# Effect of dynamical screening on the polaron cyclotron-resonance mass of a two-dimensional electron gas in GaAs-$Al_xGa_{1-x}As$ heterostructures

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The cyclotron-resonance mass of a two-dimensional electron gas interacting with bulk LO phonons is investigated as a function of the magnetic field strength and of the electron density. A theory is presented and worked out numerically, for the first time, which includes a full dynamical screening of the electron-phonon interaction for arbitrary magnetic field strength. It is found that near the magnetophonon resonance a dynamical screening theory and a static screening theory give, within 1%, the same contribution to the polaron cyclotron-resonance mass renormalization in the region of experimentally relevant electron densities for a GaAs-$Al_xGa_{1-x}As$ heterostructure.

The cyclotron-resonance of a two-dimensional (2D) electron gas interacting with bulk LO phonons has recently been the subject of a number of studies, both theoretical[1-8] and experimental.[9-14] It is found experimentally that the polaron effects in the 2D electron systems in GaAs-$Al_xGa_{1-x}As$ heterostructures are smaller than in the three-dimensional GaAs system. Earlier theoretical calculations, however, which were based on a *one-polaron* approximation, predict an *enhancement* of the polaron effects in those 2D systems in comparison with bulk systems. This puzzle has now been partly solved. It has been realized[5-16] that (1) the nonzero extent of the 2D electron gas in the direction perpendicular to the 2D electron layer reduces the polaron effect considerably, and (2) the many-particle effects, i.e., Fermi-Dirac statistics and screening of the electron-phonon interaction, also reduce the polaron renormalization of the electron effective mass.

In this paper we will investigate the effect of *screening* of the electron-phonon interaction on the cyclotron-resonance mass of the electron. The screening of the electron-phonon interaction has already been studied in the zero magnetic field case.[15,16] When a strong magnetic field is applied perpendicular to the 2D electron gas the density of states of the electrons splits up into a series of δ functions. This singular nature of the energy spectrum will manifest itself in a screening of the electron-electron interaction and the electron-phonon coupling which will be different from the zero magnetic field case. In Ref. 3 Larsen addressed the importance of Fermi-Dirac statistics (i.e., the occupation effect) in describing the electron-phonon interaction in 2D electron systems. In Ref. 6 Lassnig studied the mixing of the magnetoplasmon and phonon modes and the occupation effect. Neither in Ref. 3 nor in Ref. 6 was a full dynamical screening theory for the electron-phonon interaction in a strong magnetic field presented.

We are interested here in a *full* dynamical screening theory of the polaron cyclotron-resonance mass. The screening which arises from the interaction between the electrons will modify the interaction between the electrons, between electrons and impurities, and between electrons and phonons. The screening of the electron-impurity interaction has been well studied[17] while the role played by the screening in the electron-phonon interaction has been neglected in most of the studies when a strong magnetic field is present.

In a recent paper[7] we studied the effect of Fermi-Dirac statistics and the effect of static screening on the cyclotron-resonance mass of polarons. It was found that most of the reduction of the polaron effect is due to the nonzero width of the 2D electron gas and due to the occupation effect. Static screening also reduces the polaron effects, but the latter reduction is small compared to the effects of nonzero layer width and Fermi-Dirac statistics. A comparison between the theoretical results and the experimental data[7,8] does not allow us to assess the effect of screening quantitatively because some parameters (e.g., the depletion charge density and the band mass of the electron) have to be fitted. From an earlier work on the screening of the electron-phonon interaction in the case of zero magnetic field,[16] it was found that a static screening approach overestimates the effect of screening in the region of relevant electron densities ($n_e \sim 10^{11} \rightarrow 10^{12}$ cm$^{-2}$) for GaAs-$Al_xGa_{1-x}As$ heterostructures. It is necessary to go beyond the static screening approach and to include the full dynamical aspects of screening in the calculation.

