Tensor-product structure of a new electromagnetic propagator for nonlocal surface optics of metals

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With emphasis on the physical interpretation, the structure of a recently constructed electromagnetic propagator describing, within the framework of the semiclassical infinite-barrier model, the nonlocal optical properties of adjacent vacuum-metal half-spaces, is analyzed. The tensor-product structure of the rotational-free and divergence-free parts of the propagator is determined, and the direct, the indirect, and the self-field contributions are identified. On the basis of a plane-wave expansion, the physical role of the different propagator terms is studied. The contributions to the propagator from the collective polariton and plasmon excitations are separated from those stemming from the electron-hole pair excitations. Via a damped-wave picture of the propagator, contact with a new local propagator formalism of Sipe is established. The flexibility of the present formalism is demonstrated by investigating (i) the excitation of nonlocal surface waves by an oscillating dipole and (ii) the concept of surface-dressed dipole polarizability in the nonlocal regime.

I. INTRODUCTION

When studying, for instance, the optical properties of metals or the behavior of light-excited atoms and molecules near the surface of a metal it is often necessary to use a nonlocal formalism to describe the response of the metal to the electromagnetic field.[1,2] A very popular and physically appealing, nonlocal description is based on the so-called semiclassical, infinite-barrier (SCIB) model.[3,4] Some problems in metal optics, however, require a description which goes beyond that of the SCIB model, and other problems in which light-induced "external" current densities are added to the bare metal are often cumbersome to describe even within the framework of the SCIB model.[1–4] In order to deal with this class of more difficult problems, a new type of screened electromagnetic Green's function associated with wave propagation in adjacent, jellium-vacuum half spaces was constructed, recently.[5] This propagator, constructed within the framework of the SCIB model, was obtained from a linear integro-differential equation, and incorporates in its propagation characteristics screening effects stemming from polariton, plasmon, and single-particle excitations and deexcitations in the jellium. To describe in particular the influence from plasmon and single-particle effects on the propagation properties of the electromagnetic field a nonlocal formalism is required as is well known.[1–4] Within the framework of this nonlocal SCIB description based on (i) the assumption that the electrons of the jellium are scattered specularly at the surface, and (ii) the neglect of quantum interference between the incoming and reflected parts of the wave function of a given electron, also the reflection and transmission properties of the field at the sharp-boundary surface were accounted for.[5]

In the present work, we shall analyze the structure of the SCIB propagator in order to gain physical insight in the propagation characteristics associated with it. Also, we shall point out, by applying the formalism to a few problems, the flexibility of the present propagator description and the clear picture it offers of the physics involved in a given problem. However, let me emphasize that the applications of the propagator formalism to a major analysis of the nonlinear (and linear) electromagnetic field inside a jellium selvedge and to a detailed study of the nonlinear (and linear) self-consistent interaction of two oscillating dipoles placed inside or outside the metal surface will be postponed to a forthcoming paper.[6]

The present paper is organized as follows. In Sec. II, first we consider the time and translational invariance of the propagator. Next, we discuss its tensor-product structure and various classification schemes. Finally, we analyze in detail the fourteen terms of the electromagnetic Green's function. Rotational-free and divergence-free so-called direct, indirect, and self-field contributions are identified and a physical picture based on superposition of plane-wave states is established. In Sec. III, the propagator contributions stemming from collective excitations, i.e., polaritons and plasmons, are investigated, and a damped-wave picture of the Green's tensor is given. We close the section by making contact to a recently established *local* propagator formalism of Sipe.[7] In Sec. IV, we outline some applications of the present dyadic propagator description. Thus, we consider the excitation of nonlocal surface waves by an oscillating dipole, we mentioned the study of elastic and inelastic light scattering from static and moving surface ripples, and we discuss the surface-dressed dipole polarizability. The agreement with the results of others in various limits is demonstrated.

II. BASIC PROPERTIES OF THE SCIB PROPAGATOR

A. Time and translation invariance

Let us consider the general expression for the dyadic Green's function $\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',t,t')$ associated with adjacent jellium-vacuum half-spaces. Under the assumption that the light-unperturbed state of the jellium is time invariant, i.e., $t,t' \Longrightarrow t-t'$, t' and t denoting the time of emission of the field from space point $\mathbf{r}'$ in the source region and the time of observation at space point $\mathbf{r}$, respectively, a Fourier-integral analysis in time, i.e.,

$$\overleftrightarrow{G}(\mathbf{r},\mathbf{r}',t-t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)e^{-i\omega(t-t')}d\omega \ , \tag{2.1}$$

is appropriate for our investigation. Taking advantage of the assumption that the jellium-vacuum system in consideration exhibits translational invariance under arbitrary vectorial displacements parallel to the surface plane, which we assume coincides with the xy plane of a Cartesian xyz-coordinate system, a Fourier analysis in the x and y coordinates gives

$$\overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty} \overleftrightarrow{S}^{-1}\cdot\overleftrightarrow{G}(z,z';q_{\parallel},\omega)\cdot\overleftrightarrow{S}e^{i\mathbf{q}_{\parallel}\cdot(\mathbf{r}_{\parallel}-\mathbf{r}'_{\parallel})}d^2q_{\parallel} \ , \tag{2.2}$$

where $\mathbf{r}_{\parallel}=(x,y,0)$, $\mathbf{r}'_{\parallel}=(x',y',0)$, and $\mathbf{q}_{\parallel}=(q_{\parallel,x},q_{\parallel,y},0)$. Denoting the magnitude of $\mathbf{q}_{\parallel}$ by $q_{\parallel}$, the explicit expression for the rotation matrix $\overleftrightarrow{S}$ is given by

$$\overleftrightarrow{S} = \frac{1}{q_{\parallel}}\begin{bmatrix} q_{\parallel,x} & q_{\parallel,y} & 0 \\ -q_{\parallel,y} & q_{\parallel,x} & 0 \\ 0 & 0 & q_{\parallel} \end{bmatrix} . \tag{2.3}$$

One should notice that the propagator $\overleftrightarrow{G}(z,z';q_{\parallel},\omega)$ depends on the magnitude of $\mathbf{q}_{\parallel}$ only, and has the general form

$$\overleftrightarrow{G}(z,z';q_{\parallel},\omega) = \begin{bmatrix} G_{xx} & 0 & G_{xz} \\ 0 & G_{yy} & 0 \\ G_{zx} & 0 & G_{zz} \end{bmatrix} . \tag{2.4}$$

In proportion to the original analysis in Ref. 5, we have made two minor changes in the mathematical notation. Thus, since we want here to multiply the propagator with the external current density from the right the old propagator has to be transposed (T), cf. Eq. (4.1) of Ref. 5. Also, the chosen sign convention of the exponents in Eq. (2.2) is opposite to that used in Ref. 5, Eq. (3.31). The two changes in the notation imply that the components of our new propagator $\overleftrightarrow{G}(z,z';q_{\parallel},\omega)$ are related to the old ones as follows: $G_{xx}^{\text{new}}=G_{xx}^{\text{old}}$, $G_{yy}^{\text{new}}=G_{yy}^{\text{old}}$, $G_{zz}^{\text{new}}=G_{zz}^{\text{old}}$, $G_{xz}^{\text{new}}=-G_{zx}^{\text{old}}$, and $G_{zx}^{\text{new}}=-G_{xz}^{\text{old}}$, as demonstrated in Appendix A.

In many contexts one wants to utilize the propagator formalism to study the solution to field problems which exhibit invariance under arbitrary translations parallel to the y axis. As described in Appendix B, the effective Green's function for such field problems is of the form

$$\overleftrightarrow{G}(x-x',z,z';\omega) = \int_{-\infty}^{\infty} \overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)dy' \ . \tag{2.5}$$

By inserting Eq. (2.2) into Eq. (2.5) and performing the integrations over y' and $q_{\parallel,y}$ it turns out, cf. Appendix B, that the effective propagator is given by

$$\overleftrightarrow{G}(x-x',z,z';\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overleftrightarrow{G}(z,z';q_{\parallel},\omega) \times e^{iq_{\parallel}(x-x')}dq_{\parallel} \ , \tag{2.6}$$

with the *reinterpretation* that $q_{\parallel}$ is the x component of the wave vector, cf. *ipso facta* $-\infty < q_{\parallel} < \infty$ in the equation above.

B. Tensor products and classification schemes

Substantial, physical insight in the properties of the propagating, electromagnetic field can be obtained by splitting the propagator $\overleftrightarrow{G}(z,z';q_{\parallel},\omega)$ into a superposition of special tensor products. In this subsection, we shall undertake a fundamental discussion of the tensor-product structure of the Green's function, and in the following subsection, we shall present the explicit expressions for the different components of the propagator and investigate the physics hidden in these expressions.

Resolved into appropriate tensor products ($\otimes$) it turns out that the propagator can be written as

$$\overleftrightarrow{G}(z,z';q_{\parallel},\omega) = \sum\!\!\!\!\!\!\int_n^{R(z,z')} \mathbf{\Gamma}_O^{(n)}(z;q_{\parallel},\omega) \otimes \mathbf{\Gamma}_S^{(n)}(z';q_{\parallel},\omega) \ , \tag{2.7}$$

where we have introduced a generalized summation symbol $\sum\!\!\!\!\!\!\int_n$ to indicate that the tensor-product superposition contains both a summation $\sum_n$ over discrete n values and an integration $\int dn$ over a continuous n spectrum. We have added a superscript $R(z,z')$ to $\sum\!\!\!\!\!\!\int$ to indicate that z and z' in the individual tensor products are *restricted (R)* to specified intervals (see Sec. II C). If there were no restrictions on z and z', the selvedge-field problem in nonlocal metal optics could be solved exactly by analytic methods, cf. the analysis in Ref. 8. Despite the presence of the restriction $R(z,z')$, obvious advantages are achieved by the division of the individual tensor product into vectors $\mathbf{\Gamma}_O^{(n)}(z;q_{\parallel},\omega)$ and $\mathbf{\Gamma}_S^{(n)}(z';q_{\parallel},\omega)$ which are functions of the "observation" (O) coordinate z, and the "source" (S) coordinate z', only. The reason that this very important separation in z and z' can be achieved stems from the fact that we are considering the semi-infinite half space geometry. To realize this by a qualitative argument one notes that a point source located at $\mathbf{r}'$ leads to a propagator $\sim |\mathbf{r}-\mathbf{r}'|^{-1}\exp(i\kappa|\mathbf{r}-\mathbf{r}'|)$ which due to the factor $|\mathbf{r}-\mathbf{r}'|^{-1}$ cannot be separated into a single product of functions of $\mathbf{r}$ and $\mathbf{r}'$, alone. For an infinitely extended, plane source located at z', however, the associated propagator $\sim\exp(i\kappa|z-z'|)$ *is* separable in restricted intervals.

Of interest in the following is also the transposed (T) propagator

$$[\overleftrightarrow{G}(z,z';q_\parallel,\omega)]^T = \sum\!\!\!\!\!\!\int_n^{R(z,z')} \mathbf{\Gamma}_S^{(n)}(z';q_\parallel,\omega) \otimes \mathbf{\Gamma}_O^{(n)}(z;q_\parallel,\omega) . \quad (2.8)$$

To make progress for instance, in the investigation of selvedge fields in nonlinear[5,6,8] and linear[9-12] nonlocal optics on the basis of the present propagator formalism, it is of importance to study the substructure of the individual tensor products from a physical point of view. Hence, we divide each of the vectors $\mathbf{\Gamma}_O^{(n)}$ and $\mathbf{\Gamma}_S^{(n)}$ into two pieces (T and L), i.e.,

$$\mathbf{\Gamma}_\alpha^{(n)} = \mathbf{\Gamma}_{\alpha,T}^{(n)} + \mathbf{\Gamma}_{\alpha,L}^{(n)}, \quad \alpha = O \text{ or } S . \quad (2.9)$$

The two vectors $\mathbf{\Gamma}_{O,T}^{(n)}$ and $\mathbf{\Gamma}_{O,L}^{(n)}$ which are functions of the observation coordinate z are chosen to obey the conditions

$$\left[iq_\parallel \mathbf{e}_x + \mathbf{e}_z \frac{\partial}{\partial z} \right] \cdot \mathbf{\Gamma}_{O,T}^{(n)}(z;q_\parallel,\omega) = 0 \quad (2.10)$$

and

$$\left[iq_\parallel \mathbf{e}_x + \mathbf{e}_z \frac{\partial}{\partial z} \right] \times \mathbf{\Gamma}_{O,L}^{(n)}(z;q_\parallel,\omega) = \mathbf{0} , \quad (2.11)$$

where $\mathbf{e}_x$ and $\mathbf{e}_z$ are unit vectors in the x and z directions. Unit vectors in the y direction will be denoted by $\mathbf{e}_y$ in the following. The requirements in Eqs. (2.10) and (2.11) can be given a straightforward interpretation. Thus, if one considers the vector field $\mathbf{\Gamma}_O^{(n)}(z)\exp(iq_\parallel x)$ it readily appears that the conditions in Eqs. (2.10) and (2.11) ensure that $\mathbf{\Gamma}_{O,T}^{(n)}(z)\exp(iq_\parallel x)$ and $\mathbf{\Gamma}_{O,L}^{(n)}(z)\exp(iq_\parallel x)$ are the divergence-free (T) and the rotational-free (L) parts of the field, respectively. For the vector field $\mathbf{\Gamma}_S^{(n)}(z')\exp(-iq_\parallel x')$ in the source coordinates, the division into a divergence-free part, $\mathbf{\Gamma}_{S,T}^{(n)}(z')\exp(-iq_\parallel x')$, and a rotational-free part, $\mathbf{\Gamma}_{S,L}^{(n)}(z')\exp(-iq_\parallel x')$, is provided by the requirements

$$\left[-iq_\parallel \mathbf{e}_x + \mathbf{e}_z \frac{\partial}{\partial z'} \right] \cdot \mathbf{\Gamma}_{S,T}^{(n)}(z';q_\parallel,\omega) = 0 \quad (2.12)$$

and

$$\left[-iq_\parallel \mathbf{e}_x + \mathbf{e}_z \frac{\partial}{\partial z'} \right] \times \mathbf{\Gamma}_{S,L}^{(n)}(z';q_\parallel,\omega) = \mathbf{0} \quad (2.13)$$

assuming the source and observation coordinate systems to coincide.

