*(\frac{1}{2})$ is again a partially enhanced value since the level will only be half full.

The particular advantage of these methods is that they are made using a fixed position of the Fermi level, and are thus least likely to be subject to systematic errors due to variations in linewidths and densities of states. In the subsequent analysis the magnitudes of the conductivities and resistivities are usually plotted as a function of $1/\cos\theta$, for any given feature. In the case of a fixed g factor this is equivalent to a linear variation of the ratio of the spin to Landau splittings, and so a constant g factor should lead to all features corresponding to a given coincidence condition occuring at the same value of $1/\cos\theta$.

III. RESULTS

Typical experimental recordings of σ_{xx} are shown in Fig. 3 for the Corbino geometry at 0.37 K, for angles

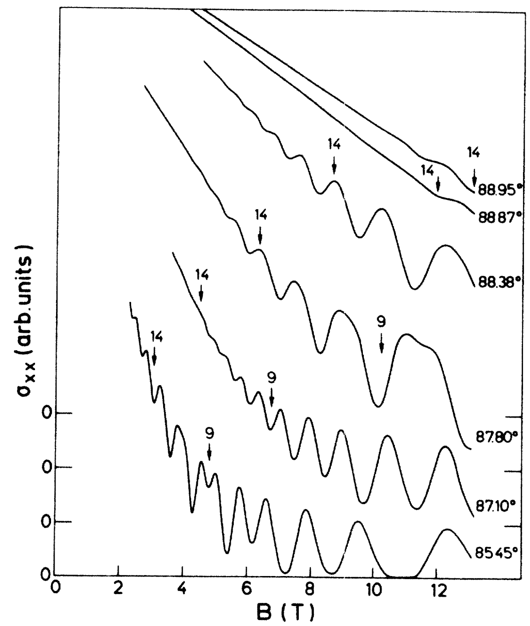

FIG. 3. Conductivity measurements of σ_{xx} as a function of the total magnetic field for six different tilt angles. The temperature is 0.37 K. The angles correspond approximately to the coincidence conditions (a) $r=\frac{1}{2}$, Eq. (6), 85.45°; (b) $r=\frac{1}{2}$, Eq. (3), 87.10°; (c) $r=1$, Eq. (4), 87.80°; (d) $r=1$, Eq. (2), 88.38°; (e) $r=\frac{3}{2}$, Eq. (2), 88.87°. The positions of $\nu=9$ and 14 are shown by the arrows. The zeros for the upper three traces (88.38°, 88.87°, and 88.9°) are the same.

close to the coincidence conditions defined above. In order to determine the precise positions of the coincidences the absolute magnitudes of the maxima and minima in conductivity were plotted as a function of $1/\cos\theta$, as determined from the oscillation period. Typical examples of this are shown in Figs. 4 and 5 for the minima and maxima in σ_{xx}, for several different occupation numbers ν. The same analysis was used on the resistivity ρ_{xx} deduced from the Hall geometry sample. As expected the values of the minima (Fig. 4) pass through a clear minimum value as a function of $1/\cos\theta$, and this occurs at progressively larger angles as the filling factor increases, corresponding to smaller g factors. The position of the minimum is quite well defined, and allows us to measure the mean g factor $[g^*(1)+g^*(0)]/2$ from Eq. (4), to an accuracy of $\pm 2\%$ in the best case. But this error can be as high as $\pm 10\%$ for some minima, which are less well defined and more asymmetric, as can be seen from Figs. 4 and 5. Probably the largest source of errors is due to systematic errors related to the exact interpretation as discussed below. The values are given in Table I for both samples. It is of course necessary to know the effective mass in order to deduce the g factors, and this has been taken as $0.067m_0$. This value was deduced from cyclotron resonance experiments on the same wafer,[22] and is slightly above the band-edge value due to nonparabolicity.

