# Temperature dependence of the polaron mass in a GaAs-$Al_xGa_{1-x}As$ heterostructure

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The temperature dependence of the electron-phonon interaction correction to the electron effective mass in a GaAs-$Al_xGa_{1-x}As$ heterostructure is investigated in the case of zero magnetic field. The electron screening with full frequency, wave vector, and temperature dependence is taken into account. It is found that the polaron mass increases with temperature up to about 100 K and starts to decrease for higher temperatures. This temperature behavior agrees qualitatively with experiment but quantitatively the theoretical polaron-mass renormalization at 100 K is almost a factor of 2 smaller than observed experimentally.

In a recent paper Brummell *et al.*[1] reported a cyclotron-resonance measurement on GaAs-$Al_xGa_{1-x}As$ heterostructures. They found that at a magnetic field of about 6 T the measured cyclotron-resonance mass increases with increasing temperature for $T < 100$ K. For higher temperatures, $T > 100$ K, the cyclotron mass seems to start to decrease. For a GaAs-$Al_xGa_{1-x}As$ heterostructure the resonant magnetopolaron effects are observed in a cyclotron-resonance experiment at a magnetic field of about 21 T and thus the experiment of Ref. 1 is far below the resonant condition.

Both the electron–LO-phonon coupling and the nonparabolicity of the electron energy band are known to contribute to a modification of the electron cyclotron mass.[2] From the measured cyclotron-resonance mass one could subtract the contribution from the electron energy band nonparabolicity and obtain the mass renormalization as due to the electron-phonon interaction in the system: the so-called *polaron-mass renormalization.* This is done in Fig. 1, where the polaron-mass renormalization is plotted as a function of the temperature as deduced from the experimental result of Brummell *et al.*[1] (upper part of the figure). The increase of the polaron mass at low temperature ($T < 100$ K) was interpreted in Ref. 1 as being caused by the temperature dependence of the screening of the electron-phonon interaction: the electron-phonon interaction would be screened out at very low temperature, but with increasing temperature the screening becomes less effective and the polaron-mass renormalization will reappear. The aim of the present paper is to investigate the validity of this interpretation by an explicit calculation of the polaron mass for arbitrary temperature with inclusion of full dynamical screening.

In deducing the polaron-mass renormalization we have assumed that the contribution from the polaron effect and the band nonparabolicity can be treated within a local parabolic band approximation[2]

$$m^*_{\mathrm{CR}} = \left[1+\frac{\Delta m_{\mathrm{np}}}{m_b}\right]\left[1+\frac{\Delta m_{\mathrm{pol}}}{m_b}\right]m_b \ , \tag{1}$$

with $\Delta m_{\mathrm{np}}$ ($\Delta m_{\mathrm{pol}}$) the mass correction due to the band nonparabolicity (polaron effect), and $m_b$ the electron mass. In principle, $\Delta m_{\mathrm{pol}}$ would also contain contributions from other effects like impurity and acoustical-phonon scattering. But it is found that the impurity and acoustical-phonon scattering give very small contributions. Therefore $\Delta m_{\mathrm{pol}}$ may be attributed purely to the

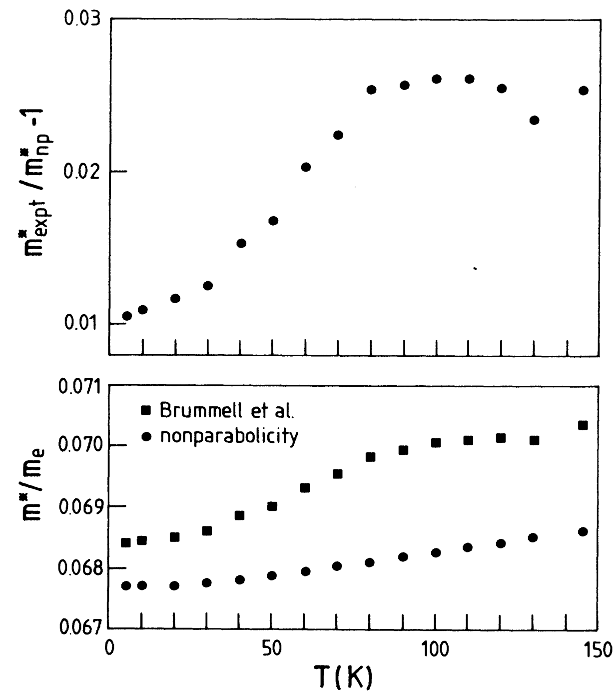


FIG. 1. In the upper part of the figure, the electron-phonon interaction correction to the electron effective mass is plotted as a function of the temperature. The solid circles are deduced from the experimental data of Ref. 1, where the contribution from the band nonparabolicity is subtracted. The sample electron density is $9\times10^{10}$ cm$^{-2}$. In the lower part of the figure the measured cyclotron-resonance mass $m^*_{\mathrm{expt}}$ (squares) is plotted together with the calculated mass $m^*_{\mathrm{np}}$ (solid circles) as due to the band nonparabolicity only.