The cyclotron-resonance mass of an electron interacting with LO phonons can be obtained from the magneto-optical absorption spectrum. In the present study the broadening of the Landau levels will be neglected for simplicity. The cyclotron-resonance frequency is given by the solution of the nonlinear equation

$$\omega-\omega_c-\mathrm{Re}\Sigma(\omega)=0\ , \tag{1}$$

where $\omega_c=eH/m_b c$ is the unperturbed cyclotron-resonance frequency with $m_b$ the electron band mass and $\Sigma(\omega)$ the memory function. The electron–LO phonon interaction is treated as a perturbation, which is valid for the system under study (i.e., the electron-phonon coupling constant for GaAs is $\alpha=0.068 \ll 1$). The real part of the memory function can be written as[7]

$$\mathrm{Re}\Sigma(\omega)=\sum_{\mathbf{k}}\frac{k_\parallel^2}{n_e m_b\omega}\frac{|V_{\mathbf{k}}|^2}{v(k_\parallel)}\frac{\omega^2}{\pi}\int_{-\infty}^{+\infty}dx\frac{[1+n(x)]\mathrm{Im}\epsilon^{-1}(k_\parallel,x)}{[(x+\omega_{\mathrm{LO}})^2-\omega^2](x+\omega_{\mathrm{LO}})}$$
$$+n(\omega_{\mathrm{LO}})\sum_{\mathbf{k}}\frac{k_\parallel^2}{n_e m_b\omega}\frac{|V_{\mathbf{k}}|^2}{v(k_\parallel)}\tfrac{1}{2}[\mathrm{Re}\epsilon^{-1}(k_\parallel,\omega+\omega_{\mathrm{LO}})+\mathrm{Re}\epsilon^{-1}(k_\parallel,\omega-\omega_{\mathrm{LO}})-2\,\mathrm{Re}\epsilon^{-1}(k_\parallel,\omega_{\mathrm{LO}})]\ , \tag{2}$$

where $\epsilon(k,\omega)$ is the dielectric function of the 2D electron gas and $n(x)=(e^{\beta\hbar x}-1)^{-1}$. $V_{\mathbf{k}}$ describes the interaction between electrons and LO phonons.[5] $v(k)$ represents the interaction between the electrons.

The collective effects of the 2D electron gas enter the memory function via the dielectric function $\epsilon(k,\omega)$, which has been studied extensively within the random-phase approximation (RPA).[18] In the zero-temperature limit the RPA gives

$$[1+n(\omega)]\mathrm{Im}\left[\frac{1}{\epsilon(k,\omega)}\right]=-\pi\frac{v(k)}{\epsilon^2(k)}\frac{m_b\omega_c}{\pi\hbar^2}f_{n_0}(1-f_{n_0})$$
$$\times C_{n_0,n_0}(k)\delta(\omega)$$
$$-\sum_{n=1}^{\infty}A_n(k)\delta(\omega-\omega_n(k))\ , \tag{3}$$

where $\omega_n(k)$ $(n=1,2,\ldots)$ are the magnetoplasmon frequencies (also called Bernstein modes) which are determined by the equation $\mathrm{Re}\epsilon(k,\omega)=0$. These magnetoplasmon modes have oscillator strengths

$$A_n(k)=\pi\left|\frac{\partial}{\partial\omega}\mathrm{Re}\epsilon(k,\omega)\right|^{-1}_{\omega=\omega_n(k)}\quad(n=1,2,\ldots)\ .$$

$n_0=[\nu]$ is the integer part of the filling factor $\nu$ and $f_{n_0}=\nu-n_0$ is the electron occupation of Landau level $n_0$. The filling factor is defined as $\nu=(n_e/2)(2\pi\hbar/m_b\omega_c)$, and

$$C_{n_0,n_0}(k)=\exp(-k^2l_0^2)[L_{n_0}(k^2l_0^2)]^2$$

with $L_n(x)$ the Laguerre polynomial of index $n$ and $l_0=(\hbar/2m_b\omega_c)^{1/2}$ the magnetic length. The magnetoplasmon frequencies and the corresponding oscillator strengths are shown in Fig. 1 as a function of the wave vector for $n=1,2,3,4$ at a magnetic field $\omega_c/\omega_{\mathrm{LO}}=0.8$ and a filling factor $\nu=0.5$, which corresponds to an electron density of $n_e=4\times10^{11}$ cm$^{-2}$ in the case of a GaAs-Al$_x$Ga$_{1-x}$As heterostructure. Note that for $k=0$ and $k\to\infty$, $\omega_n(k)=n\omega_c$. Furthermore, the oscillator strength is $A_n(k)=0$ when $k=0$ or $k\to\infty$, and $A_n(k)$ decreases with increasing $n$. After substituting Eq. (3) into Eq. (2) the real part of the memory function becomes