In order to deduce the magnitude of the exchange enhanced g factor $g^*(1)$, we need to know the value of $g^*(0)$. In their original calculations for silicon MOS de-

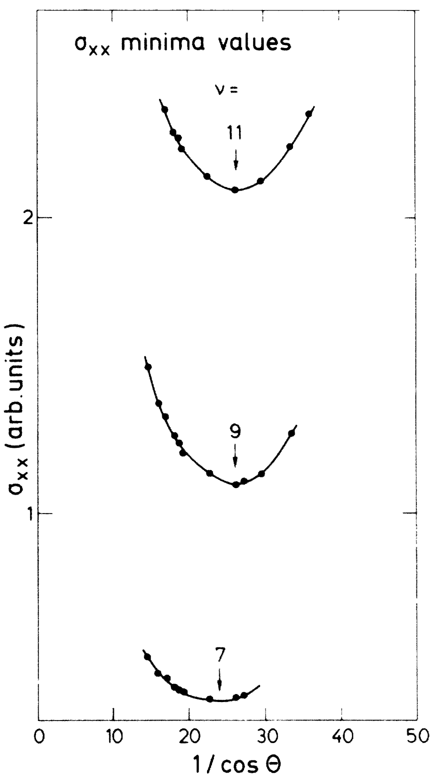

FIG. 4. Values of the minima in σ_{xx} for the filling factors $\nu=7,9,11$ as a function of $1/\cos\theta$ with θ the tilt angle. The arrows show the values used to calculate the g factors.

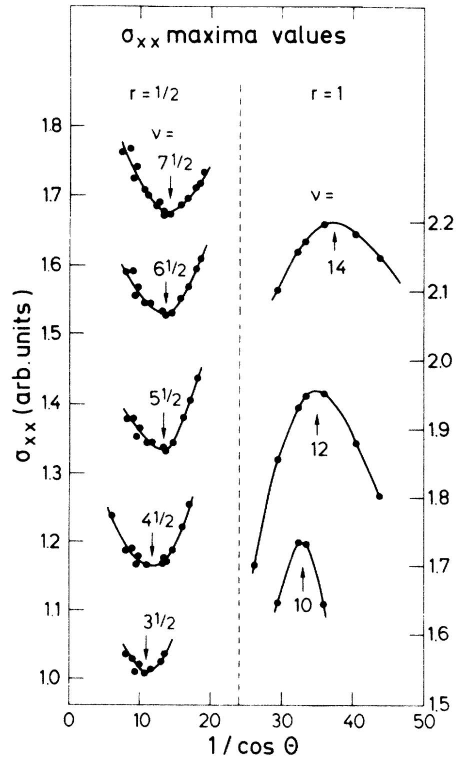

FIG. 5. Values of the maxima in σ_{xx} for several filling factors ν as a function of $1/\cos\theta$ with θ the tilt angle. The two different conditions $r=\frac{1}{2}$ and $r=1$ are separated by a broken line. The arrows show the values used to calculate the g factors.

vices Ando and Uemura[4] suggested that one could ignore inter-Landau-level exchange, so that spin population differences in other Landau levels were unimportant when determining the exchange enhancement for a given level. Providing that this assumption holds then the splitting due to $g^*(0)$ should be equal to the bare spin splitting. The band-edge g factor in GaAs is -0.44,[17] however nonparabolicity decreases this value much more rapidly than the effective mass[17,23] and so we estimate that the kinetic energy due to binding in the heterojunction and the finite Fermi energy cause a reduction to -0.40. This value has also been found from photoconductivity measurements of ESR in the same sample.[8,9] Using this value for $g^*(0)$ we may now deduce the values for the full exchange-enhanced g factor $g^*(1)$, also shown in Table I. It now becomes apparent how dramatically the exchange enhancement has increased the effective g factor, apparently by more than a factor of 5 in some circumstances.

