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# Critical currents of the quantum Hall effect in the mesoscopic regime

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The critical currents associated with the breakdown of the quantum Hall effect have been measured in high-mobility, one-dimensional channels fabricated in GaAs-$Al_xGa_{1-x}As$ heterostructures. The results differ in several respects from the breakdown in macroscopic samples. The onset of dissipation shows little hysteresis and depends on the direction of the current with respect to the magnetic field. The value of the critical current is also independent of the filling factor across a large fraction of the Hall plateau. These observations indicate that mesoscopic considerations must be included in any theory of breakdown on this size scale.

The role of dissipation in the quantum transport is a topic of much current interest.[1-6] In particular, the mechanism for the onset of dissipation on the Hall-effect resistance plateaux remains to be determined. Several models[7-11] have been proposed to account for the rapid increase of the transverse magnetoresistance above a critical current $I_c$, but the comparison with experiment has been hampered by lack of control over the two-dimensional electron gas density over large areas. By constructing smaller samples it is hoped that the underlying quantum processes of the breakdown can be differentiated from mechanisms involving macroscopic sheet density inhomogenities. Small samples are also useful for investigating any mesoscopic phenomena that may affect the breakdown.

This work reports on studies of the breakdown of the integer Hall effect in electron waveguides fabricated in GaAs-$Al_xGa_{1-x}As$ that satisfy $W \simeq \lambda_f \ll l_e \simeq L \leq L_\phi$.[12] Here $W \simeq 80$–$200$ nm is the electrical width, $\lambda_f \simeq 60$ nm is the Fermi wavelength, $l_e \simeq 1.6$–$2.0$ $\mu$m is the elastic mean-free-path, $L \simeq 0.7$–$2.5$ $\mu$m is the sample length in the direction of the current, and $L_\phi \simeq 5$–$7$ $\mu$m is the phase-breaking length. This represents a new regime of transport in which to investigate the breakdown in a well-defined geometry.

In the regime where $W \simeq \lambda_f \ll \simeq L \leq L_\phi$ there are only $\sim 10$–$100$ current-carrying states (see below). This is to be contrasted with $L/\pi l_B \sim 10^5$ such states in a 300-$\mu$m-wide control sample on the $i=2$ Hall plateau. Here we have used the free-electron model for the quantum Hall effect in a homogeneous sample.[13] The waveguide samples also satisfy $W \simeq 10 l_B$, where $l_B = (\hbar/eB)^{1/2}$ is the magnetic length, for fields on the same plateau. In this situation, the current-carrying edge states fill the entire sample. The macroscopic sample, on the other hand, possesses both bulk and edge states. Another consideration is the mesoscopic nature of the small samples. The sensitivity of the transport coefficients to the anisotropic impurity/boundary configuration due to the extensive phase coherence is evidenced by the observation of negative (dynamic) resistance[12] and a second-harmonic generation.[14] The nonlocality of voltage measurements in this case requires us to treat four point resistances in the form $R_{ij,kl}$, where $i,j$ label voltage leads and $k,l$ are current leads.[15] These differ from the usual $R_{xx}$ and $R_{xy}$. On a Hall plateau, however, $R_{xx} = \frac{1}{2}[R_{ij,kl}(H) + R_{kl,ij}(H)]$, below $I_c$. Since we found experimentally that the breakdown was independent of the field direction, we may take the measured $R_{ij,kl}$ to be the same as $R_{xx}$, for current levels at or below $I_c$.

The shape of our samples [see Fig. 3(b)] allowed us to define better than other workers the areas in which the breakdown was measured. References 5 and 6, for example, measured the breakdown between two macroscopic voltage pads as a current was passed through a constriction between them. As distinct from this technique, our voltage probes were extensions of the microscopic current channel. Hence, the area in which the breakdown was measured included the voltage leads and had a size no greater than $\sim L_\phi$. This may have reduced the effects of the energy-relaxing two-dimensional electron reservoirs on the breakdown measurement. In this sense, we have measured the onset of dissipation in a few current-carrying states on a length scale less than $L_\phi$. This dissipation should not be confused with the spatial separation of elastic and inelastic scattering processes characteristic of mesoscopic systems. Rather, the breakdown measured here occurs when the current density $\mathbf{j}$ no longer lies perpendicular to the local electric field $\mathbf{E}$. As soon as breakdown occurs, $L_\phi$ shortens abruptly.