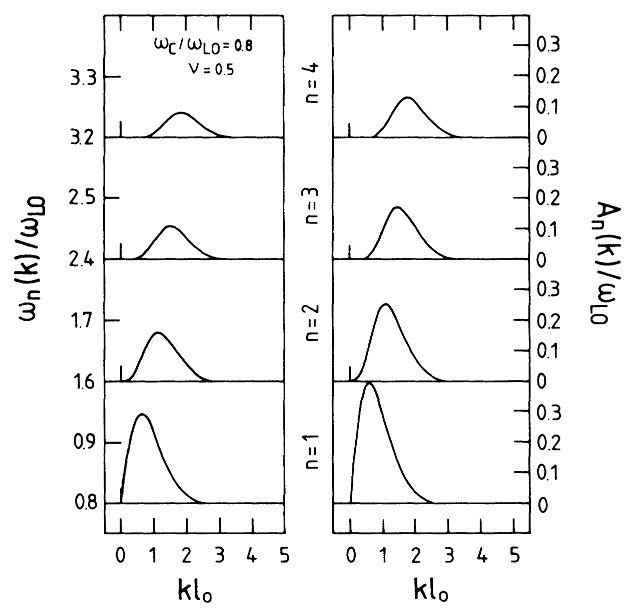


FIG. 1. The dispersion of the magnetoplasmon modes and its corresponding oscillator strength for the lowest four branches. The electron density is $4\times10^{11}$ cm$^{-2}$ and the magnetic field is such that $\omega_c/\omega_{\mathrm{LO}}=0.8$ [$l_0=(\hbar/2m_b\omega_c)^{1/2}$ is the magnetic length].

$$\mathrm{Re}\Sigma(\omega)=-\sum_{\mathbf{k}}\frac{k_{\parallel}^2}{\nu m_b\hbar\omega}\frac{|V_{\mathbf{k}}|^2}{\epsilon^2(k_{\parallel})}\frac{\omega^2 C_{n_0,n_0}(k_{\parallel})}{\omega_{\mathrm{LO}}(\omega_{\mathrm{LO}}^2-\omega^2)}f_{n_0}(1-f_{n_0})$$
$$-\sum_{\mathbf{k}}\frac{k_{\parallel}^2}{\pi n_e m_b\omega}\frac{|V_{\mathbf{k}}|^2}{v(k_{\parallel})}\sum_{n=1}^{\infty}\frac{\omega^2 A_n(k_{\parallel})}{[\omega_n(k_{\parallel})+\omega_{\mathrm{LO}}]\{[\omega_n(k_{\parallel})+\omega_{\mathrm{LO}}]^2-\omega^2\}}\ . \quad (4)$$

In our numerical calculation we find that it is sufficient to include the contributions from the lowest 40 branches of the magnetoplasmon mode. The higher branches give a negligible contribution to $\mathrm{Re}\Sigma(\omega)$ when $\nu<2$.

In Fig. 2 the electron-phonon interaction correction to the cyclotron-resonance mass of an electron is plotted as a function of the magnetic field strength. All physical parameters are chosen as corresponding to the GaAs-$Al_xGa_{1-x}As$ heterostructures. The electron density is $4\times10^{11}$ cm$^{-2}$. In Fig. 2 the calculation is performed for an ideal 2D system (i.e., the 2D electron layer has zero width) where one can clearly see the difference between the different approximations. Including the dynamical screening, the polaron cyclotron mass correction is reduced as compared to a calculation where only the occupation effect is considered. The reduction of the polaron cyclotron mass correction is about $0.02m_b$ at $\omega_c/\omega_{\mathrm{LO}}=0.6$ (this corresponds to a magnetic field of about 12.5 T). This reduction increases to $0.035m_b$ at $\omega_c/\omega_{\mathrm{LO}}=0.9$ (19 T). Note that the reduction in $\Delta m^*$ as a consequence of the occupation effect as compared to the zero electron density limit (not shown in this figure) is $0.03m_b$ and $0.05m_b$, respectively, for the above magnetic field values.