electron—LO-phonon interaction. The band mass is obtained by fitting a zero-temperature cyclotron-resonance theory developed by the present authors[3,4] to the experimental result at 5 K. We found the band mass $m_b = 0.066 m_e$. In the lower part of Fig. 1 the measured cyclotron-resonance mass $m^*_{\rm expt}$ (squares) is plotted together with the calculated mass $m^*_{\rm np}$ (solid circles) with inclusion of the band nonparabolicity only. $m^*_{\rm np}$ is calculated at experimentally determined magnetic field values in a way[5] similar to the calculation of $m^*_{01}$ in Ref. 1.

There exists a considerable number of theoretical studies on the electron-phonon interaction in 2D electron systems. Most of the theoretical studies on the temperature dependence of the electron-phonon interaction, with or without screening, deal with the electron mobility.[6] Only a few investigations are devoted to the problem of the polaron correction to the electron effective mass. In Ref. 7 Lei studied the polaron correction to the electron effective mass and found that for a GaAs-$Al_xGa_{1-x}As$ heterostructure the polaron-mass correction is very small ($\Delta m / m_b < 0.002$) and is almost independent of the temperature for the considered temperature range (i.e., $T < 60$ K). His approach is based on a calculation of the self-energy of the electron at zero magnetic field where electron screening is considered dynamically.

To the best of our knowledge there exists no theory on the temperature dependence of the polaron cyclotron-resonance mass in the presence of a strong magnetic field in which the electron screening is treated in a dynamical way. The reason for the absence of such a theory may be rooted in the fact that the density of states of a 2D electron gas in the presence of a strong magnetic field is highly singular. One has to treat the problem in a self-consistent way, i.e., the broadening of the Landau levels and the screening affect each other (for instance, see Ref. 8 and references therein). Such a task, even in the case of zero temperature,[9] is very involved.

In the present study we will limit ourselves to the temperature dependence of the electron-phonon interaction in the case of zero magnetic field; the screening will be treated with full frequency, wave vector, and temperature dependence. The experiment of Ref. 1 was performed for a magnetic field such that $\omega_c / \omega_{\rm LO} \approx 0.34$, which means considerably below the magnetopolaron resonance. Therefore one may expect that the zero magnetic field approximation will be sufficient to make a *qualitative* comparison with the experimental data and to gain insight into the temperature dependence of the electron-phonon interaction and more specific insight into the role played by the screening.

In this paper we will use an approach which is different from Ref. 7. Instead of calculating the electron-self-energy, the dynamical conductivity of the 2D electron gas $\sigma(\omega)$ (i.e., the actually measured quantity) will be calculated within a memory-function approach (see Ref. 10 and references therein):

$$\sigma(\omega) = \frac{i n_e e^2 / m_b}{\omega - \Sigma(\omega)} , \tag{2}$$

where $n_e$ is the electron density, $m_b$ the band mass, and $\Sigma(\omega)$ the so-called memory function. By writing Eq. (1) into the Drude form the polaron-mass renormalization is given by[10]

$$\frac{\Delta m}{m_b} = - \lim_{\omega \to 0} \frac{\mathrm{Re}\Sigma(\omega)}{\omega} . \tag{3}$$

This mass renormalization is identical to the polaron cyclotron-resonance mass in the limit of zero magnetic field, and therefore the definition of the polaron-mass renormalization given by Eq. (3) is consistent with what is observed experimentally. The real part of the memory function is given by (for an analogous calculation see Refs. 3 and 10)

$$\begin{aligned} \mathrm{Re}\Sigma(\omega) = {} & \sum_{\mathbf{k}} \frac{k_\parallel^2}{n_e m_b \omega} \frac{|V_{\mathbf{k}}|^2}{v(k_\parallel)} \frac{\omega^2}{\pi} \int_{-\infty}^{+\infty} dx \frac{[1+n(x)]\,\mathrm{Im}\epsilon^{-1}(k_\parallel, x)}{[(x+\omega_{\rm LO})^2 - \omega^2](x+\omega_{\rm LO})} \\ & + n(\omega_{\rm LO}) \sum_{\mathbf{k}} \frac{k_\parallel^2}{n_e m_b \omega} \frac{|V_{\mathbf{k}}|^2}{v(k_\parallel)} \tfrac{1}{2} [\mathrm{Re}\epsilon^{-1}(k_\parallel, \omega + \omega_{\rm LO}) + \mathrm{Re}\epsilon^{-1}(k_\parallel, \omega - \omega_{\rm LO}) - 2\,\mathrm{Re}\epsilon^{-1}(k_\parallel, \omega_{\rm LO})] , \end{aligned} \tag{4}$$

where $\epsilon(k,\omega)$ is the dielectric function of the 2D electron gas and $n(x) = (e^{\beta \hbar x} - 1)^{-1}$. $\epsilon(k,\omega)$ will be calculated within the random-phase approximation.[11] $V_{\mathbf{k}}$ describes the interaction between electrons and LO phonons.[2-3] $v(k)$ represents the interaction between the electrons.