By inserting the tensor-product superposition of Eq. (2.7) into Eq. (2.6) and utilizing the division of the vector fields given in Eq. (2.9), it is realized that the propagator in Eq. (2.6) can be written as follows:

$$\begin{aligned} \overleftrightarrow{G}(x-x',z,z';\omega) = {} & \overleftrightarrow{G}_{TT}(x-x',z,z';\omega) \\ & + \overleftrightarrow{G}_{TL}(x-x',z,z';\omega) \\ & + \overleftrightarrow{G}_{LT}(x-x',z,z';\omega) \\ & + \overleftrightarrow{G}_{LL}(x-x',z,z';\omega) , \end{aligned} \quad (2.14)$$

where the individual terms on the right-hand side of the equation fulfill the conditions

$$\nabla' \cdot [\overleftrightarrow{G}_{TT}]^T = \nabla \cdot \overleftrightarrow{G}_{TT} = \mathbf{0} , \quad (2.15)$$

$$\nabla' \cdot [\overleftrightarrow{G}_{TL}]^T = \nabla \times \overleftrightarrow{G}_{TL} = \mathbf{0}\ (\overleftrightarrow{0}) , \quad (2.16)$$

$$\nabla' \times [\overleftrightarrow{G}_{LT}]^T = \nabla \cdot \overleftrightarrow{G}_{LT} = \overleftrightarrow{0}\ (\mathbf{0}) , \quad (2.17)$$

$$\nabla' \times [\overleftrightarrow{G}_{LL}]^T = \nabla \times \overleftrightarrow{G}_{LL} = \overleftrightarrow{0} . \quad (2.18)$$

Note that the different parts of the propagator each have been given two subscripts. The subscript to the left indicates whether the propagator part in question is divergence free (index T) or rotational free (index L) with respect to the source coordinates (x',z'). The subscript to the right classifies the propagator part as being either divergence free (index T) or rotational free (index L) with respect to the observation coordinates (x,y).

In terms of the tensor-product decomposition one has in explicit form

$$\overleftrightarrow{G}_{TT}(x-x',z,z';\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum\!\!\!\!\!\!\int_n^{R(z,z')} \mathbf{\Gamma}_{O,T}^{(n)}(z;q_\parallel,\omega) \otimes \mathbf{\Gamma}_{S,T}^{(n)}(z';q_\parallel,\omega) \times e^{iq_\parallel(x-x')} dq_\parallel , \quad (2.19)$$

$$\overleftrightarrow{G}_{TL}(x-x',z,z';\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum\!\!\!\!\!\!\int_n^{R(z,z')} \mathbf{\Gamma}_{O,L}^{(n)}(z;q_\parallel,\omega) \otimes \mathbf{\Gamma}_{S,T}^{(n)}(z';q_\parallel,\omega) \times e^{iq_\parallel(x-x')} dq_\parallel , \quad (2.20)$$

$$\overleftrightarrow{G}_{LT}(x-x',z,z';\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum\!\!\!\!\!\!\int_n^{R(z,z')} \mathbf{\Gamma}_{O,T}^{(n)}(z;q_\parallel,\omega) \otimes \mathbf{\Gamma}_{S,L}^{(n)}(z';q_\parallel,\omega) \times e^{iq_\parallel(x-x')} dq_\parallel , \quad (2.21)$$

and

$$\overleftrightarrow{G}_{LL}(x-x',z,z';\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\sum_n^{R(z,z')}\Gamma_{O,L}^{(n)}(z;q_\parallel,\omega)\otimes\Gamma_{S,L}^{(n)}(z';q_\parallel,\omega)\times e^{iq_\parallel(x-x')}dq_\parallel \ . \tag{2.22}$$

C. The fourteen terms of the propagator

In the preceding subsection it was suggested that restrictions $[R(z,z')]$ exist which limit the allowed values of z and z' in the individual tensor products. The restrictions can be grouped into two main categories. Thus, the membership of the one category stems from the division of space into metal and vacuum half-spaces. The restrictions of the other category, which are those associated with the so-called direct propagator contributions in either vacuum or metal, are necessary to distinguish between source points to the left $(z'<z)$ and the right $(z'>z)$ of a given point of observation. In the context of selvedge-field calculations it is the members of this last category which make the field-theoretical problem of the jellium selvedge so difficult to tackle.[6,8]

To distinguish between the parts of the propagator which belong to different combinations of vacuum and metal domains, we introduce the following superscripts on the Green's function $\overleftrightarrow{G}(z,z';q_\parallel,\omega)$: $<<$ $(z'<0,z<0)$, $<>$ $(z'<0, z>0)$, $><$ $(z'>0,z<0)$, and $>>$ $(z'>0,z>0)$. Thus, keeping the dependence of the propagator on $q_\parallel$ and ω implicit in the notation one has

$$\begin{aligned}\overleftrightarrow{G}(z,z')=&\Theta(-z')\Theta(-z)\overleftrightarrow{G}^{<<}(z,z')\\&+\Theta(-z')\Theta(z)\overleftrightarrow{G}^{<>}(z,z')\\&+\Theta(z')\Theta(-z)\overleftrightarrow{G}^{><}(z,z')\\&+\Theta(z')\Theta(z)\overleftrightarrow{G}^{>>}(z,z') ,\end{aligned} \tag{2.23}$$

Θ being the Heaviside unit step function.

Let us investigate now the explicit tensor-product structure of the propagator in the different domains. In the domain $z'<0,z<0$, one obtains on the basis of the result in Ref. 5 after a slight rewriting, and use of Eq. (A5)

$$\overleftrightarrow{G}^{<<}(z,z')=\overleftrightarrow{D}_{TT}^{<<}(z-z')+\overleftrightarrow{I}_{TT}^{<<}(z+z')+\overleftrightarrow{g}_{LL}^{<<}(z-z') , \tag{2.24}$$

where

$$\overleftrightarrow{D}_{TT}^{<<}(z-z')=\frac{e^{iq_\perp^0|z-z'|}}{2iq_\perp^0}[\mathbf{e}_y\otimes\mathbf{e}_y+\Theta(z-z')\mathbf{e}_i\otimes\mathbf{e}_i+\Theta(z'-z)\mathbf{e}_r\otimes\mathbf{e}_r] , \tag{2.25}$$

$$\overleftrightarrow{I}_{TT}^{<<}(z+z')=\frac{e^{-iq_\perp^0(z+z')}}{2iq_\perp^0}(r^s\mathbf{e}_y\otimes\mathbf{e}_y+r^p\mathbf{e}_r\otimes\mathbf{e}_i) , \tag{2.26}$$

and

$$\overleftrightarrow{g}_{LL}^{<<}(z-z')=\left[\frac{c_0}{\omega}\right]^2\delta(z-z')\mathbf{e}_z\otimes\mathbf{e}_z . \tag{2.27}$$

In Eqs. (2.25) and (2.26) we have introduced $q_\perp^0=[(\omega/c_0)^2-q_\parallel^2]^{1/2}$, and the unit vectors

$$\mathbf{e}_i=\frac{c_0}{\omega}(q_\perp^0,0,-q_\parallel) , \tag{2.28}$$

$$\mathbf{e}_r=\frac{c_0}{\omega}(-q_\perp^0,0,-q_\parallel) . \tag{2.29}$$

The quantities r^s and r^p in Eq. (2.26) denote the relevant amplitude reflection coefficients for s- and p-polarized light, respectively. The explicit expressions for r^s and r^p are given in Appendix C.

The result in Eqs. (2.24)–(2.27) can be interpreted in a physically appealing way. Thus, by inserting Eq. (2.24), with (2.25) and (2.26), into Eq. (B8) and this equation in turn into Eq. (B2), one obtains after performing the integrations over x' and z', for $z\neq z'$, the following relations between the Fourier amplitudes of the appropriate electric fields and different parts of the external current density:

$$\mathbf{E}(q_\parallel,q_\perp^0,\omega)-\mathbf{E}^{(0)}(q_\parallel,q_\perp^0,\omega)=-\frac{\mu_0\omega}{2q_\perp^0}(\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_i\otimes\mathbf{e}_i)\cdot\mathbf{J}_{\text{ext}}^{(-\infty|z)}(q_\parallel,q_\perp^0,\omega) , \tag{2.30}$$

$$\begin{aligned}\mathbf{E}(q_\parallel,-q_\perp^0,\omega)-\mathbf{E}^{(0)}(q_\parallel,-q_\perp^0,\omega)=&-\frac{\mu_0\omega}{2q_\perp^0}[(\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_r\otimes\mathbf{e}_r)\cdot\mathbf{J}_{\text{ext}}^{(z|0)}(q_\parallel,-q_\perp^0,\omega)\\&+(r^s\mathbf{e}_y\otimes\mathbf{e}_y+r^p\mathbf{e}_r\otimes\mathbf{e}_i)\cdot\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)],\end{aligned} \tag{2.31}$$

where

$$\mathbf{J}_{\text{ext}}^{(-\infty|z)}(q_\parallel,q_\perp^0,\omega)=\int_{-\infty}^{\infty}\Theta(z-z')\times\mathbf{J}_{\text{ext}}(x',z';\omega)\times e^{-i(q_\parallel x'+q_\perp^0 z')}dx'dz',\quad z<0 \tag{2.32}$$

$$\mathbf{J}_{\text{ext}}^{(z|0)}(q_\parallel,-q_\perp^0,\omega)=\int_{-\infty}^{\infty}\Theta(z'-z)\Theta(-z')\times\mathbf{J}_{\text{ext}}(x',z';\omega)\times e^{-i(q_\parallel x'-q_\perp^0 z')}dx'dz' , \tag{2.33}$$

$$\mathbf{J}_{\rm ext}^{(-\infty\,|\,0)}(q_\parallel,q_\perp^0,\omega)=\int_{-\infty}^{\infty}\Theta(-z')\mathbf{J}_{\rm ext}(x',z';\omega)\times e^{-i(q_\parallel x'+q_\perp^0 z')}dx'dz' . \qquad (2.34)$$

The relations in Eqs. (2.30) and (2.31) demonstrate that the tensor-product structure in Eqs. (2.25) and (2.26) is associated with the propagation of different plane-wave excitations between the source point (x',z') and the observation point (x,z). Thus, Eq. (2.30), which stems from the first two terms on the right-hand side of Eq. (2.25), is related to the *direct (D)* propagation of the plane-wave excitation $(q_\parallel,0,q_\perp^0)$ between the two points, assuming $z'<z$. The source strength is given by the Fourier transform of the part of the external current-density distribution lying to the *left* of the plane of observation, i.e., $z'<z$ [cf. Eq. (2.32)]. The dyadic notation shows that only the *transverse* part of $\mathbf{J}_{\rm ext}^{(-\infty\,|\,z)}$ is responsible for the *transverse* field propagation in vacuum. The first and third term on the right-hand side of Eq. (2.25) is associated with direct field propagation from (x',z') to (x,z) in the case where $z'>z$. Since $q_\parallel$ is a fixed quantity, the relevant wave vector is of course $(q_\parallel,0,-q_\perp^0)$ in this case. The associated Fourier-transformed relation is part of Eq. (2.31). As one would expect, this part of the equation relates the transverse part of the Fourier transform of the external current-density distribution lying in *vacuum* and to the *right* of the plane of observation, i.e., $z'>z$ [cf. Eq. (2.34)], to the Fourier amplitude of the transverse vacuum field. The terms of the propagator in Eq. (2.26) describe *indirect (I)* field propagation, i.e., propagation involving a reflection of the field at the surface. It is realized from the Fourier-space description [Eq. (2.31)] that these terms relate the transverse parts of the field of the wave vector $(q_\parallel,0,-q_\perp^0)$ and the external current density of the "reflected" wave vector $(q_\parallel,0,q_\perp^0)$. The entire distribution of external current-density sources in vacuum $(z'<0)$ contributes in this case, as one would expect [cf. Eq. (2.34)]. We have added the subscripts TT to the direct and indirect propagator terms to indicate that these are divergence free in the source and observation coordinates. The absence of rotational-free propagator components in the domain $z'<0, z<0$, stems from the fact that vacuum can support only transversely polarized wave propagation. A schematic illustration of the propagation characteristics associated with the different tensor products in $\overleftrightarrow{D}_{TT}^{\lessgtr <}$ and $\overleftrightarrow{I}_{TT}^{\lessgtr <}$ is shown in Fig. 1.

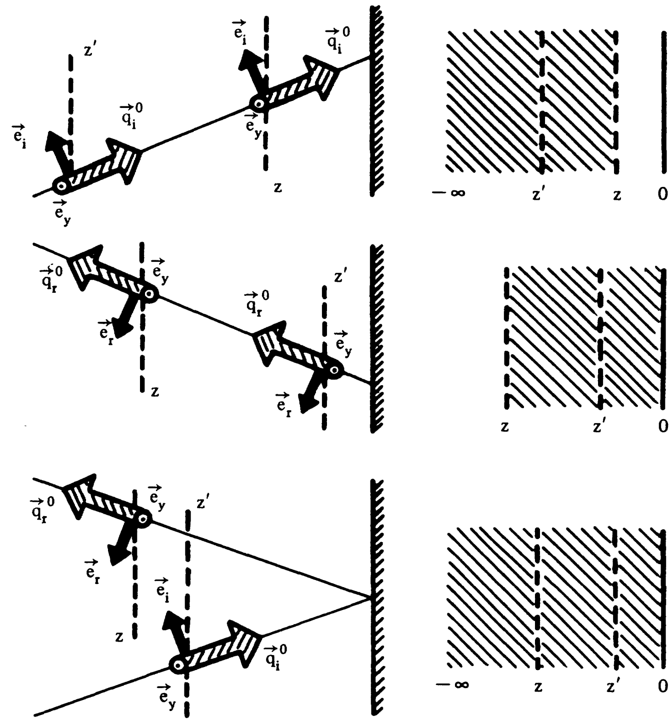

FIG. 1. Schematic diagrams illustrating the propagator characteristics associated with the propagation of transversely polarized plane waves between the source (z') and observation (z) plane in the case where both planes lie in vacuum. In the upper and center panels are shown the direct propagator contributions, $\overleftrightarrow{D}_{TT}^{\lessgtr <}$, for $z'<z$ and $z'>z$, respectively. In the lower panels figure the indirect propagator, $\overleftrightarrow{I}_{TT}^{\lessgtr <}$, characteristics are illustrated. The hatched areas to the right of the individual figures indicate from which domains in space the external current-density source distribution contributes in the different cases.

The singular behavior of the propagator at $z=z'$ is determined by the Green's function $\overleftrightarrow{g}_{LL}^{\lessgtr <}$ of Eq. (2.27) which is rotational free in both z' and z. Using the same procedure as that leading to Eqs. (2.30) and (2.31) one obtains the relation

$$\mathbf{E}(z;q_\parallel,\omega)-\mathbf{E}^{(0)}(z;q_\parallel,\omega)=-\frac{i}{\epsilon_0\omega}\mathbf{e}_z\otimes\mathbf{e}_z\cdot\mathbf{J}_{\rm ext}(z;q_\parallel,\omega) \qquad (2.35)$$

between the Fourier amplitudes in $q_\parallel$ and ω of the electric field and the external current density. The propagator $\overleftrightarrow{g}_{LL}^{\lessgtr <}$, which relates $\mathbf{E}$ and $\mathbf{J}_{\rm ext}$ at the *same* plane $z'=z$, is the so-called self-field part of the Green's tensor. For conciseness, subscript LL is given to the self-field propagator.