The conductivity maxima show a very clear maximum on rotation (Fig. 5), which may be used to deduce the g-factor from Eq. (2), with $r=1$. In essence this condition is identical to that just used above, with the important difference that now the Fermi level lies within two partially filled levels. The results are again shown in Table I for both samples. It is apparent that the g factors deduced in this case are still substantially enhanced over the bulk value, since there is still a spin population difference for each Landau level considered separately, although the total spin populations are equal. The values are however, substantially lower than those deduced from the minima in σ_{xx} and ρ_{xx}. This is direct evidence for the existence of oscillations in the g factor as a function of filling factor. This difference is also apparent in the original traces shown in Fig. 3 for the two conditions. At 87.8°, close to the $r=1$ condition for

TABLE I. g factors deduced for $r=1$.

	Corbino sample				Hall sample			
ν	B (T)	$\bar{g}$	$g^*(1)$	$g^*(\frac{1}{2})$	B (T)	$\bar{g}$	$g^*(1)$	$g^*(\frac{1}{2})$
7	11.49	1.24	2.08		10.02	1.37	2.34	
9	9.75	1.14	1.88		8.61	1.24	2.08	
10	11.05	0.90		0.90	11.56	0.83		0.83
11	8.04	1.13	1.86		7.34	1.19	1.98	
12	9.77	0.85		0.85	10.25	0.78		0.78
14	8.90	0.80		0.80				

$\nu=9$ and 11, we have the Fermi level between two spin states (ν an odd integer), giving the minimum value of σ_{xx}, but it is also clear that there is no coincidence of the levels when there is no spin population difference since the peaks for even integers (e.g., $\nu=8$) still show the signs of residual spin splitting. At 88.38° we are close to the coincidence for the even integer filling at $\nu=10$ and 12, by which time the conductivity minima have risen substantially.

As a further measure of the g factor we have used Eq. (6), resulting from a minimum overlap of different spin states ($r=\frac{1}{2}$), which we identify by studying the values of the conductivity maxima for half filled and fully resolved levels. This can be seen in Fig. 5 to give a minimum height for the peaks at angles in the range 85°–86°, depending upon the peak studied. In this case the mean g values are again greater than 1, indicating substantial exchange enhancement. The experimental values are given in Table II, together with the values of $g^*(\frac{1}{2})$ calculated from Eq. (6), using the bare value of 0.4 for $g^*(0)$.

On further rotation it was found that the amplitude of the Shubnikov–de Haas oscillations passes through a zero. This is due to the spin states from adjacent Landau levels passing through each other and again forming an equally spaced ladder. This corresponds to the $r=\frac{3}{2}$ condition [Eq. (2)]. Since this occurs for rather high filling factors ($\nu>12$), the broadening of the levels is enough to cause the oscillations to vanish. The position of this zero in amplitude may again be used to calculate the g factors, as given also in Table II. The values deduced are consistent with those found from the other coincidence conditions.

A general feature of all of the measurements given in Tables I and II is that there is a steady decrease of the g factor with increasing filling factor. This is a consequence of the fact that increasing filling factor corresponds to a decrease in the magnetic field at which the determination is made. As a result the energetic splitting of the spin states is smaller, leading to a smaller difference in spin population because the levels are broadened.

It is also interesting to analyze the data using the method of Eq. (3), where minima are compared for different filling factors. This is a technique for finding the $r=\frac{1}{2}$ condition for equally spaced levels. The g factors deduced this way are also shown in Table II. These give the rather surprising result that they are considerably lower than found from the $r=1$ σ_{xx} minima condition, although they correspond to the full spin-split value $g^*(1)$. This may be a consequence of the fact that they are measured at rather smaller values of the total field, where the absolute magnitude of the spin splitting is smaller. As a result there may be a smaller spin population difference thus leading to a smaller exchange enhancement.