The narrow samples were patterned using electron beam lithography and dry etching techniques on modulation doped, molecular-beam-epitaxy- (MBE)-grown GaAs substrates.[16] They were cooled to $T \simeq 280$ mK in a top-loading $^3$He sorption pumped cryostat, and ac (17-Hz) four-probe $I$-$V$ measurements were performed. The zero-field mobility in a 300-$\mu$m-wide sample was 300 000 cm$^2$/V sec with a two-dimensional sheet density of $n_2 \simeq 3.89 \times 10^{11}$ cm$^{-2}$, as determined by Hall and Shubnikov-deHaas measurements. Because of surface depletion, the lithographic width $W_L \simeq 0.47$–$0.50$ $\mu$m exceeded the electrical width $W$. The latter was estimated by requiring a simple particle-in-a-box model with an infinite confining potential to reproduce the measured low-field carrier density and the number of Shubnikov-deHaas minima. The reduction in carrier density in the

 

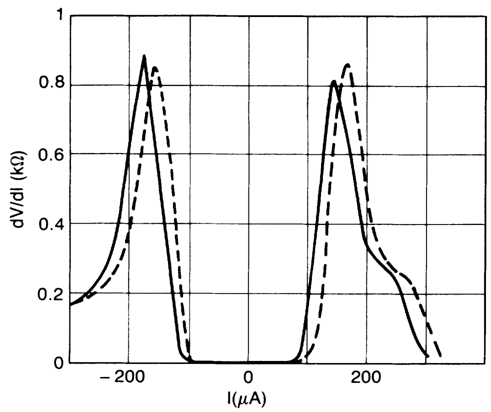


FIG. 1. Differential resistance vs current for a 300-$\mu$m-wide sample on the $i=2$ plateau at 276 mK. Notice the large amount of hysteresis.

narrow samples implied that $W$ ranged over 80–200 nm.

Figures 1 and 2 show results for the macroscopic (control) sample: $W \simeq 300$ $\mu$m fabricated from the same substrate as the narrow samples. The curves in Fig. 3 clearly show the hysteretic effects observed by others.[2] Figure 2 gives the dependence of the critical current on the $i=2$ plateau as a function of filling factor $\nu = 2\pi l_B^2 n_2$ for two sections of the Hall bar. It displays the "triangle" pattern previously measured by other researchers, with the critical current rising to a sharp maximum near the center of the Hall plateau.[1–3,5,6] Throughout this work $I_c$ is defined as the excitation necessary to produce $dV/dI > 0$ with a signal-to-noise ratio $\simeq 1$; this typically occurred when $dV/dI \sim 100$ Ω. The critical current was found to be independent of the direction of the current with respect to the magnetic field in the large sample. The fact that two different sections of the sample generate nonidentical triangle patterns, as shown in Fig. 2, indicates that the mechanism involved in breakdown is local on a 600-$\mu$m scale.

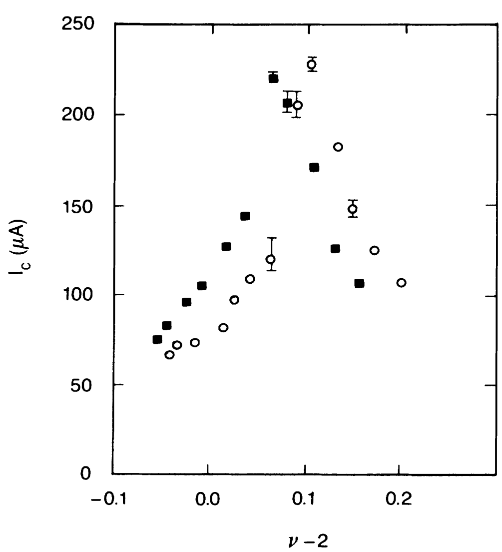


FIG. 2. Critical current as a function of filling factor for the same sample as Fig. 1 for two adjoining sections on the Hall bar.