Most surprisingly we find that the static screening approach gives almost the same results as the dynamical screening treatment. The difference is very small for large magnetic fields ($\omega_c/\omega_{\mathrm{LO}}>0.5$). Only for small magnetic fields does the difference become noticeable on the scale of our plot. This can be understood by looking at the first term on the right-hand side of Eq. (4), which gives the dominant contribution in the resonant region $\omega_c\sim\omega_{\mathrm{LO}}$. This contribution, which contains the static screening factor $\epsilon(k)$, turns out to be equal to the dominant contribution in the memory function as calculated within the static screening approximation. Therefore we may conclude that when the broadening of the Landau levels is neglected the static screening approach gives

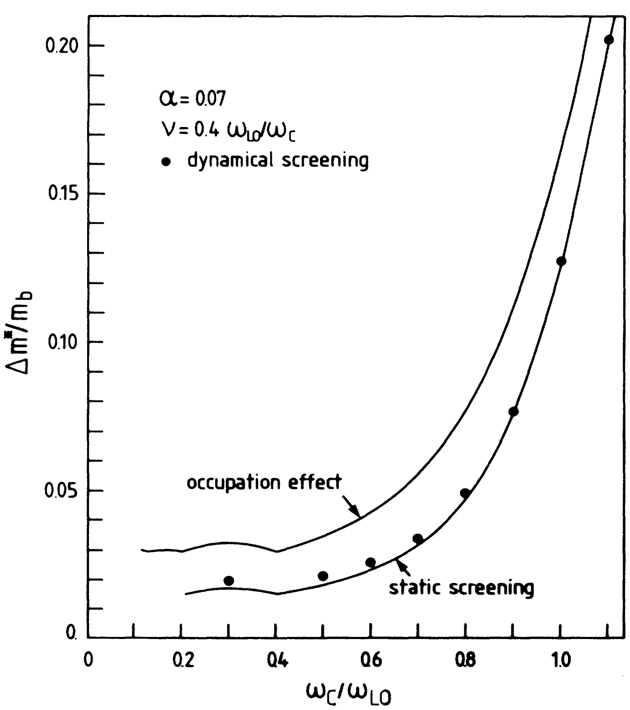


FIG. 2. The polaron correction to the cyclotron-resonance mass is plotted as a function of the magnetic field strength. The many-particle effects are treated within different approximations (occupation effect, static screening, and dynamical screening). The electron density is $4\times10^{11}$ cm$^{-2}$ and all other parameters are taken corresponding to a GaAs-$Al_xGa_{1-x}As$ heterostructure.

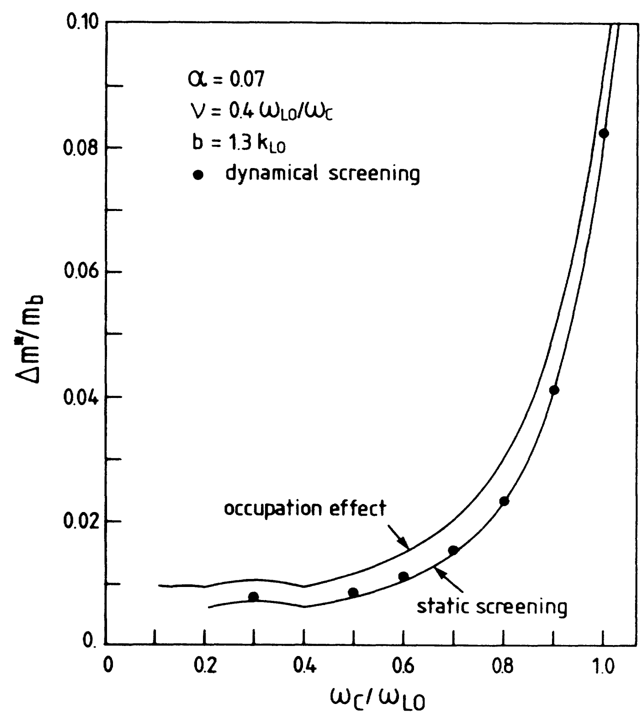


FIG. 3. The same as Fig. 2 but now for a quasi-two-dimensional system. The nonzero width of the 2D electron gas is considered by taking into account the lowest electric subband which is of the Stern-Fang-Howard type.

