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Aharonov-Bohm effect in the hopping conductivity of a small ring

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By a direct numerical simulation of variable-range-hopping conduction we study the magnetoconductance oscillation in a strongly disordered small ring as a function of the magnetic flux through its aperture. We find well-defined Aharonov-Bohm oscillations with flux periodicity $\phi_0 = h/e$ and $\phi_0/2$ in the hopping conductance provided the ring is small and the temperature low. In the large-ring limit and for higher temperature we find that only the $\phi_0/2$ oscillations survive, which is consistent with the recent experimental finding of Poyarkov *et al.*

There has been a lot of recent interest[1] in the Aharonov-Bohm–type interference phenomena in small condensed matter systems such as metal rings of mesoscopic dimensions. Most of the current work has been on metallic systems where electron motion is diffusive[2–5] (with the system size being much larger than the elastic mean free path). Some very recent publications[6–8] deal with interference effects in semiconductor microstructures where electron motion is ballistic. The subject of this paper is to discuss the Aharonov-Bohm effect in a still different transport regime, namely conduction in a strongly disordered system where the electronic transport is via the hopping process.

In an earlier paper, Nguyen, Spivak, and Shklovskii[9] considered theoretically the issue of Aharonov-Bohm oscillations in the hopping conductivity of a strongly disordered system. Based on strictly zero-temperature simulations of a simplified model Nguyen *et al.*[9] claimed that there is a phase transition in the period of oscillation from the flux period h/e (Aharonov-Bohm) to $h/2e$ (Sharvin and Sharvin) as a function of the degree of compensation x of the system. For $x < x_c = 5\%$ they found pure Aharonov-Bohm oscillation (h/e period) whereas for $x > 5\%$ the period was found to be $h/2e$ in their numerical study. The value of x_c, the critical disorder, was found to be independent of the system size in their simulation at $T=0$. We emphasize that the tight-binding Anderson model simulation of Ref. 9 is necessarily restricted to $T=0$ where, in principle, no hopping conduction can take place.

We have carried out a numerical simulation of the finite-temperature hopping conductivity in a ring geometry which we believe to be more appropriate for studying the Aharonov-Bohm effect than the grid geometry used in Ref. 9. In contrast to the simulation of Ref. 9 where only the overlap integral between perfect lead terminals was calculated within a simple tight-binding Anderson model on a square lattice, we carry out an actual variable-range-hopping (VRH) transport calculation using the critical path percolation model of Ambegaokar, Halperin, and Langer.[10] This model has recently been used successfully[11] to study fluctuations in the low-temperature hopping conduction in narrow silicon metal-oxide-semiconductor field-effect transistors.[12] Our results in the ring geometry disagree with the results (in the grid geometry) of Nguyen *et al.*[9] in one important aspect—we find that at $T=0$ the value x_c of the critical disorder depends on the system size and goes to zero for infinitely large rings. At higher temperatures our hopping transport simulation indicates that x_c is always zero. Thus, our results indicate that there is no phase transition, and *only* the $h/2e$ flux-periodic oscillations survive at finite temperatures (or, for larger rings at $T=0$) in the presence of any finite disorder ($x \neq 0$). We believe that our finite-temperature results are valid for other geometries with equivalent topology. Our results are consistent with the very recent experimental findings of Poyarkov *et al.*[13] and may explain why they observe *only* the $h/2e$ period oscillations in the magnetoresistance of disordered oxidized PbTe films and *no* h/e period. We have also done an analysis of the temperature dependence of the ensemble-averaged conductivity of the system, and find that, in agreement with the experimental results, Mott's variable range hopping law is obeyed (we may add that the simplified model of Ref. 9 is inadequate for studying the temperature dependence of conductivity and, in this sense, is a rather poor model since conduction by hopping can take place only at nonzero temperatures.) We point out, however, that any comparison between our theoretical simulation and the experimental work of Ref. 13 could be somewhat inappropriate because of the difference in the two geometries.

Our model consists of a ring (connecting the perfect source and sink current leads) along which electron transport takes place via the variable range hopping mechanism. Each arm of the ring is assumed to be one dimensional (i.e., single channel) for simplicity. Transport takes place via hopping of an electron through localized states distributed along the ring. The effect of the magnetic field is incorporated by assuming that the electronic wave function along one arm of the ring has an additional phase factor $e^{i\phi/\phi_0}$ compared with that in the other arm where ϕ is the flux through the aperture of the ring and $\phi_0 = h/e$. We assume that the magnetic field does not penetrate the ring itself (i.e., the ring is taken to be ultrathin) which allows us to neglect Zeeman effect or other such direct effects of the magnetic field itself on the hopping process.[14]

The numerical simulation of the variable range hopping

is carried out in the standard fashion[11] by using the Miller-Abrahams resistive network model. Instead of obtaining an exact numerical solution of the resistive network we follow the percolation model suggested by Ambegaokar, Halperin, and Langer[10] and calculate the resistance of the network by using the critical resistance technique. We have also carried out an analysis similar to that of Nguyen *et al.*[9] by using an Anderson model to calculate the overlap integral between the source and the sink for our ring geometry. We do not find any phase transition around $x=x_c\approx 0.05$ (where the flux periodicity changes from h/e to $h/2e$) for large rings. For smaller rings, we find, in contrast to the results of Ref. 9, a systematic dependence of x_c on ring size, and x_c tends to vanish for large rings indicating that only the $h/2e$ oscillations exist. These results (at $T=0$) based on the Anderson model are consistent with our more rigorous finite-temperature variable-range-hopping calculations described below.