Figure 3 shows a plot of $dV/dI$ against $I$ on the $i=2$ Hall plateau for a sample of 0.5 $\mu$m lithographic width. Now the hysteresis is insignificant. Furthermore, there is no well-defined breakdown resistance for $I > I_c$. This observation is in disagreement with the work of Refs. 5 and 8, where it is claimed that the breakdown of the Hall effect proceeds via a quantized breakdown resistance. In contrast with the triangle pattern of Fig. 2 for the 300-$\mu$m-wide sample, the corresponding measurement of $I_c$ for $W \sim 0.1$ $\mu$m displays a saturation effect over a large fraction of the $i=2$ plateau (see Fig. 4). The critical current barely saturates on the $i=4$ plateau. Figure 5 demonstrates that $I_c$ depends on the *current direction* as well as $\nu$. The current anisotropy occurred for about 75% of the runs on the 3 different samples, whereas the data were reproducible within the experimental precision on any given run. $I_c$ did not depend on the choice of voltage probes used to measure it, as long as they were situated along a channel whose lithographic width remained constant. This indicates that the variations in $I_c$ due to *macroscopic* sheet density gradients *along* the wire length have been eliminated. (In a 1D wire there must be density gradients across the device.)

The following technique was used to ensure that the ex-

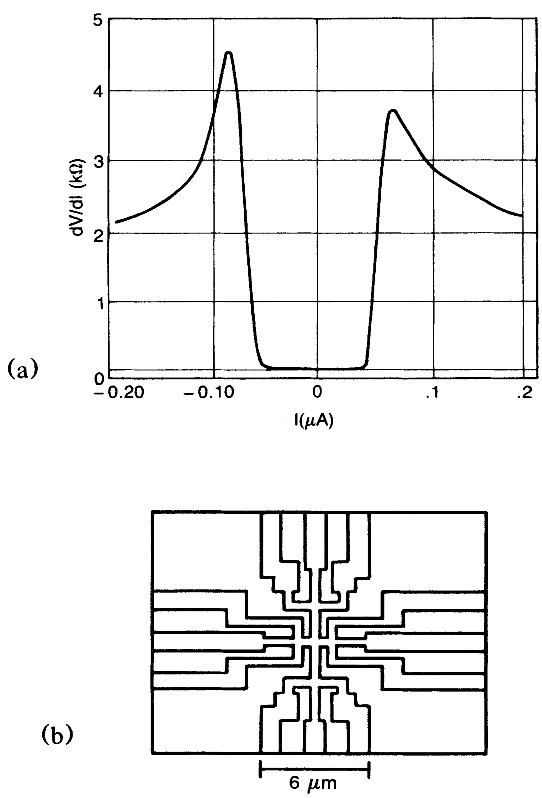


FIG. 3. (a) $dV/dI$ vs $I$ for a sample of 0.47 $\mu$m lithographic width on the $i=2$ plateau. The hysteresis has disappeared. (b) shows a typical geometry for a sample of 0.5 $\mu$m lithographic width.

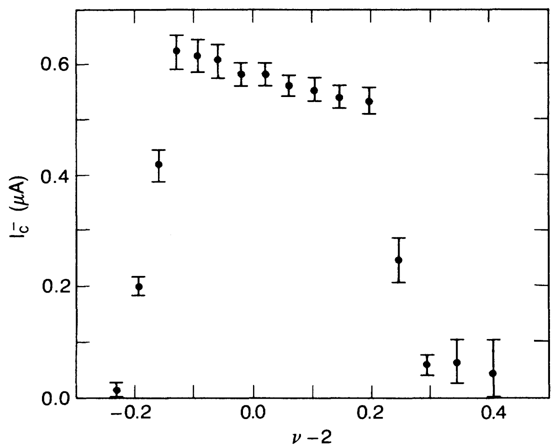


FIG. 4. Dependence of $I_c$ on $\nu$ for the same sample as Fig. 3. Note the saturation of $I_c$ near the plateau center.