The tensor-product structure of the Green's function in the domain $z'>0$, $z<0$ takes the following form:

$$\overleftrightarrow{G}^{><}(z,z')=\overleftrightarrow{G}_{TT}^{><}(z,z')+\overleftrightarrow{G}_{LT}^{><}(z,z') , \qquad (2.36)$$

with

$$\overleftrightarrow{G}_{TT}^{><}(z,z')=\frac{e^{-iq_\perp^0 z}}{2\pi}\left[(1-r^s)\mathbf{e}_y\otimes\mathbf{e}_y\int_{-\infty}^{\infty}\frac{e^{iq_\perp z'}}{N_T(q)}dq_\perp+\frac{\omega(1+r^p)}{c_0 q_\perp^0}\mathbf{e}_r\otimes\int_{-\infty}^{\infty}\mathbf{e}_l^T(q_\parallel,q_\perp)\frac{q_\perp e^{iq_\perp z'}}{qN_T(q)}dq_\perp\right] , \qquad (2.37)$$

and

$$\overleftrightarrow{G}_{LT}^{><}(z,z')=-\frac{\omega(1+r^p)}{2\pi c_0 q_\perp^0}e^{-iq_\perp^0 z}\mathbf{e}_r \otimes \int_{-\infty}^{\infty}\mathbf{e}_I^L(q_\parallel,q_\perp)\frac{q_\parallel e^{iq_\perp z'}}{qN_L(q)}dq_\perp \ . \tag{2.38}$$

In Eqs. (2.37) and (2.38), we have introduced the well-known quantities[5]

$$N_T(q,\omega)=\left[\frac{\omega}{c_0}\right]^2\left[1+\frac{i}{\epsilon_0\omega}\sigma_T(q,\omega)\right]-q^2 \tag{2.39}$$

and

$$N_L(q,\omega)=\left[\frac{\omega}{c_0}\right]^2\left[1+\frac{i}{\epsilon_0\omega}\sigma_L(q,\omega)\right] , \tag{2.40}$$

where σ_T and σ_L are the Fourier-transformed transverse[13] and longitudinal[14] conductivity response functions, respectively, and $q=(q_\parallel^2+q_\perp^2)^{1/2}$. The notation of the added subscripts TT and LT follows the convention made in Eqs. (2.15) and (2.17). For conceptual clarity also the unit vectors

$$\mathbf{e}_I^T(q_\parallel,q_\perp)=\frac{1}{q}(-q_\perp,0,-q_\parallel) \tag{2.41}$$

and

$$\mathbf{e}_I^L(q_\parallel,q_\perp)=\frac{1}{q}(q_\parallel,0,-q_\perp) \tag{2.42}$$

have been introduced, cf. the discussion below. As one would have anticipated, the amplitude transmission coefficients $1-r^s$ and $1+r^p$ for s- and p-polarized light occur in Eqs. (2.37) and (2.38).

Following the same procedure as that which lead to Eqs. (2.30) and (2.31), one obtains via Eqs. (2.36)–(2.38) the formal Fourier-transformed equation

$$\begin{aligned}\mathbf{E}(q_\parallel,-q_\perp^0,\omega)=&\mathbf{E}^{(0)}(q_\parallel,-q_\perp^0,\omega)-\frac{i\mu_0\omega}{2\pi}(1-r^s)\mathbf{e}_y\otimes\mathbf{e}_y\cdot\int_{-\infty}^{\infty}\frac{1}{N_T(q)}\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)dq_\perp\\&-\frac{i\mu_0\omega^2}{2\pi c_0 q_\perp^0}(1+r^p)\mathbf{e}_r\otimes\int_{-\infty}^{\infty}\frac{q_\perp}{qN_T(q)}\mathbf{e}_I^T(q_\parallel,q_\perp)\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)dq_\perp\\&+\frac{i\mu_0\omega^2}{2\pi c_0 q_\perp^0}(1+r^p)\mathbf{e}_r\otimes\int_{-\infty}^{\infty}\frac{q_\parallel}{qN_L(q)}\mathbf{e}_I^L(q_\parallel,q_\perp)\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)dq_\perp ,\end{aligned} \tag{2.43}$$

or equivalently

$$\begin{aligned}\mathbf{E}(q_\parallel,-q_\perp^0,\omega)=&\mathbf{E}^{(0)}(q_\parallel,-q_\perp^0,\omega)\\&-\frac{i\mu_0\omega}{2\pi}\int_{-\infty}^{\infty}\left[\frac{1-r^s}{N_T(q)}\mathbf{e}_y\otimes\mathbf{e}_y+\frac{\omega(1+r^p)}{c_0 q_\perp^0}\mathbf{e}_r\otimes\left[\frac{q_\perp}{qN_T(q)}\mathbf{e}_I^T(q_\parallel,q_\perp)\right.\right.\\&\left.\left.-\frac{q_\parallel}{qN_L(q)}\mathbf{e}_I^L(q_\parallel,q_\perp)\right]\right]\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)dq_\perp ,\end{aligned} \tag{2.44}$$

where

$$\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)=\int_{-\infty}^{\infty}\Theta(z')\mathbf{J}_{\text{ext}}(x',z';\omega)\times e^{-i(q_\parallel x'-q_\perp z')}dx'dz' \ . \tag{2.45}$$

Equations (2.43) [or (2.44)] show, as expected, that the propagator parts in Eqs. (2.38) and (2.39) are associated with the propagation of plane waves between the source and observation points. The last part of the field propagation from the surface to the point of observation lying in vacuum is described by a *single* plane wave vector $(q_\parallel,0,-q_\perp^0)$, whereas the first part of the propagation, i.e., from the point of observation inside the metal to the surface, is effected by a *coherent superposition* of plane waves. This superposition of plane waves is characterized by the fact that all the waves have the same wave-vector component $(q_\parallel)$ along the surface. The actual spectrum of wave-vector components $(-q_\perp)$ perpendicular to the surface depends on the structure of the conductivity response functions. The wave vector of a given mode in the spectrum is $(q_\parallel,0,-q_\perp)$. It readily appears from Eqs. (2.44) and (2.45) that the entire external current-density distribution in the metal domain contributes as source for the field propagation described by the propagator $\overleftrightarrow{G}^{><}(z,z')$. Furthermore, it is realized from the dyadic notation of Eq. (2.43) [or (2.44)], and from the fact that $\mathbf{e}_I^T(q_\parallel,q_\perp)$ and $\mathbf{e}_I^L(q_\parallel,q_\perp)$, are unit vectors perpendicular (T) and parallel (L) to the wave vector $(q_\parallel,0,-q_\perp)$ of the, towards the surface from the metal side, incident (I) mode in consideration that it is the transverse, via $\mathbf{e}_y\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)$ and $\mathbf{e}_I^T(q_\parallel,q_\perp)\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)$, and the longitudinal, via $\mathbf{e}_I^L(q_\parallel,q_\perp)\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp,\omega)$, parts of each of the plane-wave components of the Fourier-transformed external current density which act as sources for the field propagation described by $\overleftrightarrow{G}_{TT}^{><}(z,z')$ and $\overleftrightarrow{G}_{LT}^{><}(z,z')$, respectively. The transverse nature of the field received at the observation point, here placed in vacuum, is of course also accounted for in the structure of Eq. (2.44). A schematic illustration of the characteristics of the propagators $\overleftrightarrow{G}{}_{TT}^{><}$ and $\overleftrightarrow{G}{}_{LT}^{><}$ is shown in Fig. 2.

Now, we analyze the tensor-product structure of the propagator associated with the domain $z'<0, z>0$. From the results of Ref. 5 we obtain after some rewriting

$$\overleftrightarrow{G}{}^{<>}(z,z')=\overleftrightarrow{G}{}_{TT}^{<>}(z,z')+\overleftrightarrow{G}{}_{TL}^{<>}(z,z') , \tag{2.46}$$

where

$$\overleftrightarrow{G}{}_{TT}^{<>}(z,z')=\frac{1}{4\pi}e^{-iq_\perp^0 z'}\left[(1-r^s)\mathbf{e}_y\otimes\mathbf{e}_y\int_{-\infty}^{\infty}\frac{1+L(q_\parallel,\omega)q_\perp}{N_T(q)}e^{iq_\perp(z-0^+)}dq_\perp + \frac{\omega}{c_0 q_\perp^0}(1+r^p)\left[\int_{-\infty}^{\infty}\frac{q_\perp+M(q_\parallel,\omega)q^2}{qN_T(q)}\mathbf{e}_R^T(q_\parallel,q_\perp)e^{iq_\perp(z-0^+)}dq_\perp\right]\otimes\mathbf{e}_i\right] , \tag{2.47}$$

and

$$\overleftrightarrow{G}{}_{TL}^{<>}(z,z')=\frac{\omega(1+r^p)}{4\pi c_0 q_\perp^0}e^{-iq_\perp^0 z'}\left[\int_{-\infty}^{\infty}\frac{q_\parallel}{qN_L(q)}\mathbf{e}_R^L(q_\parallel,q_\perp)e^{iq_\perp(z-0^+)}dq_\perp\right]\otimes\mathbf{e}_i , \tag{2.48}$$

with the usual convention for the subscripts *TT* and *TL*.

The explicit expressions for $L(q_\parallel,\omega)$ and $M(q_\parallel,\omega)$ are given in Eqs. (C8) and (C10). For brevity, use has been made of the notation $\lim_{z'\to 0^+}\int F(q_\perp,z-z')dq_\perp \equiv \int F(q_\perp,z-0^+)dq_\perp$, also. For a *p*-polarized plane wave of wave vector $(q_\parallel,0,-q_\perp)$ incident (I) on the surface from the metal side the appropriate unit vectors for the description of the transverse and longitudinal parts of the electric field are $\mathbf{e}_I^T$ and $\mathbf{e}_I^L$, respectively [see Eq. (2.41) and (2.42)]. The corresponding transverse and longitudinal unit field vectors for the reflected (R) field [wave vector $(q_\parallel,0,q_\perp)$] are

$$\mathbf{e}_R^T(q_\parallel,q_\perp)=\frac{1}{q}(q_\perp,0,-q_\parallel) \tag{2.49}$$

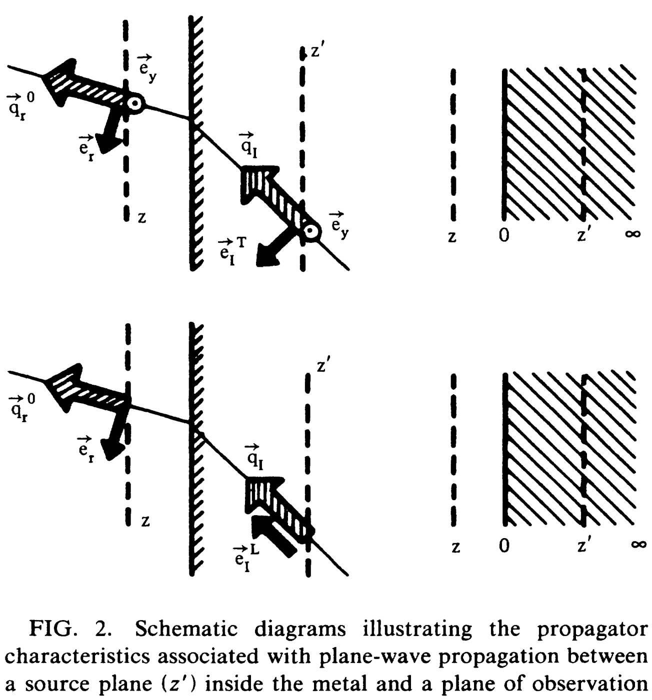

FIG. 2. Schematic diagrams illustrating the propagator characteristics associated with plane-wave propagation between a source plane (z') inside the metal and a plane of observation (z) in vacuum. In the upper panel is shown the contribution $\overleftrightarrow{G}{}_{TT}^{><}$ stemming from divergence-free waves in both metal and vacuum, and in the lower panel, the contribution $\overleftrightarrow{G}{}_{LT}^{><}$ associated with rotational-free and divergence-free wave propagation in metal and vacuum, respectively. The external current density in the hatched domains of space, shown to the right, contributes as source for the field radiation.

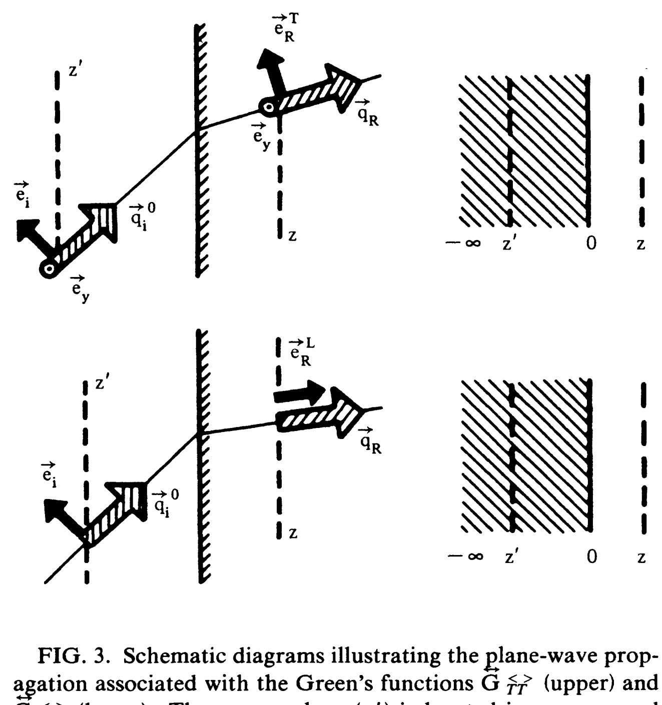

FIG. 3. Schematic diagrams illustrating the plane-wave propagation associated with the Green's functions $\overleftrightarrow{G}{}_{TT}^{<>}$ (upper) and $\overleftrightarrow{G}{}_{TL}^{<>}$ (lower). The source plane (z') is located in vacuum and the observation plane (z) inside the metal. In vacuum the field is divergence free and in the metal it is either divergence free (upper) or rotational free (lower). The distribution in space of the external current density sources contributing to the field radiation in this case is indicated by the hatched areas to the right of the figures.

and

$$\mathbf{e}_R^L(q_\parallel,q_\perp)=\frac{1}{q}(q_\parallel,0,q_\perp)\ , \tag{2.50}$$

respectively. It is obvious that just these unit vectors must occur in Eqs. (2.47) and (2.48). As one would expect, the propagator terms in Eqs. (2.47) and (2.48) are proportional to either the amplitude transmission coefficient $1-r^s$ or the transmission coefficient $1+r^p$. Further insight into the structure of $\overleftrightarrow{G}^{<>}(z,z')$ is obtained by Fourier analyzing Eq. (B2), utilizing for the propagator Eq. (B8) with the appropriate expressions in Eqs. (2.46)–(2.48) inserted. Thus, one finds

$$\begin{aligned}\mathbf{E}(q_\parallel,q_\perp,\omega)&=\mathbf{E}^{(0)}(q_\parallel,q_\perp,\omega)\\&\quad-\frac{i\mu_0\omega}{2}e^{-iq_\perp 0^+}\Bigg[(1-r^s)\frac{1+L(q_\parallel,\omega)q_\perp}{N_T(q)}\mathbf{e}_y\otimes\mathbf{e}_y\\&\quad+\frac{\omega(1+r^p)}{c_0q_\perp^0}\left[\frac{q_\perp+M(q_\parallel,\omega)q^2}{qN_T(q)}\mathbf{e}_R^T(q_\parallel,q_\perp)+\frac{q_\parallel}{qN_L(q)}\mathbf{e}_R^L(q_\parallel,q_\perp)\right]\otimes\mathbf{e}_i\Bigg]\cdot\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)\ .\end{aligned} \tag{2.51}$$

The equation above gives the relation between the Fourier amplitudes of the electric field $[\mathbf{E}(q_\parallel,q_\perp,\omega)]$ and the external current density $[\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)]$ belonging to the entire vacuum half space, cf. Eq. (2.34). Due to the transverse nature of electromagnetic fields *in vacuo,* only the transverse part of $\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)$ contributes to the electric field at the point of observation, as is obvious from the dyadic notation. The dyadic notation of Eq. (2.51) also demonstrates that the plane-wave component $(q_\parallel,0,q_\perp)$ of the field $[\mathbf{E}(q_\parallel,q_\perp,\omega)]$ has a transverse [via $\mathbf{e}_y$ and $\mathbf{e}_R^T(q_\parallel,q_\perp)$] as well as a longitudinal [via $\mathbf{e}_R^L(q_\parallel,q_\perp)$] component. One should be aware of the fact that a *single* plane-wave component of the external current density, say $\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)$, in general generates a *spectrum* of plane-wave components, $\mathbf{E}(q_\parallel,q_\perp,\omega)$, in the field inside the metal. All the waves in the spectrum have the same component of the wave vector along the surface, namely, $q_\parallel\mathbf{e}_x$. The z dependence of the electric field stemming from the "background" field $\mathbf{E}^{(0)}(z;q_\parallel,\omega)$ and from the external current density $\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)$ thus is given by

$$\mathbf{E}(z;q_\parallel,\omega)=\mathbf{E}^{(0)}(z;q_\parallel,\omega)+\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\gamma(q_\parallel,q_\perp,\omega)e^{iq_\perp z}dq_\perp\right]\cdot\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)\ , \tag{2.52}$$

where $\gamma(q_\parallel,q_\perp,\omega)$ is defined as the factor to $\mathbf{J}_{\text{ext}}^{(-\infty|0)}(q_\parallel,q_\perp^0,\omega)$ in Eq. (2.51). A schematic illustration of the propagation characteristics of $\overleftrightarrow{G}_{TT}^{<>}$ and $\overleftrightarrow{G}_{TL}^{<>}$ is shown in Fig. 3.