IV. ACTIVATION MEASUREMENTS

Since the coincidence method may be used to measure spin splittings only at particular angles, we have also studied the activation energies deduced from the temperature dependence of the conductivity for spin split peaks. A typical example of the plots obtained is shown in Fig. 6 for $\nu=1$, for angles between 0 and 74.1°. It can be seen that there is a well-behaved activation plot for temperatures below 2.5 K, from which we can deduce

TABLE II. g factors deduced for $r=\frac{1}{2}$.

From Eq. (6), Corbino sample

ν	B (T)	$\bar{g}$	$g^*(\frac{1}{2})$
$3\frac{1}{2}$	10.53	1.35	1.67
$4\frac{1}{2}$	8.71	1.27	1.53
$5\frac{1}{2}$	7.92	1.15	1.40
$6\frac{1}{2}$	7.06	1.09	1.32
$7\frac{1}{2}$	6.43	1.06	1.28

From Eq. (3)

	Corbino sample			Hall sample		
ν	B (T)	$\bar{g}$	$g^*(1)$	B (T)	$\bar{g}$	$g^*(1)$
7	8.28	0.86	1.32	8.83	0.78	1.16
9	7.02	0.79	1.18	7.33	0.73	1.06
11	5.94	0.77	1.13	6.24	0.70	1.00

From Eq. (2) and $r=3/2$

	Corbino sample		Hall sample	
ν	B (T)	$\bar{g}$	B (T)	$\bar{g}$
13	11.31	1.02	12.26	0.904
15	10.7	0.94	11.03	0.87
17			10.22	0.83

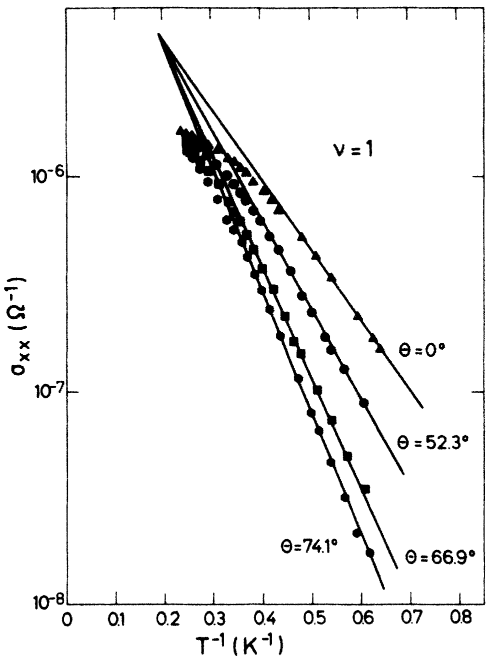

FIG. 6. Temperature dependence of the conductivity at $\nu=1$ for four different tilt angles.

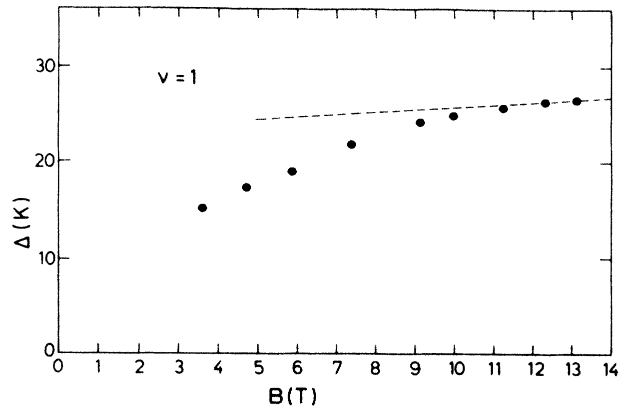

FIG. 7. The measured spin splittings as deduced from the activation energies as a function of the total magnetic field. The dashed line shows a constant energy plus the bare spin splitting.

the spin splitting (Δ) as a function of magnetic field, as shown in Fig. 7. At first sight exponentially activated behavior is a little surprising, since one would expect a temperature dependence which is rather stronger than exponential because the exchange enhancement of the splitting will be increased by the increasing spin population difference as the temperature decreases. This effect has probably saturated by 2.5 K however, since the energy gaps measured are considerably in excess of this (15–26 K). We would expect the activation measurements to be accurate only for large energy gaps however, since the finite width of the band of extended states in each level will tend to give activation energies smaller than the level separation.