In Fig. 2 the electron-phonon interaction correction to the electron effective mass is plotted as a function of the temperature for two values of the electron density. The calculation is performed for an ideal 2D system, i.e., the 2D electron layer has a zero width. The result corresponding with the zero electron density is the 2D analogue of the Feynman-Hellwarth-Iddings-Platzman (FHIP) theory for 3D polarons.[12] All physical parameters are taken corresponding to a GaAs-$Al_xGa_{1-x}As$ heterostructure. The polaron-mass correction is found to increase first with increasing temperature ($T < 80$ K) and to reach a maximum around 90 K. For still higher temperature it starts to decrease. Apparently, the experimentally observed polaron-mass renormalization is smaller than that predicted by the ideal 2D calculation (see also Fig. 1). Dynamical screening reduces the polaron-mass renormalization over the whole temperature region as expected. However, it is found that the absolute reduction of the polaron-mass correction due to the electron screening is larger at 100 K than at zero temperature. This is opposite to the intuitive argument of Ref. 1. Note that the general temperature behavior of the polaron-mass renormalization is not due to many-particle effects but already shows up in the one-polaron behavior. The increase of the mass correction at low temperature is due to the nonparabolicity induced by the electron-phonon interaction as was explained earlier (see Ref. 13 and references therein) in the case of 3D polarons.

In Fig. 3 the polaron-mass renormalization is plotted as a function of the temperature for different values of the electron density. But now the calculation is performed for a quasi-2D electron system which corresponds more closely to the experimental system. Only the lowest subband is taken into account, using a Stern-Fang-Howard variational wave function.[6] The inset shows the mass correction as a function of the electron density in the zero-temperature limit. Again it is found that the reduction of the polaron-mass correction due to the screening is larger around 100 K than at zero temperature. The position of the maximum of the polaron-mass renormalization shifts to lower temperature as the electron density increases. The temperature dependence of the polaron mass found in the present study is quite different from the result of Lei,[7] where the mass renormalization is found to be almost independent of the temperature below 60 K (in Ref. 7 no data were given for $T > 60$ K). The reason for this contradiction with the present theoretical result may be attributed to the difference in definition of the polaron mass.

In the zero-temperature limit for electron density $10^{11}$ $cm^{-2}$ the polaron-mass correction is $\Delta m / m_b = 0.007$, while experimentally it is about $\Delta m / m_b = 0.01$. This difference is due to the fact that the cyclotron-resonance experiment is performed at a nonzero magnetic field. If the magnetic field is taken equal to the experimental value, the experimental value could be reobtained.[4] For the same electron density ($10^{11}$ $cm^{-2}$) the maximum value of the polaron-mass renormalization is only about $\Delta m / m_b = 0.014$ (at temperature around 100 K) which is almost a factor of 2 smaller than the experimentally measured value of $\Delta m / m_b = 0.026$. This quantitative discrepancy cannot be explained as due to the effect of a nonzero magnetic field.

Next, possible explanations for the discrepancy between the present theoretical results and the experimental data will be briefly discussed. In a cyclotron-resonance experiment, besides electron–LO-phonon coupling there are also electron-impurity and electron–acoustic-phonon interactions. The interaction between electrons and impurities will shift the cyclotron-resonance frequency, which may be interpreted as a mass renormalization.[6] However, this correction will be mixed with band nonparabolicity and is difficult to assess experimentally, at least at low temperature. A zero magnetic field calculation[14] for a Si-inversion layer suggests that the mass renormalization due to the electron-impurity interaction decreases with increasing temperature. Furthermore, for the GaAs-$Al_xGa_{1-x}As$ heterostructures used in Ref. 1, the mass renormalization due to impurity scattering will be very small,[15] i.e., much smaller than the mass renormalization due to the