We close this section by a discussion of the tensor-product structure of the Green's tensor associated with the domain $z'>0$, $z>0$. Utilizing the divisions in Eqs. (2.15)–(2.18) and the result obtained in Ref. 5, one obtains after somewhat tedious but straightforward calculations

$$\overleftrightarrow{G}^{>>}(z,z')=\overleftrightarrow{D}_{TT}^{>>}(z-z')+\overleftrightarrow{D}_{LL}^{>>}(z-z')+\overleftrightarrow{I}_{TT}^{>>}(z,z')+\overleftrightarrow{I}_{TL}^{>>}(z,z')+\overleftrightarrow{I}_{LT}^{>>}(z,z')+\overleftrightarrow{I}_{LL}^{>>}(z,z')+\overleftrightarrow{g}_{LL}^{>>}(z-z')\ , \tag{2.53}$$

where

$$\begin{aligned}\overleftrightarrow{D}_{TT}^{>>}(z-z')&=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_R^T(q_\parallel,q_\perp)\otimes\mathbf{e}_R^T(q_\parallel,q_\perp)]\frac{e^{iq_\perp(z-z')}}{N_T(q)}dq_\perp\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}[\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_I^T(q_\parallel,q_\perp)\otimes\mathbf{e}_I^T(q_\parallel,q_\perp)]\frac{e^{iq_\perp(z'-z)}}{N_T(q)}dq_\perp\ ,\end{aligned} \tag{2.54}$$

and

$$\begin{aligned}\overleftrightarrow{D}_{LL}^{>>}(z-z')&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_R^L(q_\parallel,q_\perp)\otimes\mathbf{e}_R^L(q_\parallel,q_\perp)\frac{e^{iq_\perp(z-z')}}{N_L(q)}dq_\perp\\&=\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_I^L(q_\parallel,q_\perp)\otimes\mathbf{e}_I^L(q_\parallel,q_\perp)\frac{e^{iq_\perp(z'-z)}}{N_L(q)}dq_\perp\ .\end{aligned} \tag{2.55}$$

The two contributions to the propagator given above can be rewritten in an appealing way as follows:

$$\begin{aligned}\overleftrightarrow{D}_{TT}^{>>}(z-z')&=\frac{1}{2\pi}\mathbf{e}_y\otimes\mathbf{e}_y\int_{-\infty}^{\infty}\frac{e^{iq_\perp(z-z')}}{N_T(q)}dq_\perp+\frac{\Theta(z-z')}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_R^T(q_\parallel,q_\perp)\otimes\mathbf{e}_R^T(q_\parallel,q_\perp)\frac{e^{iq_\perp(z-z')}}{N_T(q)}dq_\perp\\&\quad+\frac{\Theta(z'-z)}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_I^T(q_\parallel,q_\perp)\otimes\mathbf{e}_I^T(q_\parallel,q_\perp)\frac{e^{iq_\perp(z'-z)}}{N_T(q)}dq_\perp\ ,\end{aligned} \tag{2.56}$$

and

$$\overleftrightarrow{D}^{>>}_{LL}(z-z')=\frac{\Theta(z-z')}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_R^L(q_\parallel,q_\perp)\otimes\mathbf{e}_R^L(q_\parallel,q_\perp)\frac{e^{iq_\perp(z-z')}}{N_L(q)}dq_\perp+\frac{\Theta(z'-z)}{2\pi}\int_{-\infty}^{\infty}\mathbf{e}_I^L(q_\parallel,q_\perp)\otimes\mathbf{e}_I^L(q_\parallel,q_\perp)\frac{e^{iq_\perp(z'-z)}}{N_L(q)}dq_\perp\ , \tag{2.57}$$

making use of the fact that $N_T(q)$ and $N_L(q)$ are even functions of $q_\perp$ and that the substitution $q_\perp\rightarrow -q_\perp$ implies $\mathbf{e}_I^T\rightarrow\mathbf{e}_R^T$ and $\mathbf{e}_I^L\rightarrow\mathbf{e}_R^L$. The explicit expressions for the remaining terms of the propagator $\overleftrightarrow{G}^{>>}(z,z')$ are

$$\begin{aligned}\overleftrightarrow{I}^{>>}_{TT}(z,z')=&\frac{1-r^s}{(2\pi)^2 i}\mathbf{e}_y\otimes\mathbf{e}_y\left[\int_{-\infty}^{\infty}(q_\perp^0-q_\perp)\frac{e^{iq_\perp(z-0^+)}}{N_T(q)}dq_\perp\right]\left[\int_{-\infty}^{\infty}\frac{e^{iq_\perp z'}}{N_T(q)}dq_\perp\right]\\&-\left[\frac{\omega}{2\pi c_0}\right]^2\frac{1+r^p}{iq_\perp^0}\left[\int_{-\infty}^{\infty}\mathbf{e}_R^T(q_\parallel,q_\perp)\frac{q_\perp-(c_0/\omega)^2q^2q_\perp^0}{qN_T(q)}e^{iq_\perp(z-0^+)}dq_\perp\right]\otimes\left[\int_{-\infty}^{\infty}\mathbf{e}_I^T(q_\parallel,q_\perp)\frac{q_\perp}{qN_T(q)}e^{iq_\perp z'}dq_\perp\right]\ ,\end{aligned} \tag{2.58}$$

$$\overleftrightarrow{I}^{>>}_{TL}(z,z')=-\left[\frac{\omega}{2\pi c_0}\right]^2\frac{1+r^p}{iq_\perp^0}\left[\int_{-\infty}^{\infty}\mathbf{e}_R^L(q_\parallel,q_\perp)\frac{q_\parallel e^{iq_\perp(z-0^+)}}{qN_L(q)}dq_\perp\right]\otimes\left[\int_{-\infty}^{\infty}\mathbf{e}_I^T(q_\parallel,q_\perp)\frac{q_\perp e^{iq_\perp z'}}{qN_T(q)}dq_\perp\right]\ , \tag{2.59}$$

$$\overleftrightarrow{I}^{>>}_{LT}(z,z')=\left[\frac{\omega}{2\pi c_0}\right]^2\frac{1+r^p}{iq_\perp^0}\left[\int_{-\infty}^{\infty}\mathbf{e}_R^T(q_\parallel,q_\perp)\frac{q_\perp-(c_0/\omega)^2q^2q_\perp^0}{qN_T(q)}e^{iq_\perp(z-0^+)}dq_\perp\right]\otimes\left[\int_{-\infty}^{\infty}\mathbf{e}_I^L(q_\parallel,q_\perp)\frac{q_\parallel}{qN_L(q)}e^{iq_\perp z'}dq_\perp\right]\ , \tag{2.60}$$

$$\overleftrightarrow{I}^{>>}_{LL}(z,z')=\left[\frac{\omega}{2\pi c_0}\right]^2\frac{1+r^p}{iq_\perp^0}\left[\int_{-\infty}^{\infty}\mathbf{e}_R^L(q_\parallel,q_\perp)\frac{q_\parallel e^{iq_\perp(z-0^+)}}{qN_L(q)}dq_\perp\right]\otimes\left[\int_{-\infty}^{\infty}\mathbf{e}_I^L(q_\parallel,q_\perp)\frac{q_\parallel e^{iq_\perp z'}}{qN_L(q)}dq_\perp\right]\ , \tag{2.61}$$

and

$$\overleftrightarrow{g}^{>>}_{LL}(z-z')=\left[\frac{c_0}{\omega}\right]^2\epsilon^{-1}(\omega)\delta(z-z')\mathbf{e}_z\otimes\mathbf{e}_z\ . \tag{2.62}$$

In Eq. (2.62) $\epsilon(\omega)=1+i\lim_{q\rightarrow 0}\sigma_T(q,\omega)/(\epsilon_0\omega)$ is the relative dielectric constant in the long-wavelength limit. Note that $\lim_{q\rightarrow 0}\sigma_T(q,\omega)=\lim_{q\rightarrow 0}\sigma_L(q,\omega)$. The subscripts, TT, LL, TL, and LT added to the terms in Eq. (2.53) follow the convention of Eqs. (2.15)–(2.18). The propagators $\overleftrightarrow{D}^{>>}_{TT}(z-z')$ and $\overleftrightarrow{D}^{>>}_{LL}(z-z')$ of Eqs. (2.56) and (2.57) describe the direct (D) propagation of the electromagnetic field between the source and observation points, and $\overleftrightarrow{D}^{>>}_{TT}(z-z')$ and $\overleftrightarrow{D}^{>>}_{LL}(z-z')$ take care of the transfer of the divergence-free (TT) and the rotational-free (LL) parts of the field, respectively. For the p-polarized part of the field one conveniently distinguishes between the cases $z'<z$ and $z'>z$. Thus, for $z'<z$ the appropriate dyadics are those associated with the "reflected (R)" field, i.e., $\mathbf{e}_R^T\otimes\mathbf{e}_R^T$ and $\mathbf{e}_R^L\otimes\mathbf{e}_R^L$. For $z'>z$ the relevant dyadics, $\mathbf{e}_I^T\otimes\mathbf{e}_I^T$ and $\mathbf{e}_I^L\otimes\mathbf{e}_I^L$, are those belonging to the "incident (I)" field. The direct terms are illustrated in Fig. 4 in a way that supports the considerations above. The Green's functions $\overleftrightarrow{I}^{>>}_{TT}(z,z')$, $\overleftrightarrow{I}^{>>}_{TL}(z,z')$, and $\overleftrightarrow{I}^{>>}_{LL}(z,z')$ represent the indirect contributions to the field propagation between the source and observation points. Various combinations of transfer via the divergence-free and the rotational-free parts of the field occur in these propagator terms which all incorporate a field reflection at the metal-vacuum surface. Thus, $\overleftrightarrow{I}^{>>}_{TT}(z,z')$ and $\overleftrightarrow{I}^{>>}_{LL}(z,z')$ describe indirect processes where either the divergence-free (TT) or the rotational-free (LL) part of the electromagnetic field is responsible for the transfer process *both before and after* the reflection at the surface. The tensor $\overleftrightarrow{I}^{>>}_{TL}(z,z')$ describes a situation where a divergence-free (T) field traveling towards the surface is combined with a rotational-free (L) field traveling away from the surface after reflection. In the propagation characteristics associated with the Green's tensor $\overleftrightarrow{I}^{>>}_{LT}(z,z')$ the incident field is rotational free (L) and the reflected field divergence free (T). A schematic illustration of the characteristics of the four indirect propagator terms are shown in Fig. 5. With the foregoing analyses of this section in mind we need not a lengthy discussion of the physics displayed in the structure of Eqs. (2.58)–(2.61). Hence, let us just make two comments. First, it is obvious from the appearance of the integrals that *both* the incident (I) and the reflected (R) field consists of coherent superpositions of plane-wave components. Secondly, it appears from the unit field vectors drawn in Fig. 5 that the unit vectors occurring in the

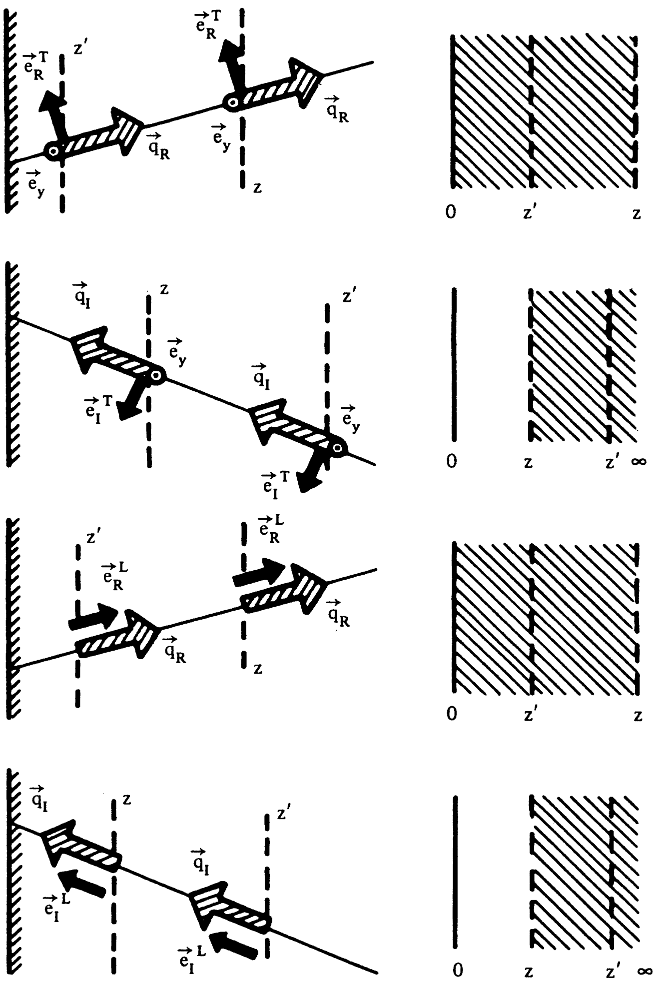

FIG. 4. Schematic diagrams illustrating the divergence-free (two upper diagrams) and the rotational-free (two lower diagrams) plane-wave propagation associated with the propagators $\overleftrightarrow{D}^{>>}_{TT}$ and $\overleftrightarrow{D}^{>>}_{LL}$. The propagation between the source (z') and observation (z) planes both located inside the metal is direct. Both of the cases $z'<z$ and $z'>z$ are shown. The distributions of external current-density sources contributing to the radiation in the individual cases are indicated by the hatched domains to the right of the actual diagrams.