The interesting and unusual feature about Fig. 7 is the apparent saturation of the splitting with increasing total magnetic field. At $\theta=0°$, the splitting corresponds to a g factor of 6.23, 15 times greater than the bare value and much larger than found in the coincidence measurements, but the increase of the splitting with total field is strongly sublinear. At high fields the splitting appears to have an asymptotic behavior of a constant value plus a linear increase characterized by the bare g factor, as shown by the solid line. As a result the apparent g factor at $\theta=74.1°$ (13.1 T) has fallen to 3.0, which is much more consistent with some of the values deduced from the coincidence measurements. Such behavior is consistent with a complete saturation of the (two-dimensional) exchange contribution to the splitting.

Measurements were also made for the higher filling factor minima at $\nu=3$ and 5, giving g factors of 5.3, and 2.5, and the spin splitting for these minima was also a noticeably sublinear function of total field, although it was not considered that the accuracy of these measurements was sufficient to warrant detailed discussion.

V. DISCUSSION

In the measurements above we have presented unequivocal evidence for a very strong exchange contribution to the spin splitting in 2D systems based on GaAs. This is very interesting as a model for Coulomb interactions in 2D systems, which are also responsible for the appearance of the fractional quantum Hall effect,[24,25] and which has also been studied as a function of tilt angle in this sample.[26] There has always been some uncertainty over the exact magnitude of the exchange enhancement in silicon,[1,10–13] and its existence at all has been questioned[27] along with the validity of the coincidence method, due to complications from either variation in the Hall field during the measurement, or the presence of a valley splitting in silicon. The work of Englert *et al.*,[13] studying absolute conductivity peaks as a function of total field, as is done here, showed that the Hall field was not influencing the results, while the present study is not complicated by the presence of the valley splitting.

Subtracting the value of the bare splitting from the total spin splitting shown in Fig. 7, we are able to make a direct plot of the exchange energy (E_{ex}) as a function of total field, for $\nu=1$, as shown in Fig. 8. The saturation in the energy suggests a saturation in the spin population difference. This has been modeled semiempirically by the introduction of a Gaussian-broadened density of states [proportional to $\exp(-E^2/\Gamma^2)$], and the assumption that the exchange energy is of the form

$$E_{ex}=E_{ex}^{0}(n_{\uparrow}-n_{\downarrow}) , \tag{7}$$

where E_{ex}^0 is a constant and $n_\uparrow$ and $n_\downarrow$ are the relative populations in the two spin states of a given Landau level. This is based on the original calculated form of Ando and Uemura[4]

$$E_{ex}=\sum_{q}\sum_{N'}\frac{V(q)}{\varepsilon(q)}J_{NN'}(q)(n_{N'_\uparrow}-n_{N'_\downarrow}) , \tag{8}$$

where the overlap terms corresponding to exchange between electrons in different Landau levels (i.e., for

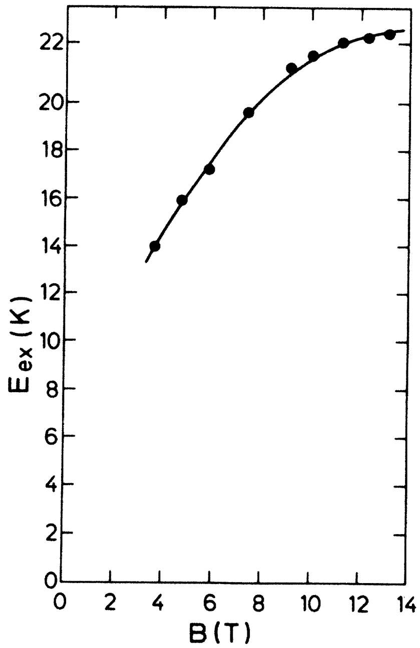

FIG. 8. The exchange energy calculated from the values in Fig. 7 as a function of the magnetic field. The solid line is calculated using Eq. (7) together with the spin splittings shown in Fig. 7.