The conductivity in the presence of the magnetic flux goes as $\sigma\sim|A_1+A_2e^{2\pi i\phi/\phi_0}|^2$, where A_1 (A_2) is the amplitude from the upper (lower) arm of the ring. For the calculation of $|A_i|^2$ ($i=1,2$), we use the standard Miller-Abrahams resistive network model and the percolation technique as stated above. The percolation model does not give us the sign of A_i which is important for interference effects. We obtain the sign by noting that at $T=0$,

$$\sin(A_i)\,|\,i=1,2=\sin[\pi_i(E_j-\mu)]\,|\,i=1,2\ ,$$

where E_j is the energy of the random impurities. In our calculation we make the reasonable assumption that this sign is unaltered at low but finite temperatures.

We show our simulation results based on the percolation model in Figs. 1–3. In Fig. 1 we show the ensemble averaged magnetoconductance as a function of the flux through the ring aperture at a fixed low temperature $k_BT=0.001$ (energy units are such[11] that the localized electron levels are distributed randomly around the chemical potential in an energy window between ± 1 with x defining the fraction of these states below the chemical potential). The ensemble averaging has been done by averaging over about 1000 different rings. In Fig. 1(a) we show our results for a very small ring with the length of each arm of the ring being $L=200$ (200 impurities along each arm of the ring). One can clearly see that for small $x=0.002$ the period of oscillation is h/e whereas for the larger value of $x=0.1$ the period of oscillation is $h/2e$. Thus, our results for a small ring are consistent with that of Ref. 9. However, in Fig. 1(b) we show our results for a larger ring with $L=2000$ where one has to go to much smaller values of compensation ($x=0.0002$) before h/e oscillations show up. Thus, in contrast to the results of Nguyen *et al.*[9] we find a systematic dependence of the critical disorder x_c, on the system size L and, in fact, for large L, x_c vanishes. We show this in Fig. 2 where we plot our calculated value of x_c against L which shows that for large rings only the $h/2e$ Sharvin-Sharvin–type oscillations can be seen in the hopping conductivity.

We believe that for finite-temperature hopping transport $\phi_0/2$ is the only allowed period for the following

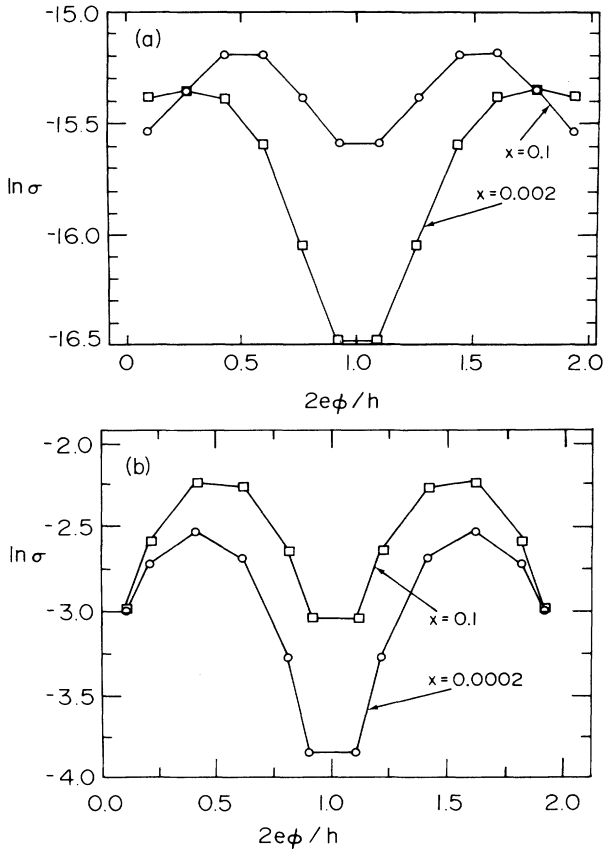

FIG. 1. $\ln\sigma$ as a function of the magnetic flux at $T=0.001$ for rings with lengths $L=200$ (a) and 2000 (b). Results for two different values of disorder x are shown so that both ϕ_0 (low x) and $\phi_0/2$ (high x) oscillations can be seen. Note that one needs much lower x in (b) to see ϕ_0 oscillations.