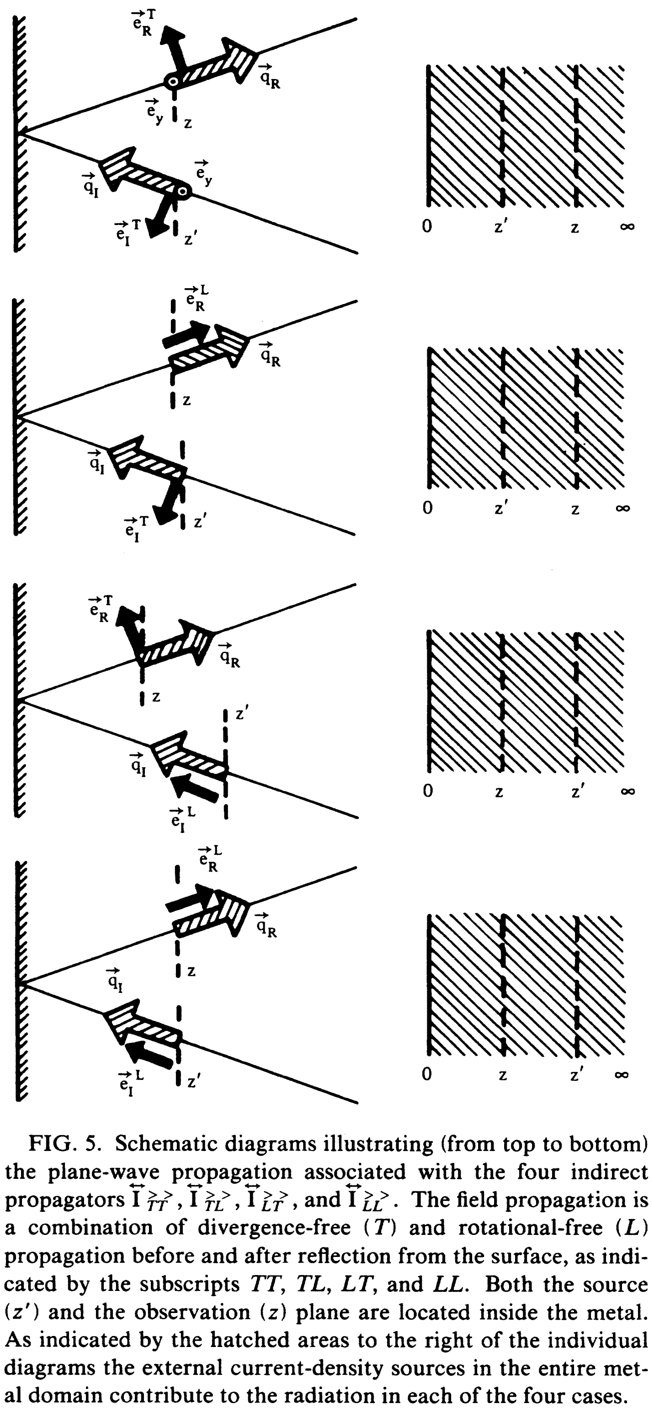

FIG. 5. Schematic diagrams illustrating (from top to bottom) the plane-wave propagation associated with the four indirect propagators $\overleftrightarrow{I}^{>>}_{TT}$, $\overleftrightarrow{I}^{>>}_{TL}$, $\overleftrightarrow{I}^{>>}_{LT}$, and $\overleftrightarrow{I}^{>>}_{LL}$. The field propagation is a combination of divergence-free (T) and rotational-free (L) propagation before and after reflection from the surface, as indicated by the subscripts TT, TL, LT, and LL. Both the source (z') and the observation (z) plane are located inside the metal. As indicated by the hatched areas to the right of the individual diagrams the external current-density sources in the entire metal domain contribute to the radiation in each of the four cases.

different tensor products of Eqs. (2.58)–(2.61) are the natural ones.

It is instructive to investigate the relation between a given Fourier amplitude of the field at the observation point and the weighted Fourier spectrum of the external current density contributing to it. Thus, for a plane-wave component of the field propagating *towards* the surface one obtains

$$\mathbf{E}(q_\parallel,-q_\perp,\omega)=\mathbf{E}^{(0)}(q_\parallel,-q_\perp,\omega)-i\mu_0\omega\left[\frac{\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_I^T(q_\parallel,q_\perp)\otimes\mathbf{e}_I^T(q_\parallel,q_\perp)}{N_T(q)}+\frac{\mathbf{e}_I^L(q_\parallel,q_\perp)\otimes\mathbf{e}_I^L(q_\parallel,q_\perp)}{N_L(q)}\right]\cdot\mathbf{J}_{\rm ext}^{(z\mid\infty)}(q_\parallel,-q_\perp,\omega)\ , \tag{2.63}$$

with

$$\mathbf{J}_{\rm ext}^{(z\mid\infty)}(q_\parallel,-q_\perp,\omega)=\int_{-\infty}^{\infty}\Theta(z'-z)\mathbf{J}_{\rm ext}(x',z';\omega)e^{-i(q_\parallel x'-q_\perp z')}dx'dz',\quad z>0\ . \tag{2.64}$$

To derive the result in Eq. (2.63) we have split the s-polarized contribution to $\overleftrightarrow{D}{}^{>>}_{TT}(z-z')$ in two by means of the relation $1=\Theta(z-z')+\Theta(z'-z)$, and utilized that $N_T(q)$ is an even function of $q_\perp$. It follows from Eq. (2.64) that only the external current density lying to the right of the observation point (i.e., $z'>z$) contributes in the present case, cf. Fig. 4. Also, it appears from Eq. (2.63) that $\mathbf{E}(q_\parallel,q_\perp,\omega)$ is related to a *single* (and the same) Fourier component in the current density, namely, $\mathbf{J}_{\text{ext}}^{(z|\infty)}(q_\parallel,-q_\perp,\omega)$. That this must necessarily be so is a direct consequence of the fact that only the direct term [Eqs. (2.56) and (2.57)] can contribute to the field propagation towards surface. For a plane-wave component of the field propagating *away* from the surface the relation to the external current density is a bit more complicated. Thus, the result is the following:

$$\mathbf{E}(q_\parallel,q_\perp,\omega)=\mathbf{E}^{(0)}(q_\parallel,q_\perp,\omega)+\mathbf{E}^{D}(q_\parallel,q_\perp,\omega)+\mathbf{E}^{I}(q_\parallel,q_\perp,\omega)\ , \tag{2.65}$$

where $\mathbf{E}^{D}(q_\parallel,q_\perp,\omega)$ and $\mathbf{E}^{I}(q_\parallel,q_\perp,\omega)$ are the fields stemming from direct (D) and indirect (I) processes. The direct contribution

$$\mathbf{E}^{D}(q_\parallel,q_\perp,\omega)=-i\mu_0\omega\left[\frac{\mathbf{e}_y\otimes\mathbf{e}_y+\mathbf{e}_R^T(q_\parallel,q_\perp)\otimes\mathbf{e}_R^T(q_\parallel,q_\perp)}{N_T(q)}+\frac{\mathbf{e}_R^L(q_\parallel,q_\perp)\otimes\mathbf{e}_R^L(q_\parallel,q_\perp)}{N_L(q)}\right]\cdot\mathbf{J}_{\text{ext}}^{(0|z)}(q_\parallel,q_\perp,\omega) \tag{2.66}$$

relates the field to a *single* Fourier component of the external current density lying to the left of the observation point ($z'<z$) and inside the metal ($z'>0$), i.e.,

$$\mathbf{J}_{\text{ext}}^{(0|z)}(q_\parallel,q_\perp,\omega)=\int_{-\infty}^{\infty}\Theta(z')\Theta(z-z')\mathbf{J}_{\text{ext}}(x',z';\omega)e^{-i(q_\parallel x'+q_\perp z')}dx'dz'\ . \tag{2.67}$$

The indirect contribution given by

$$\begin{aligned}\mathbf{E}^{I}(q_\parallel,q_\perp,\omega)=-\mu_0\omega e^{-iq_\perp 0^+}\Bigg[&\frac{(1-r^s)(q_\perp^0-q_\perp)}{N_T(q)}\mathbf{e}_y\otimes\mathbf{e}_y\cdot\int_{-\infty}^{\infty}\frac{\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp',\omega)}{N_T(q')}dq_\perp'\\&+\left[\frac{\omega}{c_0}\right]^2\frac{1+r^p}{q_\perp^0}\left[\frac{q_\perp-(c_0/\omega)^2q^2q_\perp^0}{qN_T(q)}\mathbf{e}_R^T(q_\parallel,q_\perp)+\frac{q_\parallel}{qN_L(q)}\mathbf{e}_R^L(q_\parallel,q_\perp)\right]\\&\otimes\int_{-\infty}^{\infty}\left[\frac{q_\parallel}{q'N_L(q')}\mathbf{e}_I^L(q_\parallel,q_\perp')-\frac{q_\perp'}{q'N_T(q')}\mathbf{e}_I^T(q_\parallel,q_\perp')\right]\cdot\mathbf{J}_{\text{ext}}^{(0|\infty)}(q_\parallel,-q_\perp',\omega)dq_\perp'\Bigg]\end{aligned} \tag{2.68}$$

relates, as expected, the field to a weighted Fourier spectrum of the external current density in the entire metal domain. For brevity, we have introduced $q'=[q_\parallel^2+(q_\perp')^2]^{1/2}$ in Eq. (2.68).

The singular behavior of the propagator $\overleftrightarrow{G}{}^{>>}(z,z')$ at $z=z'$ leads to the following (in z) local relation between field and external current density:

$$\mathbf{E}(z;q_\parallel,\omega)=\mathbf{E}^{(0)}(z;q_\parallel,\omega)-\frac{i}{\epsilon_0\omega}\epsilon^{-1}(\omega)\mathbf{e}_z\otimes\mathbf{e}_z\cdot\mathbf{J}_{\text{ext}}(z;q_\parallel,\omega)\ . \tag{2.69}$$

We conclude by pointing out that Eqs. (2.23)–(2.27), (2.36)–(2.38), (2.46)–(2.48), and (2.53) and (2.56)–(2.62) constitute the main result of this section.

III. THE PROPAGATOR IN THE POLARITON PLUS PLASMON-POLE APPROXIMATION

The screened propagator described in the preceding section incorporates optical phenomena associated with collective polariton and plasmon excitations and with noncollective electron-hole pair excitations. On the basis of the Lindhard-Mermin dielectric functions,[13,14] the collective and the single-particle excitations can be identified by studying the analytical properties of the dielectric functions in the complex $q_\perp$ plane, as is well known.[15] In a number of investigations in nonlocal optics, qualitative correct answers can be obtained retaining only the collective excitations in the description.[2] Mathematically, the polariton and plasmon excitations appear via the pole structure of the dielectric functions. Being interested in the collective excitations only it is often advantageous to use simpler response functions which do not contain the electron-hole pair excitations, e.g., the hydrodynamic response functions.[2] In the present section we shall study the electromagnetic propagator in the collective-mode approximation, and make contact with the tensor-product Green's-function formalism developed quite recently by Sipe[7] within the framework of local optics.

A. Residues and reflection coefficients

To determine the electromagnetic propagator in the polariton plus plasmon-pole approximation, the integrals along the real $q_\perp$ axis appearing in Eqs. (2.37), (2.38), (2.47), (2.48), and (2.56)–(2.61) are performed by contour integration in the complex $q_\perp$ plane. The locations $\kappa_\perp^T$ and $\kappa_\perp^L$ of the polariton (T) and plasmon (L) poles are determined by the dispersion relations

$$N_T(q_\parallel,\kappa_\perp^T,\omega)=0 \tag{3.1}$$

and

$$N_L(q_\parallel,\kappa_\perp^L,\omega)=0\ , \tag{3.2}$$

respectively. The residue ($\mathcal{R}_T^+$) associated with the polariton pole is given by [see Eqs. (C4) and (C6)]

$$\mathcal{R}_T^+=-\frac{1}{2\kappa_\perp^T} \tag{3.3}$$

if single-particle excitations are neglected. To obtain the residue

$$\mathcal{R}_L^+=\lim_{q_\perp\to\kappa_\perp^L}\left[\frac{q_\perp-\kappa_\perp^L}{N_L(q_\parallel,q_\perp,\omega)}\right] \tag{3.4}$$

for the plasmon pole, we take as a starting point the exact result

$$\frac{2}{\pi}\lim_{z'\to 0^+}\int_0^\infty\frac{1}{N_L(q)}\frac{\sin q_\perp z'}{q_\perp}dq_\perp=\left[\frac{c_0}{\omega}\right]^2 . \tag{3.5}$$

The relation in Eq. (3.5) is readily obtained by noting that $N_L(q)\to(\omega/c_0)^2$ for $q_\perp\to\infty$ and by comparison to Eq. (C4). Since $N_L(q)$ is an even function of $q_\perp$, Eq. (3.5) can be rewritten as follows:

$$\left[\frac{c_0}{\omega}\right]^2=\frac{1}{\pi i}\lim_{z'\to 0^+}\int_{-\infty}^{\infty}\frac{e^{iq_\perp z'}}{q_\perp N_L(q)}dq_\perp\ . \tag{3.6}$$

Now, the right-hand side of Eq. (3.6) can be evaluated by contour integration (neglecting eventual branch cuts) noting that the poles are located at $q_\perp=0$ and $q_\perp=\pm\kappa_\perp^L$ in the complex $q_\perp$ plane. Choosing the contour as shown in Fig. 6 one obtains

$$\left[\frac{c_0}{\omega}\right]^2=\frac{2\mathcal{R}_L^+}{\kappa_\perp^L}+\frac{1}{N_L(q_\parallel,q_\perp=0,\omega)}\ . \tag{3.7}$$

Since $N_L(q_\parallel,q_\perp,\omega)=(\omega/c_0)^2\epsilon_L(q_\parallel,q_\perp,\omega)$, where $\epsilon_L=1+i\sigma_L/(\epsilon_0\omega)$ is the longitudinal dielectric response function, one finds for the plasmon residue (assuming no particle-hole excitations)

$$\mathcal{R}_L^+=\frac{1}{2}\left[\frac{c_0}{\omega}\right]^2\kappa_\perp^L[1-\epsilon_L^{-1}(q_\parallel,\omega)]\ , \tag{3.8}$$

where $\epsilon_L(q_\parallel,\omega)=\epsilon_L(q_\parallel,q_\perp=0,\omega)$. For the hydrodynamic model, in which particle-hole excitations are absent *a priori*, and where

$$N_L(q_\parallel,q_\perp,\omega)=(\omega/c_0)^2[q_\perp^2-(\kappa_\perp^L)^2]/[q_\perp^2-(\kappa_\perp^L)^2-\omega_p^2/D(\omega)]\ ,$$

$D(\omega)$ being the diffusion coefficient and ω_p the plasma frequency, one gets via Eq. (3.8) the exact and well-known result