$N' \neq N$) are ignored. The spin population difference is calculated using the measured activation energy gaps. The solid line in Fig. 8 shows the result of such a calculation, using the parameters $E_{ex}^{0}=38$ K (3.3 meV) and $\Gamma=22$ K (1.9 meV), which appears to describe the data very well.

The magnitude of the exchange energy used is quite reasonable, since the scale of the interaction can be estimated from the Coulomb energy of the system $e^2/4\pi\varepsilon\varepsilon_0 l_B$, where l_B is the cyclotron radius, which is equal to ≈ 9 meV at 3.35 T. Rather more surprising perhaps, is the magnitude of the broadening needed to account for the field dependence. This is very large compared to the width of the extended states in the Landau level, as judged by the field at which Shubnikov–de Haas oscillations begin to appear ($\Gamma \approx 3$ K ≈ 0.26 meV), or to the density-of-states width from self-consistent Born approximation (SCBA) calculations and the mobility ($\Gamma \approx 4.4$ K ≈ 3.8 meV) (Ref. 4) or to zero-field lifetime uncertainty (0.44 K ≈ 0.04 meV), or to the cyclotron resonance linewidth (0.3 K ≈ 0.026 meV).[22] Recent measurements of the density-of-states broadening from the de Haas–van Alphen effect,[28] specific heat[29] and capacitance[30] measurements on high-quality heterojunctions, have found values of order 12 K (1 meV), although these measurements have still been performed on samples with somewhat lower mobilities. It has recently been suggested however, that these rather large values may be due to statistical fluctuations in the local carrier density.[31] It would be rather surprising if long-range fluctuations were responsible for modulating the exchange interaction, which is in essence a short-range effect. The only possible mechanism for this would seem to be the resulting fluctuations in the local occupancy which would lead to fluctuations in the spin population difference, and might then amplify any disorder potential. It should be borne in mind that any systematic increase of the exchange energy upon tilting may lead us to overestimate the value of the exchange energy E_{ex}^{0}, and consequently to overestimate the broadening. We may set a lower limit on the exchange energy by assuming that the spin splitting seen in Fig. 7 is asymptotically approaching a constant value for fully resolved levels plus the bare splitting, which is shown as the dotted line in Fig. 7. This then gives $E_{ex}^{0}=22.5$ K.

It is quite likely that the magnitude of the exchange interaction itself will be modified by the presence of the parallel magnetic field component. One effect of the parallel field is to cause a compression of the wave function in the direction perpendicular to the interface due to the additional kinetic energy terms introduced into the Hamiltonian. This effect may be estimated by a variational calculation,[32] which shows that for the sample studied in this work the mean extension of the wave function in the z direction will be decreased by approximately 35% by a parallel field component of 10 T. This will have two effects: (1) The strength of the exchange interactions will be increased, due to the increasingly two-dimensional character of the electron gas;[4] and (2) the additional kinetic energy of the electrons will reduce the bare g factor due to band nonparabolicity. Clearly these two effects will work in opposition to each other; however, we would expect the first to be dominant at high fields, for well-resolved levels. This may explain some of the increase in measured exchange energy of tilting, and would therefore require the use of a smaller value of the broadening parameter as discussed above. It is unlikely, however, to be the dominant factor, since the wave function compression should begin as a small factor, approximately proportional to B^2, and at high fields should be a sublinear but nonsaturating function of field, in contrast to the initial linear increase and subsequent saturation behavior seen in Fig. 8.