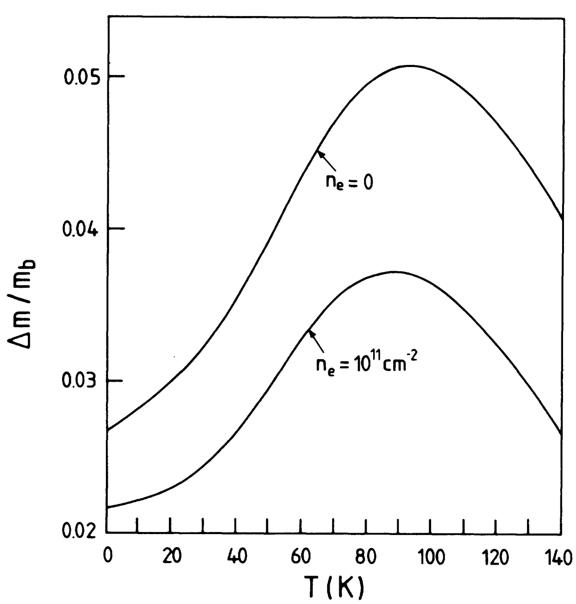


FIG. 2. The electron-phonon interaction correction to the electron effective mass is plotted as a function of the temperature for two values of the electron density. The calculation is performed for an ideal 2D system. All physical parameters are taken corresponding to a GaAs-$Al_xGa_{1-x}As$ heterostructure.

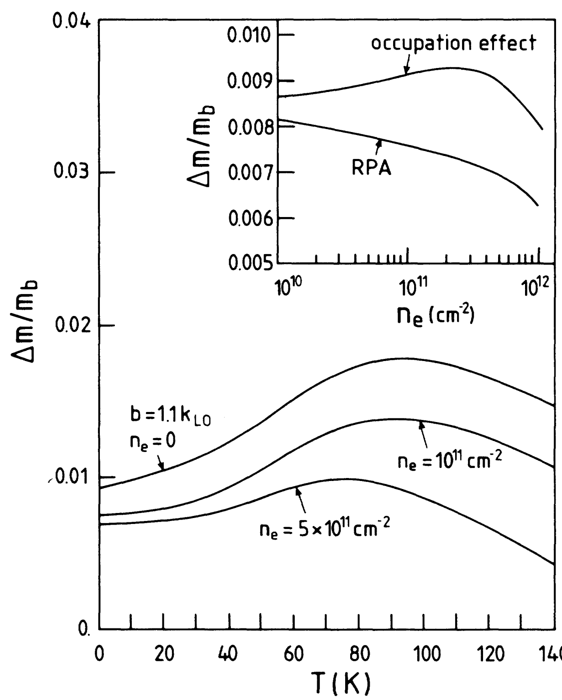


FIG. 3. The polaron-mass renormalization due to the electron-phonon coupling is plotted as a function of the temperature for different values of the electron density. The inset shows the mass correction as a function of the electron density at zero temperature. The calculation is performed for a quasi-2D electron system where only the lowest subband is taken into account. Other parameters are taken corresponding to GaAs-$Al_xGa_{1-x}As$ heterostructures.

electron–LO-phonon coupling. Therefore the electron-impurity interaction could be ruled out. For GaAs-$Al_xGa_{1-x}As$ heterostructures, coupling between electrons and acoustic phonons (deformation potential) was found to be important for limiting the electron mobility at $T<50$ K. However, because the coupling constant[16] between the electrons and acoustic phonons is of the order of about $10^{-6}$, the mass renormalization will be very small. We have verified this numerically by performing a one-particle calculation and found $|\Delta m/m_b|<0.0002$ over the whole temperature region under study here.

In the present study only the lowest subband has been taken into account. This is a good approximation at low temperature for low electron density samples. For the sample with electron density $9\times10^{10}$ $cm^{-2}$ used in Ref. 1 the difference in energy between the first two subbands is 13.3 meV,[17] which corresponds to a temperature of about 150 K. Since the total contribution to the mass renormalization is an average over contributions from all subbands one does not expect large increases of the mass correction for the temperature concerned here ($T<150$ K). To verify this assertion a numerical calculation is performed, taking into account the first two subbands, but within a single-particle picture, i.e., without screening (in doing so an upper bound for the correction is found). It is found that the total polaron-mass renormalization increases by about $0.0015m_b$ over the whole temperature region; this is still too small to explain the experimental results of Ref. 1. Higher subbands will give even smaller corrections to the polaron mass.

In conclusion, we have presented a calculation of the temperature dependence of the polaron mass at zero magnetic field which agrees qualitatively with the experimental results. On the basis of the present calculation we are able to rule out the earlier interpretation of Ref. 1, that the temperature behavior of the polaron mass as observed in the cyclotron-resonance experiment is a consequence of the temperature dependence of the screening of the electron-phonon interaction. The present theoretical results rather seem to suggest it is a one-polaron effect. Quantitatively it is found that the mass renormalization at $T=100$ K as due to the electron-phonon interaction is a factor of 2 smaller than observed experimentally.

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