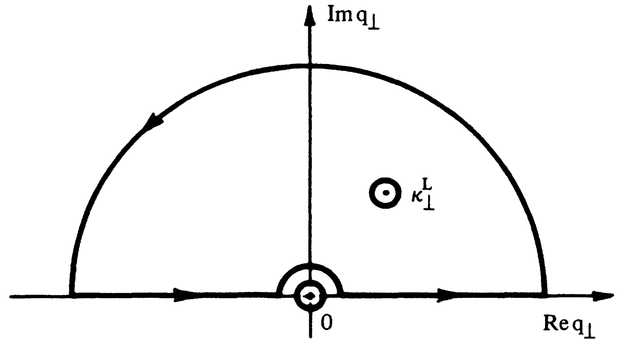

FIG. 6. Contour in the complex $q_\perp$ plane used to calculate the plasmon residue $\mathcal{R}_L^+$ in terms of the plasmon wave-vector component $\kappa_\perp^L$ and the longitudinal response function $\epsilon_L(q_\parallel,\omega)$. Poles are located at $q_\perp=0$ and $\kappa_\perp^L$ and branch cuts are neglected.

$$\mathcal{R}_L^+=-\left[\frac{\omega_p}{\omega}\right]^2\frac{c_0^2}{D(\omega)}\frac{1}{2\kappa_\perp^L}\ . \tag{3.9}$$

The amplitude reflection coefficients are determined by noting that $L(q_\parallel,\omega)$ and $M(q_\parallel,\omega)$ in Eqs. (C8) and (C10) can be written as

$$L(q_\parallel,\omega)=\frac{i}{\pi}\lim_{z'\to 0^+}\int_{-\infty}^{\infty}\frac{e^{iq_\perp z'}}{N_T(q)}dq_\perp \tag{3.10}$$

and

$$M(q_\parallel,\omega)=\frac{i}{\pi}\lim_{z'\to 0^+}\int_{-\infty}^{\infty}\left[\frac{q_\perp^2}{N_T(q)}+\frac{q_\parallel^2}{N_L(q)}\right]\frac{e^{iq_\perp z'}}{q^2}dq_\perp\ . \tag{3.11}$$

Contour integration, and use of Eqs. (3.3) and (3.9) in turn give

$$L(q_\parallel,\omega)=\frac{1}{\kappa_\perp^T} \tag{3.12}$$

and

$$M(q_\parallel,\omega)=\frac{\kappa_\perp^T}{\kappa_T^2}+\left[\frac{c_0q_\parallel}{\omega\kappa_L}\right]^2\kappa_\perp^L[\epsilon_L^{-1}(q_\parallel,\omega)-1]\ , \tag{3.13}$$

noting that the integrand in Eq. (3.11) has no poles at $q_\perp=\pm iq_\parallel$ ($q=0$). For brevity, we have introduced $\kappa_T^2=q_\parallel^2+(\kappa_\perp^T)^2$ and $\kappa_L^2=q_\parallel^2+(\kappa_\perp^L)^2$, in Eq. (3.13). By inserting Eqs. (3.12) and (3.13) into (C7) and (C9), respectively, one obtains the following expression for the amplitude reflection coefficients for *s*- and *p*-polarized light:

$$r^s(q_\parallel,\omega)=\frac{q_\perp^0-\kappa_\perp^T}{q_\perp^0+\kappa_\perp^T} \tag{3.14}$$

and

$$r^p(q_\parallel,\omega)=\frac{q_\perp^0\epsilon_T(\kappa_T,\omega)-\kappa_\perp^T-(q_\parallel/\kappa_L)^2\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]}{q_\perp^0\epsilon_T(\kappa_T,\omega)+\kappa_\perp^T+(q_\parallel/\kappa_L)^2\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]}\ , \tag{3.15}$$

where the transverse dielectric response function $\epsilon_T(\kappa_T,\omega)=1+i\sigma_T(\kappa_T,\omega)/(\epsilon_0\omega)$ has been introduced via the dispersion relation

$$\frac{c_0}{\omega}\kappa_T=\epsilon_T^{1/2}(\kappa_T,\omega)\ . \tag{3.16}$$

Note that the *form* of r^s is as in local optics. If one neglects the plasmon mode which contributes only to r^p also the *form* of r^p is as in the local approach. One should also notice the $r^p=-1$, i.e., total reflection, if only the rotational-free part of the field in the metal was considered. This is obvious since no *radiative* electromagnetic energy can be carried by a pure plasmon field in linear theory, cf. Ref. 8.

B. Damped-wave picture of the propagator

We now determine the explicit expressions for the different propagator terms in the pole approximation. For the terms $\overleftrightarrow{D}{}^{><}_{TT}(z-z')$ and $\overleftrightarrow{I}{}^{><}_{TT}(z+z')$ the correct expressions are given by Eqs. (2.25) and (2.26) provided we insert the results in Eqs. (3.14) and (3.15) for the reflection coefficients.

After contour integration along an expanding semicircle in the upper half (since $z'>0$) of the complex $q_\perp$ plane, use of Eqs. (3.3), (3.14)–(3.16), and a certain amount of algebraic manipulations, one obtains

$$\overleftrightarrow{G}{}^{><}_{TT}(z,z')=\frac{1}{2i\kappa_\perp^T}e^{i(\kappa_\perp^T z'-q_\perp^0 z)}\left[(1-r^s)\mathbf{e}_y\otimes\mathbf{e}_y+\frac{\epsilon_T^{1/2}(\kappa_T,\omega)(1-r^p)\mathbf{e}_r\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)}{1+\dfrac{\kappa_\perp^L}{\kappa_\perp^T}\left[\dfrac{q_\parallel}{\kappa_L}\right]^2\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]}\right]\ . \tag{3.17}$$

Note that a complex unit vector $\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)=(-\kappa_\perp^T,0,-q_\parallel)/[q_\parallel^2+(\kappa_\perp^T)^2]^{1/2}$ occurs in the dyadic product of Eq. (3.17). It is possible of course to write the expression for $\overleftrightarrow{G}{}^{><}_{TT}(z,z')$ in a variety of equivalent ways. Here, we have chosen the form in Eq. (3.17) because of its close relation to the adequate form in local optics, cf. the discussion in Sec. III C. The propagator $\overleftrightarrow{G}{}^{><}_{LT}(z,z')$ describing the subsequent plasmon and polariton propagation becomes after contour integration, use of Eqs. (3.8) and (3.15), and a little algebra

$$\overleftrightarrow{G}{}^{><}_{LT}(z,z')=e^{i(\kappa_\perp^L z'-q_\perp^0 z)}\mathbf{e}_r\otimes\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)\frac{c_0 q_\parallel\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[1-\epsilon_L^{-1}(q_\parallel,\omega)]}{i\omega\kappa_L\left[q_\perp^0\epsilon_T(\kappa_T,\omega)+\kappa_\perp^T+\left[\dfrac{q_\parallel}{\kappa_L}\right]^2\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]\right]}\ , \tag{3.18}$$

with a complex unit vector $\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)=(q_\parallel,0,-\kappa_\perp^L)/[q_\parallel^2+(\kappa_\perp^L)^2]^{1/2}$.

Using the same procedure as outlined above one obtains for the propagators referring to the domain $z'<0$, $z>0$ [Eqs. (2.47) and (2.48)] the following result in the collective-mode approximation:

$$\overleftrightarrow{G}{}^{<>}_{TT}(z,z')=\frac{1}{2iq_\perp^0}e^{i(\kappa_\perp^T z-q_\perp^0 z')}\left\{(1+r^s)\mathbf{e}_y\otimes\mathbf{e}_y+\frac{1+r^p}{\epsilon_T^{1/2}(\kappa_T,\omega)}\left[1+\frac{1}{2}\frac{\kappa_\perp^L}{\kappa_\perp^T}\left[\frac{q_\parallel}{\kappa_L}\right]^2\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]\right]\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_i\right\}, \tag{3.19}$$

and

$$\overleftrightarrow{G}{}^{<>}_{TL}(z,z')=e^{i(\kappa_\perp^L z-q_\perp^0 z')}\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)\otimes\mathbf{e}_i\frac{c_0 q_\parallel\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]}{2i\omega\kappa_L\left[q_\perp^0\epsilon_T(\kappa_T,\omega)+\kappa_\perp^T+\left[\dfrac{q_\parallel}{\kappa_L}\right]^2\kappa_\perp^L\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]\right]}\ , \tag{3.20}$$

with the complex unit vectors $\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)=(\kappa_\perp^T,0,-q_\parallel)/[q_\parallel^2+(\kappa_\perp^T)^2]^{1/2}$ and $\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)=(q_\parallel,0,\kappa_\perp^L)/[q_\parallel^2+(\kappa_\perp^L)^2]^{1/2}$. In passing, one should notice the close relation between the expressions for $\overleftrightarrow{G}{}^{><}_{LT}$ and $\overleftrightarrow{G}{}^{<>}_{TL}$.

To derive the equations for the direct contributions $\overleftrightarrow{D}{}^{>>}_{TT}(z-z')$ and $\overleftrightarrow{D}{}^{>>}_{LL}(z-z')$ in the pole approximation, the contour integration must be done in the upper half plane for $z>z'$ and in the lower half-plane for $z<z'$. Hence, as a result of the residue calculation one obtains

$$\overleftrightarrow{D}{}^{>>}_{TT}(z-z')=\frac{1}{2i\kappa_\perp^T}[\mathbf{e}_y\otimes\mathbf{e}_y e^{i\kappa_\perp^T|z-z'|}+\Theta(z-z')\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)e^{i\kappa_\perp^T(z-z')}$$
$$+\Theta(z'-z)\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)e^{i\kappa_\perp^T(z'-z)}]\ , \tag{3.21}$$

and

$$\overleftrightarrow{\mathrm{D}}{}^{>>}_{LL}(z-z')=\frac{i}{2}\left[\frac{c_0}{\omega}\right]^2\kappa_\perp^L[1-\epsilon_L^{-1}(q_\parallel,\omega)]$$
$$\times[\Theta(z-z')\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)\otimes\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)e^{i\kappa_\perp^L(z-z')}+\Theta(z'-z)\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)\otimes\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)e^{i\kappa_\perp^L(z'-z)}]\ . \tag{3.22}$$

Referring to Fig. 4, the physical interpretation of the results in Eqs. (3.21) and (3.22) is obvious.

For the indirect terms in Eqs. (2.58)–(2.61) we find the formulas

$$\overleftrightarrow{\mathrm{I}}{}^{>>}_{TT}(z,z')=-\frac{1}{2i\kappa_\perp^T}e^{i\kappa_\perp^T(z+z')}\left[r^s\mathbf{e}_y\otimes\mathbf{e}_y+\frac{q_\perp^0\epsilon_T(\kappa_T,\omega)-\kappa_\perp^T}{q_\perp^0\epsilon_T(\kappa_T,\omega)+\kappa_\perp^T+\kappa_\perp^L\left[\frac{q_\parallel}{\kappa_L}\right]^2\epsilon_T(\kappa_T,\omega)[\epsilon_L^{-1}(q_\parallel,\omega)-1]}\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)\right]\ , \tag{3.23}$$

and

$$\overleftrightarrow{\mathrm{I}}{}^{>>}_{TL}(z,z')=\frac{iq_\parallel\kappa_\perp^L(1+r^p)[1-\epsilon_L^{-1}(q_\parallel,\omega)]}{4q_\perp^0\kappa_L\kappa_T}\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)e^{i(\kappa_\perp^Lz+\kappa_\perp^Tz')}\ , \tag{3.24}$$

$$\overleftrightarrow{\mathrm{I}}{}^{>>}_{LT}(z,z')=\frac{iq_\parallel[q_\perp^0\epsilon_T(\kappa_T,\omega)-\kappa_\perp^T]\kappa_\perp^L(1+r^p)[1-\epsilon_L^{-1}(q_\parallel,\omega)]}{4q_\perp^0\kappa_L\kappa_T\kappa_\perp^T}\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)e^{i(\kappa_\perp^Tz+\kappa_\perp^Lz')}\ , \tag{3.25}$$

$$\overleftrightarrow{\mathrm{I}}{}^{>>}_{LL}(z,z')=\frac{i}{q_\perp^0}(1+r^p)\left[\frac{c_0q_\parallel\kappa_\perp^L[1-\epsilon_L^{-1}(q_\parallel,\omega)]}{2\omega\kappa_L}\right]^2\mathbf{e}_R^L(q_\parallel,\kappa_\perp^L)\otimes\mathbf{e}_I^L(q_\parallel,\kappa_\perp^L)e^{i\kappa_\perp^L(z+z')}\ . \tag{3.26}$$

In the following subsection, we shall consider the electromagnetic propagator in the regime of local optics, and make contact to a recent work of Sipe.[7] In the local limit the plasmon contribution vanishes and only the propagator terms $\overleftrightarrow{\mathrm{D}}{}^{<<}_{TT}$, $\overleftrightarrow{\mathrm{I}}{}^{<<}_{TT}$, $\overleftrightarrow{\mathrm{G}}{}^{><}_{TT}$, $\overleftrightarrow{\mathrm{G}}{}^{<>}_{TT}$, $\overleftrightarrow{\mathrm{D}}{}^{>>}_{TT}$, and $\overleftrightarrow{\mathrm{I}}{}^{>>}_{TT}$ survive.