As a further test of the model we have used the exchange-enhanced g factors measured by the coincidence method to estimate the exchange energy and compared this with the predictions of Eq. (7). As these are all measured at higher filling factors and consequently lower perpendicular field values ($B_\perp$), we have introduced an additional factor in the exchange interaction of $1/(B_\perp)^{1/2}$ which is proportional to $1/l_B$. This is because the energies in the problem should scale with the electrostatic interaction energy $e^2/4\pi\varepsilon\varepsilon_0 l_B$. Thus we have

$$E_{ex}^{0}=20.8(B_\perp)^{1/2}\text{ K}\ . \tag{9}$$

The resulting values for the measured and calculated exchange energies are shown in Table III, where we have again taken account of overlap using the measured energy splittings and the fact that some measurements are made with half-filled levels. The agreement can again be seen to be very good, with the exception of the values determined from Eq. (4). This is probably because of two factors. Firstly this condition may slightly overestimate the spin splitting by an amount of order kT, due to

TABLE III. Measured and calculated exchange energies (Corbino sample).

ν	$g^*(1)$	$g^*(\frac{1}{2})$	r	$g^*(p)\mu_B B$ (K)	E_{ex} (K)[a]	E_{ex} (K) [Eq. (7)]
3.5		1.67	$\frac{1}{2}$	11.8	9.0	8.8
4.5		1.53	$\frac{1}{2}$	9.0	6.6	6.7
5.5		1.40	$\frac{1}{2}$	7.5	5.3	5.3
6.5		1.32	$\frac{1}{2}$	6.3	4.4	4.2
7.5		1.28	$\frac{1}{2}$	5.5	3.8	3.6
7.0	2.09		1	16.2	13.1	9.9
	(1.32)[b]		$\frac{1}{2}$	7.3	5.1	5.2
9.0	1.88		1	12.3	9.7	7.3
	(1.18)[b]		$\frac{1}{2}$	5.6	3.7	3.5
11.0	1.86		1	10.0	7.9	5.6
	(1.13)[b]		$\frac{1}{2}$	4.5	2.9	2.6
10.0		0.90	1	6.7	3.75	3.6
12.0		0.85	1	5.6	3.0	2.9
14.0		0.80	1	4.8	2.4	2.3

[a]From $E_{ex}=g^*(p)\mu_B B - g_0\mu_B B$.
[b]These values are measured at $r=\frac{1}{2}$ using Eq. (3) and are therefore at much lower total fields than the values quoted for $r=1$.

increasing overlap of the adjacent spin levels close to the minimum condition, and secondly the neglect of inter-Landau level exchange in Eq. (8) is not necessarily correct, and this could lead to some enhancement of the g factor in the adjacent Landau level, thus reducing the value deduced for g_S^*. In summary therefore we can say that the experiments described here appear to be very well described by a single exchange energy of the form given in Eq. (9).

It is interesting to compare our estimate for the exchange energy with that given by Englert *et al.*[21] In their work the spin splitting was only clearly resolved for $\nu=3$ at 11 T, and this is probably the only point at which their measurements will give a reliable value. The functional form they use for the exchange interaction is also different from Eq. (9), but we can make a reasonable comparison for this one field and filling factor. At this field Eq. (9) will give an exchange energy of 69 K (6 meV), almost double their value of 35 K (3 meV). This difference is probably associated with the assumptions made about the level broadening, and the magnetic field dependence of the exchange energy. If for example we were to assume that this was independent of field, then Eq. (7) gives a value of 38 K (3.3 meV), but we would have much too large an exchange enhancement for our low-field measurements.

In conclusion we may say that we have presented very clear evidence for the existence of population-dependent exchange enhancement of the g factor in GaAs-$Ga_xAl_{1-x}As$ heterojunctions, and have shown this to be related to the two-dimensional character of the system. A single formula [Eq. (9)], appears to give a good description of the magnitude of the exchange energy, although it may be influenced by systematic factors.

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