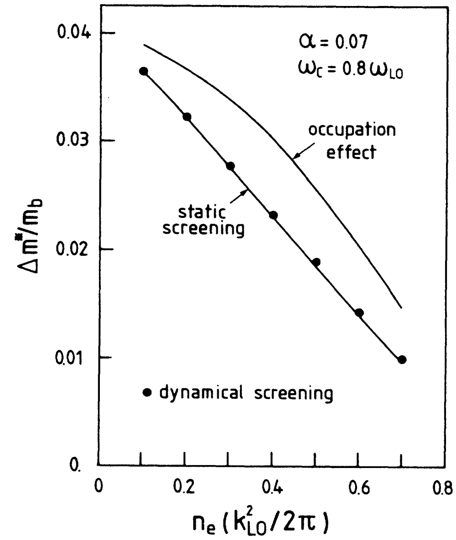


FIG. 4. The polaron cyclotron-resonance mass renormalization ($\omega_c = 0.8\omega_{LO}$) is plotted as a function of the electron density. The electron density is in units of $k_{LO}^2/2\pi$, which is $10^{12}$ $cm^{-2}$ for a GaAs-$Al_xGa_{1-x}$As heterostructure.

fairly good results for the polaron cyclotron-resonance mass renormalization in the relevant magnetic field region, i.e., $\omega_c/\omega_{LO} > 0.5$.

In Fig. 3 the polaron correction to the cyclotron-resonance mass is plotted as a function of the magnetic field strength for a quasi-2D system. The nonzero width of the 2D electron gas is considered by taking into account only the lowest subband which is described by the Stern-Fang-Howard variational wave function. The electron density is $4\times10^{11}$ $cm^{-2}$. Again, the static screening result and the dynamical screening result are very close. The reduction of the polaron effect due to screening is obvious. At $\omega_c/\omega_{LO} = 0.6$ the polaron cyclotron-resonance mass correction for a 2D electron gas with a nonzero layer thickness is reduced by about $0.025m_b$. The reduction of the mass correction due to the dynamical screening effect is about $0.005m_b$.

Up to now we have limited ourselves to one particular electron density. Both occupation effect and screening depend on electron density. Furthermore, the width of the 2D electron gas also depends on the electron density, i.e., it decreases with increasing electron density. In Fig. 4 the cyclotron-resonance mass renormalization (at $\omega_c = 0.8\omega_{LO}$) is plotted as a function of the electron density. The electron density is in units of $k_{LO}^2/2\pi$ [$k_{LO} = (2m_b\omega_{LO}/\hbar)^{1/2}$], which is $10^{12}$ $cm^{-2}$ for a GaAs-$Al_xGa_{1-x}$ heterostructure. $\omega_c = 0.8\omega_{LO}$ is chosen since we are mainly interested in the region where the polaron effects are dominant, i.e., in the region where $\omega_c$ is near to $\omega_{LO}$. As expected, the polaron effects (hence the cyclotron-resonance mass correction) decrease as the electron density increases. Note also that the dynamical screening approximation gives almost the same result as the static screening treatment over the whole region of the electron densities under study here.

In summary, the effect of screening of the electron-phonon interaction on the cyclotron-resonance mass of polarons was studied. It was found that the static screening and the dynamical screening approaches give almost the same results over a wide range of electron densities and magnetic field strengths, especially in the polaron resonant region where $\omega_c \sim \omega_{LO}$. From our study we may conclude that, as far as the polaron correction to the cyclotron-resonance mass is concerned, in practice no full dynamical screening calculation which is very involved numerically is necessary and a static screening approach will be sufficient. In the small magnetic field region dynamical screening gives slightly different results from the static screening approach, but then the polaron effect is small and experimentally difficult to assess. For higher magnetic fields (i.e., $\omega_c/\omega_{LO} > 0.5$, or $H > 10.5$ T for GaAs) where polaron effects are important and observable experimentally, the dynamical screening will give almost the same results as a static screening approach. The latter approach is much simpler and easier to handle numerically.

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