C. Propagator structure in the local regime

To write the electromagnetic propagator in a physically appealing way in the local regime we consider the elementary Fresnel coefficient identities derived in local optics, i.e.,

$$r=-R\ , \tag{3.27}$$

$$tT-rR=1\ , \tag{3.28}$$

where r and t are the amplitude reflection and transmission coefficients, respectively, for light incident on the boundary from, say, the vacuum side, and R and T are the amplitude reflection and transmission coefficients for light incident from the metal side. We have omitted the superscript s or p on the coefficients since the Fresnel coefficient identities hold for both state of polarization. In terms of the reflection coefficients r^s and r^p, the transmission coefficients from the vacuum side are

$$t^s=1+r^s\ , \tag{3.29}$$

$$t^p=\epsilon^{-1/2}(\omega)(1+r^p)\ . \tag{3.30}$$

The transmission coefficient from the reverse side, obtained by combining Eqs. (3.27)–(3.30), can be written as

$$T^s=1-r^s\ , \tag{3.31}$$

$$T^p=\epsilon^{1/2}(\omega)(1-r^p)\ . \tag{3.32}$$

Utilizing Eqs. (3.29)–(3.32), the propagators $\overleftrightarrow{\mathrm{G}}{}^{><}_{TT}$ and $\overleftrightarrow{\mathrm{G}}{}^{<>}_{TT}$ in Eqs. (3.17) and (3.19) take the following form:

$$\overleftrightarrow{\mathrm{G}}{}^{><}_{TT}(z,z')=\frac{1}{2i\kappa_\perp^T}e^{i(\kappa_\perp^Tz'-q_\perp^0z)}$$
$$\times[T^s\mathbf{e}_y\otimes\mathbf{e}_y+T^p\mathbf{e}_r\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)]\ , \tag{3.33}$$

and

$$\overleftrightarrow{\mathrm{G}}{}^{<>}_{TT}(z,z')=\frac{1}{2iq_\perp^0}e^{i(\kappa_\perp^Tz-q_\perp^0z')}$$
$$\times[t^s\mathbf{e}_y\otimes\mathbf{e}_y+t^p\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_i]\ , \tag{3.34}$$

utilizing the appropriate local-limit value of $\kappa_\perp^T$. The *forms* of $\overleftrightarrow{\mathrm{D}}{}^{<<}_{TT}$, $\overleftrightarrow{\mathrm{I}}{}^{<<}_{TT}$, and $\overleftrightarrow{\mathrm{D}}{}^{>>}_{TT}$ in the local limit are as in Eqs. (2.25), (2.26), and (3.21), and the explicit expressions need not be rewritten. Finally, the local-limit expression for $\overleftrightarrow{\mathrm{I}}{}^{>>}_{TT}$ is readily obtained via Eqs. (3.15), (3.23), and (3.27). Thus,

$$\overleftrightarrow{\mathrm{I}}{}^{>>}_{TT}(z,z')=\frac{1}{2i\kappa_\perp^T}e^{i\kappa_\perp^T(z+z')}$$
$$\times[R^s\mathbf{e}_y\otimes\mathbf{e}_y+R^p\mathbf{e}_R^T(q_\parallel,\kappa_\perp^T)\otimes\mathbf{e}_I^T(q_\parallel,\kappa_\perp^T)]\ . \tag{3.35}$$

The expressions for the dyadic propagators obtained in Eqs. (3.33)–(3.35), and in the local-limit version of Eqs. (2.25), (2.26), and (3.21), are in agreement with those determined by Sipe,[7] recently. For a detailed account of the local Green's-function formalism, and examples of its application in linear, local optics, the reader is referred to the work of Sipe.[7]

IV. SOME APPLICATIONS OF THE FORMALISM

In this terminating section, I shall outline how the present propagator theory can be useful for investigations within the domain of nonlocal optics. I shall limit myself to a description of three characteristic applications. Further examples are given in Ref. 6. In the work on surface linear-response functions by Cottam and Maradudin[16] the need for an electromagnetic Green's tensor for some particular choice of the nonlocal response tensor is stressed, and some types of studies are mentioned where the nonlocal propagator would be especially useful.

First of all, let us simplify the general expression for the Green's tensor given in Eq. (2.2). Thus, by introducing polar coordinates, i.e., $(q_{\parallel,x},q_{\parallel,y})\Longrightarrow(q_{\parallel},\alpha)$, and placing the polar axis along the direction given by $\mathbf{R}=\mathbf{r}_{\parallel}-\mathbf{r}'_{\parallel}$, one can easily carry out the angular integration over α. Doing this, one obtains

$$\overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)=\overleftrightarrow{G}(R,z,z';\omega)$$
$$=\int_0^\infty \begin{bmatrix} \frac{1}{2}[J_0(\mu)-J_2(\mu)]G_{xx}(z,z';q_{\parallel},\omega) + \frac{1}{2}[J_0(\mu)+J_2(\mu)]G_{yy}(z,z';q_{\parallel},\omega) & 0 & iJ_1(\mu)G_{xz}(z,z';q_{\parallel},\omega) \\ 0 & \frac{1}{2}[J_0(\mu)+J_2(\mu)]G_{xx}(z,z';q_{\parallel},\omega) + \frac{1}{2}[J_0(\mu)-J_2(\mu)]G_{yy}(z,z';q_{\parallel},\omega) & 0 \\ iJ_1(\mu)G_{zx}(z,z';q_{\parallel},\omega) & 0 & J_0(\mu)G_{zz}(z,z';q_{\parallel},\omega) \end{bmatrix} \frac{q_{\parallel}}{2\pi}dq_{\parallel} \;, \tag{4.1}$$

where $\mu=q_{\parallel}R$. The function $J_n(\mu)$ is the Bessel function of the first kind (J) and integer order n ($n=0$, 1, or 2 here).

A. Excitation of nonlocal surface waves by an oscillating dipole

Let us consider the special case where the external current density consists of a single, harmonically oscillating, electric dipole placed at $\mathbf{r}_0$, i.e.,

$$\mathbf{J}_{\text{ext}}(\mathbf{r}';\omega)=\mathbf{J}_0\delta(\mathbf{r}'-\mathbf{r}_0) \;, \tag{4.2}$$

where $\mathbf{J}_0=\omega\mathbf{p}_0/i$, $\mathbf{p}_0$ being the amplitude of the electric dipole moment. Inserting Eq. (4.2) into (B1), one finds that the electric field at $\mathbf{r}$ is given by

$$\mathbf{E}(\mathbf{r};\omega)=-\mu_0\omega^2\overleftrightarrow{G}(\mathbf{r},\mathbf{r}_0;\omega)\cdot\mathbf{p}_0 \;, \tag{4.3}$$

assuming no background field. If we are interested in the field just inside ($z\rightarrow0^+$) the surface at a distance R from a dipole just outside ($z'\rightarrow0^-$) the surface one has

$$\mathbf{E}(R,z\rightarrow0^+;\omega)=-\mu_0\omega^2\overleftrightarrow{G}^{<>}(R,z\rightarrow0^+,z'\rightarrow0^-;\omega)\cdot\mathbf{p}_0 \;, \tag{4.4}$$

with $\overleftrightarrow{G}^{<>}$ given by Eq. (4.1) with Eqs. (2.46)–(2.48) inserted. The field in Eq. (4.4) will be determined by collective and single-particle excitations as we have realized. Let us focus our attention on the collective contribution. Considering a complex $q_{\parallel}$ plane the poles of the integrand in Eq. (4.1) determine these collective excitations. Now, it can be shown that $G_{yy}^{<>}$ has no poles, and that the remaining elements in the propagator all have poles determined by the condition $r^p(q_{\parallel},\omega)=0$, cf. Eqs. (2.47) and (2.48). The condition $r^p=0$ is equivalent to

$$q_{\perp}^0(q_{\parallel},\omega)+\left[\frac{\omega}{c_0}\right]^2 M(q_{\parallel},\omega)=0 \;, \tag{4.5}$$

a relation which, in the pole approximation for M, takes the form

$$q_{\perp}^0\epsilon_T(\kappa_T,\omega)+\kappa_{\perp}^T=[1-\epsilon_L^{-1}(q_{\parallel},\omega)] \times\epsilon_T(\kappa_T,\omega)\left[\frac{q_{\parallel}}{\kappa_L}\right]^2\kappa_{\perp}^L \;. \tag{4.6}$$

Equation (4.6) is the nonlocal dispersion relation for surface waves in the collective regime.[17,18] In the local limit, the right-hand side of Eq. (4.6) vanishes, and we are left with the local dispersion relation for electromagnetic surface waves.[19] In the nonretarded limit ($c_0\rightarrow\infty$), Eq. (4.6) has the well-known "electrostatic" plasmon surface wave as a solution.[18,19] Using the hydrodynamic model it can be shown[20] that the dispersion relation in the frequency regime $\omega_p/\sqrt{2}\lesssim\omega\lesssim\omega_p$ has three distinct solutions, which outside the regime collapse to the two uncoupled solutions mentioned above.

In the far field, i.e., for $\mu\rightarrow\infty$, the asymptotic behavior

$$J_0(\mu)+J_2(\mu)\rightarrow0 \tag{4.7}$$

readily shows, cf. Eq. (4.1), that s-polarized *plane* surface waves cannot exist (remembering that $G_{yy}^{<>}=0$ in the pole approximation), a well-known result also in nonlocal optics.[17–19]

B. Elastic and inelastic light scattering from static and moving surface ripples on a nonlocal metal

If the external current density is related to the volumes in space bounded by the mean surface plane $z=0$ and the so-called topography function $z=\xi(x,y)$, one can on the basis of Eq. (B1) and the actual, nonlocal Green's function study the Rayleigh[21] and Brillouin scattering from surface ripples on a nonlocal metal within the framework of the SCIB model. In a forthcoming paper we shall, by means of a model where the surface topography is built using electric dipoles as elementary building stones, investigate the rough-surface scattering in the nonlocal regime. Note that a single dipole on the surface represents a very simple rough-surface topography.

C. Surface-dressed dipole polarizability

If we are interested in calculating the field acting on the dipole and arising from the dipole itself, we must, cf. Eq. (4.3), determine the propagator $\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0;\omega)$. Since the fundamental question regarding self-field effects, i.e., effects associated with the reaction on the dipole stemming from the dipole itself in the absence of any other particles, is of no concern to us here, the only contribution to $\overleftrightarrow{G}(\mathbf{r}_0,\mathbf{r}_0;\omega)$ is the indirect one. Denoting this by $\overleftrightarrow{I}(\mathbf{r}_0,\mathbf{r}_0;\omega)$, one has $\overleftrightarrow{I}=\overleftrightarrow{I}_{TT}^{<<}(\mathbf{r}_0,\mathbf{r}_0;\omega)$ if the dipole is placed outside the metal, and $\overleftrightarrow{I}=\overleftrightarrow{I}_{TT}^{>>}(\mathbf{r}_0,\mathbf{r}_0;\omega)+\overleftrightarrow{I}_{TL}^{>>}(\mathbf{r}_0,\mathbf{r}_0;\omega)+\overleftrightarrow{I}_{LT}^{>>}(\mathbf{r}_0,\mathbf{r}_0;\omega)+\overleftrightarrow{I}_{LL}^{>>}(\mathbf{r}_0,\mathbf{r}_0;\omega)$ if it is placed inside (cf. Fig. 7). In the presence of a background field $\mathbf{E}^{(0)}$, which is the sum of the incident field and the field reflected (or transmitted) from (or through) the boundary in the absence of the dipole, the self-consistent field which drives the dipole oscillator is

$$\mathbf{E}(\mathbf{r}_0;\omega)=\mathbf{E}^{(0)}(\mathbf{r}_0;\omega)-\mu_0\omega^2\overleftrightarrow{I}(\mathbf{r}_0,\mathbf{r}_0;\omega)\cdot\mathbf{p}_0 \ . \tag{4.8}$$

Assuming that the dipole has the polarizability $\overleftrightarrow{\alpha}_0$, i.e.,

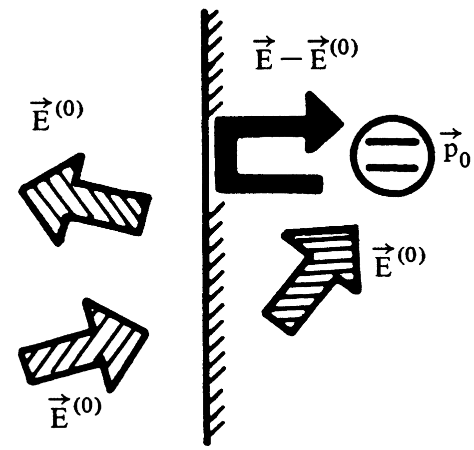

FIG. 7. Schematic illustration of the principle of surface dressing $(\mathbf{E}-\mathbf{E}^{(0)})$ of a dipole $(\mathbf{p}_0)$ placed inside a metal and exposed to the background field $(\mathbf{E}^{(0)})$. The self-consistent field is denoted by $\mathbf{E}$.

$$\mathbf{p}_0=\overleftrightarrow{\alpha}_0\cdot\mathbf{E}(\mathbf{r}_0,\omega) \ , \tag{4.9}$$

one finds by combining Eqs. (4.8) and (4.9) that

$$\mathbf{p}_0=\overleftrightarrow{\alpha}\cdot\mathbf{E}^{(0)}(\mathbf{r}_0,\omega) \ , \tag{4.10}$$

where

$$\overleftrightarrow{\alpha}(z_0;\omega)=[\overleftrightarrow{U}+\mu_0\omega^2\overleftrightarrow{\alpha}_0\cdot\overleftrightarrow{I}(z_0;\omega)]^{-1}\cdot\overleftrightarrow{\alpha}_0 \tag{4.11}$$

is the surface-dressed dipole polarizability. If the dipole is placed inside the metal, the bulk screening of the field is of course contained in $\overleftrightarrow{I}(z_0;\omega)$. In Eq. (4.11) we have stressed that the renormalized polarizability (and $\overleftrightarrow{I}$) depends on the *distance* (z_0) of the dipole from the surface only. In Eq. (4.11) $\overleftrightarrow{U}$ is the unit tensor.

Let us finally demonstrate that Eq. (4.11) contains the results obtained by others within the framework of the semiclassical infinite-barrier model or simpler models. Since $J_1(0)=J_2(0)=0$ and $J_0(0)=1$, one obtains via Eq. (4.1)

$$\overleftrightarrow{I}(R=0,z=z'=z_0;\omega)=\frac{1}{4\pi}\int_0^\infty\begin{bmatrix} I_{xx}(z_0;\omega)+I_{yy}(z_0;\omega) & 0 & 0\\ 0 & I_{xx}(z_0;\omega)+I_{yy}(z_0;\omega) & 0\\ 0 & 0 & 2I_{zz}(z_0;\omega)\end{bmatrix}q_\parallel dq_\parallel \ . \tag{4.12}$$

Now, if the dipole is outside $(z_0<0)$ the metal one gets by combining Eqs. (2.26), (2.28), and (2.29)

$$I_{TT,xx}^{<<}(z_0;\omega)+I_{TT,yy}^{<<}(z_0;\omega)=\left[r^s(q_\parallel,\omega)-\left[\frac{c_0q_\perp^0}{\omega}\right]^2 r^p(q_\parallel,\omega)\right]\frac{e^{-2iq_\perp^0z_0}}{2iq_\perp^0} \tag{4.13}$$

and

$$I_{TT,zz}^{<<}(z_0;\omega)=\left[\frac{c_0q_\parallel}{\omega}\right]^2 r^p(q_\parallel,\omega)\frac{e^{-2iq_\perp^0z_0}}{2iq_\perp^0} \ . \tag{4.14}$$

The final result for $\overleftrightarrow{\alpha}$ is obtained by inserting Eq. (4.12) with (4.13) and (4.14) into Eq. (4.11). In the nonretarded ($c_0 \to \infty$) limit, where only virtual ($\nabla \times \mathbf{E} = 0$) photons contribute, one has $N_T(q) \to -q^2$ and $q_\perp^0 \to iq_\parallel$. Then, since

$$L(q_\parallel,\omega) = -\frac{i}{\pi} \lim_{z' \to 0^+} \int_{-\infty}^{\infty} \frac{e^{iq_\perp z'} dq_\perp}{q_\parallel^2 + q_\perp^2} = \frac{1}{iq_\parallel} , \quad (4.15)$$

in this limit, as one readily shows by contour integration in the upper half-plane around the pole at $q_\perp = iq_\parallel$, one finds $r^s = 0$, and hence $I^{<,<}_{TT,yy}(z_0;\omega) = 0$, an expected result when only virtual photons are present. In turn,

$$I^{<,<}_{TT,xx}(z_0;\omega) = I^{<,<}_{TT,zz}(z_0;\omega) = -\frac{q_\parallel}{2} \left[\frac{c_0}{\omega} \right]^2 r^p(q_\parallel,\omega) e^{2q_\parallel z_0} , \quad (4.16)$$

with

$$r^p(q_\parallel,\omega) = \frac{\pi - q_\parallel \int_{-\infty}^{\infty} \frac{e^{iq_\perp 0^+} dq_\perp}{q^2 \epsilon_L(q,\omega)}}{\pi + q_\parallel \int_{-\infty}^{\infty} \frac{e^{iq_\perp 0^+} dq_\perp}{q^2 \epsilon_L(q,\omega)}} . \quad (4.17)$$

The result obtained above in the nonlocal and nonretarded regime is in complete agreement with that of Fuchs and Barrera[22] obtained by a different approach. In the local regime, the field radiated from the dipole and determined outside the surface via Eq. (4.3) with Eq. (2.2) and (2.24)–(2.26) inserted is in agreement with that given by Hellen and Axelrod,[23] and Morawitz and Philpott.[24] In turn, the renormalized polarizability agrees with that obtained in a local approach.[24]

The nonlocal propagator formalism we have developed can also be used with advantage to study the renormalized polarizability of an impurity or a "vacancy" inside the metal. I shall not discuss that subject here but refer to a forthcoming paper of mine[6] where the self-consistent, surface- and bulk-screened interaction between two such defects is studied within the framework of the semiclassical, infinite-barrier model.