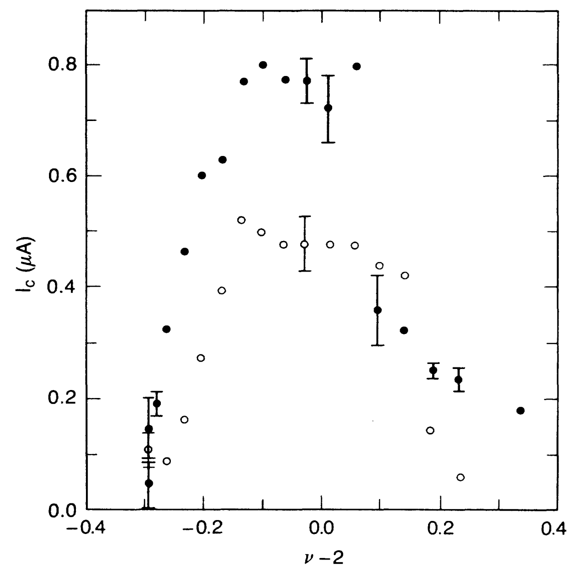


FIG. 5. Dependence of $I_c$ on $\nu$ and current direction for a sample of 0.50 $\mu$m lithographic width.

perimental conditions remained mesoscopic at current levels comparable to the observed $I_c$. The low field ($B<0.6$ T) magnetoresistance was measured for several current drives at 280 mK. Autocorrelating these traces gave a correlation field $\Delta H(I)$, from which the *low field* $L_\phi(I)$ was determined through $L_\phi(I)W\Delta H(I)\simeq 2\Phi_0$.[17] This yielded $L_\phi \simeq 4$–$8$ $\mu$m for $I \lesssim 10$ nA, degrading to $L_\phi \simeq 2$–$5$ $\mu$m for $I=500$ nA. This establishes that in *low fields* $L_\phi(I) \gtrsim L \gg W$, for the current levels up to $I_c$ values observed in the narrow samples.

One proposed model for the onset of dissipation is Zener breakdown between Landau levels caused by the Hall electric field.[7,8] In Ref. 8, Eaves and Sheard estimate a critical current density $J_c(\text{Zener}) \simeq 27.2$ A/m at the center of the $i=2$ plateau. A typical measured value on a sample with $W_L \simeq 0.47$ $\mu$m (yielding $W \simeq 80$–$120$ nm is 5.0–7.5 A/m. An alternative mechanism attributes the onset to phonon generation which is energetically allowed when the local electronic drift velocity $v_d = E/B$ exceeds a sound velocity $v_s \simeq 2.5$–$5.3$ km/sec (phonon Cherenkov radiation).[9,10] From this we may infer a critical current density $J_c = (ie^2/h)Bv_s \simeq 1.1$–$2.5$ A/m on the $i=2$ plateau, about four times smaller than the observed value. An electronic width 3–4 times greater than that calculated from the Hall traces would bring the measured critical current density down to the phonon limit, but then the electrical width would have the feature of being at least the lithographic width. However, since samples with $W_L < 0.40$ $\mu$m are nonconducting at low temperatures, the electrical width is expected to be at least $\simeq 0.40$ $\mu$m less than $W_L$. A third explanation for breakdown is that the fraction of extended states in a Landau-level increases with the Hall field.[1,11] This elegantly accounts for the triangle pattern seen in the large sample but fails to predict the saturation effect in the narrow ones. Moreover, none of the models as they stand now without mesoscopic features can explain the anisotropy of $I_c$.