reason. As we increase the temperature slightly so that VRH hopping is still the dominant transport mechanism, we expect that the sign of the conductivity amplitude remains the same as that of the weak link in the hopping path. It then follows that different rings in the ensemble give randomly different signs. Consequently, the ensem-

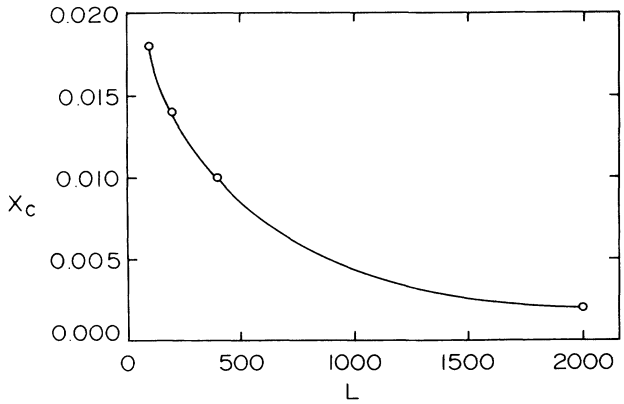

FIG. 2. The critical value of the disorder x_c (above which the ϕ_0 oscillations do not exist) as a function of the system size L.

ble averaged logarithmic conductivity $\langle \ln\sigma \rangle$ has only a $\phi_0/2$ period, the ϕ_0 period being averaged out. Thus, the curves with period ϕ_0 in Fig. 1 disappear at higher temperatures.

Finally, in Fig. 3 we plot our ensemble-average value of $\ln\sigma$ (where σ is the conductivity) against the temperature (at a fixed magnetic field) to show that one is indeed in the Mott's variable-range-hopping region. Since we have assumed the transport along each arm of the ring to be one dimensional (no interchannel hopping being allowed), we find that $\langle\sigma\rangle \sim e^{-(T/T_0)^{-1/2}}$ whereas in the experimental results of Poyarkov *et al.*,[13] the variable-range-hopping exponent is the two-dimensional value of $\frac{1}{3}$ rather than $\frac{1}{2}$ since their hopping process is along two-dimensional grids. But, as we have shown in a different context, two-dimensional simulations allowing interchannel hopping gives[11] the expected exponent $\frac{1}{3}$.

One of the puzzling issues raised by our simulation and the experimental results[13] is the basic question of the existence of long-range phase coherence in the hopping conduction. Hopping transport is an inherently inelastic process since it involves absorption and emission of phonons in allowing the strongly localized electrons to make transitions. Existence of well-defined quantum-interference oscillations (h/e or $h/2e$) implies that somehow quantum-phase memory is not completely destroyed in spite of these phonon assisted transitions.[15] Experimental results[13] clearly show that phase coherence is not destroyed completely in the hopping process. One possibility is that phase memory is destroyed exponentially as e^{-L/L_i} where L_i is some "typical" inelastic scattering length and, even in a variable-range-hopping process, phase coherence is not completely destroyed unless $L \gg L_i$. We can, for example, take $L/L_i \approx n$, where n is the number of hops the electron typically makes in traversing the distance between the source and the sink. In our simulation (even for the larger rings) we have taken care to keep $n \lesssim 3$, which ensures that substantial phase memory is maintained throughout the hopping process leading to the possibility

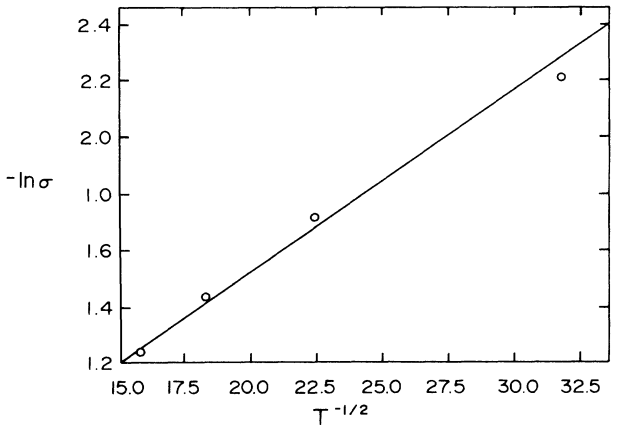

FIG. 3. The magnitude of $\ln\sigma$ as a function of $T^{-1/2}$. The straight line plot implies that the Mott's variable-range-hopping exponent is $\frac{1}{2}$.

of quantum-interference effects. One explanation[16] for this is that the destruction of the phase memory of the electrons depends on the energy changes in the hopping process. In the VRH problem, the energy changes involve long-wavelength acoustic phonons, and, are, therefore, very small causing only small deviations in the phase. This is consistent with the recent theoretical finding in Ref. 15. There are also strong experimental indications[17] that phase memory is not completely destroyed in hopping process assisted by long wavelength low-energy acoustic phonons. More experimental and theoretical investigation is needed to clarify this point.

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