APPENDIX A: THE RELATION BETWEEN THE OLD (REF. 5) AND NEW (THIS WORK) PROPAGATOR

To establish the connection between the propagator of Ref. 5 and that used in this work we take as a starting point Eq. (3.31) of Ref. 5, i.e.,

$$\overleftrightarrow{G}^{\rm old}(\mathbf{r},\mathbf{r}';\omega) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \overleftrightarrow{S}^{-1} \cdot \overleftrightarrow{G}^{\rm old}(z,z';q_\parallel,\omega) \cdot \overleftrightarrow{S} e^{i\mathbf{q}_\parallel \cdot (\mathbf{r}'_\parallel - \mathbf{r}_\parallel)} d^2 q_\parallel . \quad (A1)$$

Since $\overleftrightarrow{S}$ is unitary, i.e., $\overleftrightarrow{S}^{-1} = \overleftrightarrow{S}^T$, where $\overleftrightarrow{S}^T$ is the transpose (T) of $\overleftrightarrow{S}$, it follows that

$$[\overleftrightarrow{G}^{\rm old}(\mathbf{r},\mathbf{r}';\omega)]^T = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \overleftrightarrow{S}^{-1} \cdot [\overleftrightarrow{G}^{\rm old}(z,z';q_\parallel,\omega)]^T \cdot \overleftrightarrow{S} e^{i\mathbf{q}_\parallel \cdot (\mathbf{r}'_\parallel - \mathbf{r}_\parallel)} d^2 q_\parallel . \quad (A2)$$

Next, by making the substitution $\mathbf{q}_\parallel \Longrightarrow -\mathbf{q}_\parallel (q_{\parallel,x} \Longrightarrow -q_{\parallel,x}, q_{\parallel,y} \Longrightarrow -q_{\parallel,y})$ one obtains

$$[\overleftrightarrow{G}^{\rm old}(\mathbf{r},\mathbf{r}';\omega)]^T = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \overleftrightarrow{S}^{-1} \cdot \overleftrightarrow{G}^{\rm new}(z,z';q_\parallel,\omega) \cdot \overleftrightarrow{S} e^{i\mathbf{q}_\parallel \cdot (\mathbf{r}_\parallel - \mathbf{r}'_\parallel)} d^2 q_\parallel , \quad (A3)$$

where

$$\overleftrightarrow{G}^{\rm new}(z,z';q_\parallel,\omega) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot [\overleftrightarrow{G}^{\rm old}(z,z';q_\parallel,\omega)]^T \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \quad (A4)$$

Performing the matrix multiplications in Eq. (A4) one finds

$$\begin{bmatrix} G_{xx}^{\rm new} & 0 & G_{xz}^{\rm new} \\ 0 & G_{yy}^{\rm new} & 0 \\ G_{zx}^{\rm new} & 0 & G_{zz}^{\rm new} \end{bmatrix} = \begin{bmatrix} G_{xx}^{\rm old} & 0 & -G_{zx}^{\rm old} \\ 0 & G_{yy}^{\rm old} & 0 \\ -G_{xz}^{\rm old} & 0 & G_{zz}^{\rm old} \end{bmatrix} , \quad (A5)$$

which is just the result cited at the end of Sec. II A. Comparing Eqs. (2.2) and (A3) one also realizes that

$$\overleftrightarrow{G}^{\text{new}}(\mathbf{r},\mathbf{r}',\omega)=[\overleftrightarrow{G}^{\text{old}}(\mathbf{r},\mathbf{r}',\omega)]^T . \tag{A6}$$

APPENDIX B: THE EFFECTIVE PROPAGATOR FOR PROBLEMS EXHIBITING TRANSLATIONAL INVARIANCE ON THE y-AXIS

The fundamental field-theoretical problem in nonlocal (linear or parametrically nonlinear) metal optics usually is described on the basis of a vectorial integro-differential equation. However, using a generalized Ewald-Oseen extinction theorem it has recently been demonstrated[25] that the field-theoretical problem can be reformulated as a pure integral equation problem of the form

$$\mathbf{E}(\mathbf{r};\omega)=\mathbf{E}^{(0)}(\mathbf{r};\omega)-i\mu_0\omega\int_{-\infty}^{\infty}\overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)\cdot\mathbf{J}_{\text{ext}}(\mathbf{r}';\omega)d^3r' \tag{B1}$$

with $\overleftrightarrow{G}(\mathbf{r},\mathbf{r}';\omega)$ given by Eq. (2.2), and μ_0 being the vacuum permeability. For definiteness, the problem is formulated here for the field at the fundamental frequency (ω). In nonlocal optics the "external" current density $\mathbf{J}_{\text{ext}}$ is related to the self-consistent electric field $\mathbf{E}$ via a nonlocal, constitutive equation. Now, if we assume that $\mathbf{J}_{\text{ext}}(\mathbf{r}';\omega)$ is independent of y', an assumption which inherently necessitates that the background field $\mathbf{E}^{(0)}(\mathbf{r};\omega)$ is invariant on the y' axis, the integral equation in (B1) is reduced to the form

$$\mathbf{E}(x,z;\omega)=\mathbf{E}^{(0)}(x,z;\omega)-i\mu_0\omega\int_{-\infty}^{\infty}\overleftrightarrow{G}(x-x',z,z';\omega)\cdot\mathbf{J}_{\text{ext}}(x',z';\omega)dx'dz' \tag{B2}$$

with $\overleftrightarrow{G}(x-x',z,z';\omega)$ given by Eq. (2.5). By combining Eqs. (2.2) and (2.5) and utilizing the δ-function expansion

$$\delta(q_{\parallel,y})=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-iq_{\parallel,y}y'}dy' , \tag{B3}$$

one obtains

$$\overleftrightarrow{G}(x-x',z,z';\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{q_\parallel}\begin{bmatrix}q_{\parallel,x}&0&0\\0&q_{\parallel,x}&0\\0&0&q_\parallel\end{bmatrix}\cdot\overleftrightarrow{G}(z,z';q_\parallel,\omega)\cdot\frac{1}{q_\parallel}\begin{bmatrix}q_{\parallel,x}&0&0\\0&q_{\parallel,x}&0\\0&0&q_\parallel\end{bmatrix}e^{iq_{\parallel,x}(x-x')}dq_{\parallel,x} , \tag{B4}$$

with $q_\parallel=|q_{\parallel,x}|>0$. Now, by rewriting Eq. (B4) as follows:

$$\overleftrightarrow{G}(x-x',z,z';\omega)=\frac{1}{2\pi}\int_{-\infty}^{0}\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix}\cdot\overleftrightarrow{G}(z,z';-q_{\parallel,x},\omega)\cdot\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&1\end{bmatrix}e^{iq_{\parallel,x}(x-x')}dq_{\parallel,x}$$
$$+\frac{1}{2\pi}\int_{0}^{\infty}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\overleftrightarrow{G}(z,z';q_{\parallel,x},\omega)\cdot\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}e^{iq_{\parallel,x}(x-x')}dq_{\parallel,x} , \tag{B5}$$

and by using the symmetry relations[5]

$$G_{ii}(z,z';-q_{\parallel,x},\omega)=G_{ii}(z,z';q_{\parallel,x},\omega) ,\quad i=x,\ y,\ \text{or}\ z \tag{B6}$$

for the diagonal elements of the propagator, and

$$G_{ij}(z,z';-q_{\parallel,x},\omega)=-G_{ij}(z,z';q_{\parallel,x},\omega) ,\quad i,j=x\ \text{or}\ z\ (i\neq j) \tag{B7}$$

for the off-diagonal elements, it is a straightforward matter to show that the propagator can be written simply as

$$\overleftrightarrow{G}(x-x',z,z';\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\overleftrightarrow{G}(z,z';q_{\parallel,x},\omega)\times e^{iq_{\parallel,x}(x-x')}dq_{\parallel,x} . \tag{B8}$$

Hence, by the replacement $q_{\parallel,x}\Longrightarrow q_\parallel$ in the integrand we have obtained the result asserted in Eq. (2.6).

If the external current density and the background field consist of only a single plane-wave component along the surface they can be written in the separated form

$$\mathbf{Z}(x',z';\omega)=\mathbf{Z}(z';q_\parallel^0,\omega)e^{iq_\parallel^0x'} , \tag{B9}$$

where $q_\parallel^0$ is the actual wave number. By inserting this form and Eq. (B8) into Eq. (B2), and by using the plane-wave expansion of the Dirac δ function one obtains after integration over x' and $q_{\parallel,x}$, respectively,

$$\mathbf{E}(z;q_\parallel^0,\omega)=\mathbf{E}^{(0)}(z;q_\parallel^0,\omega)-i\mu_0\omega\int_{-\infty}^{\infty}\overleftrightarrow{G}(z,z';q_\parallel^0,\omega)\cdot\mathbf{J}_{\text{ext}}(z';q_\parallel^0,\omega)dz' . \tag{B10}$$

APPENDIX C: AMPLITUDE REFLECTION COEFFICIENTS

In the regime of nonlocal optics the amplitude reflection coefficient for s-polarized light, in the notation of Ref. 5, is given by

$$r^s(q_\parallel,\omega)=\frac{q_\perp^0\alpha(0)-\beta(0)}{q_\perp^0\alpha(0)+\beta(0)}\ , \tag{C1}$$

where

$$\alpha(0)=\frac{1}{2\pi}\lim_{z'\to 0^+}\int_{-\infty}^{\infty}\frac{e^{iq_\perp z'}}{N_T(q)}dq_\perp\ , \tag{C2}$$

and

$$\beta(0)=\frac{1}{2\pi}\lim_{z'\to 0^+}\int_{-\infty}^{\infty}\frac{q_\perp e^{iq_\perp z'}}{N_T(q)}dq_\perp\ , \tag{C3}$$

with $N_T(q)$ taken from Eq. (2.39). According to the Lindhard formalism[15] one has in the limit $q_\perp\to\infty$, $\sigma_T(q_\parallel,q_\perp,\omega)\to 1-(\omega_p/\omega)^2$, i.e., a constant value. This result implies that $\beta(0)$ can be calculated explicitly. Thus,

$$\beta(0)=\frac{i}{\pi}\lim_{z'\to 0^+}\int_0^{\infty}\frac{q_\perp^2}{N_T(q)}\frac{\sin q_\perp z'}{q_\perp}dq_\perp$$
$$=\frac{i}{\pi}\frac{\pi}{2}\lim_{q_\perp\to\infty}\left[\frac{q_\perp^2}{N_T(q)}\right]=\frac{1}{2i}\ . \tag{C4}$$

It is instructive to compare the exact result in Eq. (C4) with that obtained on the basis of models where $N_T(q)$ exhibits no branch-cut structure. With the polariton pole located at $q_\perp=\kappa_\perp^T$ in the upper half-plane of the complex $q_\perp$ plane, $\beta(0)$ is given by

$$\beta(0)=i\kappa_\perp^T\mathcal{R}_T^+\ , \tag{C5}$$

where

$$\mathcal{R}_T^+=\lim_{q_\perp\to\kappa_\perp^T}\left[\frac{q_\perp-\kappa_\perp^T}{N_T(q_\parallel,q_\perp,\omega)}\right]\ . \tag{C6}$$

Now, if one uses the so-called hydrodynamic model,[2] N_T has the local-limit form $N_T=(\kappa_\perp^T)^2-q_\perp^2$. When inserted into Eqs. (C5) and (C6), this form leads immediately to $\beta(0)=1/(2i)$ in agreement with Eq. (C4). Using instead the so-called near-local model,[26] $N_T=a[(\kappa_\perp^T)^2-q_\perp^2]$, where $a=a(\omega)$ is a complex number close to 1, one finds $\beta(0)=1/(2ia)$. The use of the near-local model, which only gives the dispersion relation $N_T(q)=0$ of a fully nonlocal model correctly around the transverse plasma edge, implies that the spatial dispersion in $\sigma_T(q_\parallel,q_\perp,\omega)$ is small. The deviation of the value of $\beta(0)$ obtained by means of the near-local approximation from the correct $1/(2i)$ value of course stems from the fact that it is the asymptotic ($q_\perp\to\infty$) value of the integrand in Eq. (C4) which determines $\beta(0)$. In the asymptotic region the near-local expression $N_T=a[(\kappa_\perp^T)^2-q_\perp^2]$ deviates from the correct one by just the factor a. For many studies of polariton modes this difference between the near-local and hydrodynamic models is of no significance. For the investigation of certain plasmon effects the difference between the two models is of crucial importance.

By inserting Eq. (C4) into Eq. (C1) one obtains

$$r^s(q_\parallel,\omega)=\frac{q_\perp^0L(q_\parallel,\omega)-1}{q_\perp^0L(q_\parallel,\omega)+1}\ , \tag{C7}$$

where

$$L(q_\parallel,\omega)=\frac{2i}{\pi}\lim_{z'\to 0^+}\int_0^{\infty}\frac{\cos q_\perp z'}{N_T(q)}dq_\perp$$
$$\equiv\frac{2i}{\pi}\int_0^{\infty}\frac{\cos(q_\perp 0^+)}{N_T(q)}dq_\perp\ . \tag{C8}$$

Inserting the result in Eq. (C4) into the equation for the reflection coefficient for p-polarized light given in Ref. 5, one obtains immediately

$$r^p(q_\parallel,\omega)=\frac{q_\perp^0-(\omega/c_0)^2M(q_\parallel,\omega)}{q_\perp^0+(\omega/c_0)^2M(q_\parallel,\omega)}\ , \tag{C9}$$

with

$$M(q_\parallel,\omega)=\frac{2i}{\pi}\int_0^{\infty}\left[\frac{q_\perp^2}{N_T(q)}+\frac{q_\parallel^2}{N_L(q)}\right]\frac{\cos(q_\perp 0^+)}{q^2}dq_\perp \tag{C10}$$

in the notation $\lim_{z'\to 0^+}\int F(q_\perp,z')dq_\perp\equiv\int F(q_\perp,0^+)dq_\perp$. The explicit expression for N_L is given in Eq. (2.40).

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