We propose a fourth model that agrees qualitatively with the experimental observations. Each current-carrying state can transport current up to $i_k \sim (1/BL) \times(\partial\epsilon/\partial x) \sim (1/4\pi\epsilon_0)(e/L)(e^2/\kappa\hbar)$ on a Hall plateau.[18] Here $\kappa \simeq 13$ is the static dielectric constant of GaAs, $x$ is the coordinate perpendicular to the average current, and $\epsilon$ is the electron energy, taken to be the Coulomb energy. If we assume a one-dimensional density-of-states, then most of the current-carrying states belong to the highest-lying Landau level/sub-band. If this level contains $N_c$ such states, then this bounds the critical current by $N_c i_k$. This is independent of $B$ for a given level index $i$. Using $I_c \simeq 600$ nA, we find that we require $N_c \sim 22$ ($L/\mu$m) current-carrying states for $i=2$. A similar value for $N_c$ results from the following argument. In Ref. 18 Heinonen and Taylor develop a numerical model for conduction on a Hall plateau in the absence of dissipation for samples a few $l_B$ wide. Including Hartree terms, they find the current to be carried by a few states in each Landau level spaced $\sim l_B$ apart. This separation arises from the Coulomb repulsion $(1/4\pi\epsilon_0)(e^2/\kappa l_B) \simeq \hbar\omega_c$ between occupied states. As the current is increased from zero the occupancy of the current-carrying states approaches one. In order to carry more total dissipationless current, more such states are required. The number of these states, however, is limited by their Coulomb repulsion, yielding $N_c \sim W/l_B \simeq 10$. For a mesoscopic sample, the spatially anisotropic potential variations make $N_c$ depend on the current direction because different directions of current are distributed differently over the sample in a magnetic field. A difference of $\sim 10$ ($L/\mu$m) states from side to side can account for the anisotropy of $I_c$. The saturation effect is absent in the large control sample because $N_c$ is then $\sim 10^3$ times bigger, and another mechanism must then be responsible for breakdown.

In conclusion, we have observed saturation and anisotropy of the critical current in the breakdown of the quantum Hall effect in narrow structures. The experiments indicate that the mesoscopic nature of these systems must be incorporated into any interpretation of the onset of dissipation in this regime.

[1]H. L. Stömer, A. M. Chang, D. C. Tsui, and J. C. M. Hwang, in *Proceedings of the 17th International Conference on the Physics of Semiconductors,* edited by D. J. Chadi and W. A. Harrison (Springer, New York, 1985).

[2]M. E. Cage, R. F. Dziuba, B. F. Field, E. R. Williams, S. M. Girvin, A. C. Gossard, D. C. Tsui, and R. J. Wagner, Phys. Rev. Lett. **51**, 1374 (1983).

[3]G. Ebert, K. von Klitzing, K. Ploog, and G. Weimann, J. Phys. C **16**, 5441 (1983).

[4]F. Kuchar, R. Meisels, G. Weimann, and H. Burkhard, in *Proceedings of the 17th International Conference on the Physics of Semiconductors,* edited by D. J. Chadi and W. A. Harrison (Springer, New York, 1985).

[5]L. Bliek, G. Hein, V. Rose, J. Niemeyer, G. Weimann, and W. Schlapp, in *Proceedings of the International Conference on the Application of High Magnetic Fields in Semiconductor Physics, Wurtzburg, West Germany, 1986,* edited by G. Landwehr (Springer-Verlag, Heidelburg, 1987).

[6]J. R. Kirtley, Z. Schlesinger, T. N. Theis, F. P. Milliken, S. I. Wright, and L. F. Palmateer, Phys. Rev. B **34**, 1384 (1986).

[7]D. C. Tsui, G. J. Dolan, and A. C. Gossard, Bull. Am. Phys. Soc. **28**, 365 (1983).

[8]L. Eaves and F. W. Sheard, Semicond. Sci. Technol. **1**, 346 (1986).

[9]O. Heinonen, P. L. Taylor, and S. M. Girvin, Phys. Rev. B **30**, 3016 (1984).

[10]P. Streda and K. von Klitzing, J. Phys. C **17**, L483 (1984).

[11]S. A. Trugman, Phys. Rev. B **27**, 7539 (1983); S. Luryi and R. F. Kazarinov, *ibid.* **27**, 1386 (1983); S. Luryi (unpublished).

[12]G. Timp, A. M. Chang, P. Mankiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, R. E. Howard, Phys. Rev. Lett. **59**, 732 (1987).

[13]B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).

[14]P. G. N. de Vegvar, G. Timp, P. M. Mankiewich, R. E. Behringer, and J. E. Cunningham (unpublished).

[15]M. Büttiker, Phys. Rev. Lett. **57**, 1761 (1986).

[16]R. E. Behringer, P. M. Mankiewich, and R. E. Howard, J. Vac. Sci. Technol. B **5**, 358 (1987).

[17]H. Baranger (private communication).

[18]O. Heinonen and P. L. Taylor, Phys. Rev. B **32**, 633 (1985).