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Backscattering of light near the optical Anderson transition

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We propose that the optical Anderson transition can be observed by studying the coherent backscattering peak near the Ioffe-Regel limit $l \simeq \lambda$. We carry out a scaling theory for the line shape of the backscattered peak above and below the transition. The line shape of the peak exhibits a marked change as the disorder increases, being different below and above the transition.

The phenomenon of weak localization[1-3] has recently been applied to photons.[4-9] For electrons, weak localization leads to quantum interference which reduces the conductivity even in the weak-disorder limit. For classical waves, it leads to a narrow backscattering peak (BSP) with a width of order λ/l, where λ is the wavelength of the scattered wave and l is the elastic transport mean free path. This phenomenon was first observed[4-6] for polystyrene spheres suspended randomly in water and later observed[7,8] for random solids which also show backscattered speckle analogous to the universal conductivity fluctuations.[10,11] After an ensemble average is performed,[7,8] the backscattered intensity reveals the same narrow BSP that is observed for random fluids. Thus, weak-localization effects are general phenomena that occur even for classical waves as a BSP. The theory for the line shape was first given[12] using a diffusion approach, but taking into account the optical time-reversal trajectories, which leads[12] to

$$I = I_0 \iiint dz\, dz'\, d^2R\, F(z,z') Q(\bar{r},\bar{r}')[1+\cos\bar{q}\cdot(\bar{r}-\bar{r}')] , \tag{1}$$

where $Q(\bar{r},\bar{r}')$ is the random-walk probability[8,12] in an absorbing plane (which we define later) and $F(z,z')$ is defined by Eq. (1) in Ref. 12. For general boundary conditions, the theory[12] predicts a line shape which crosses over from a $1/q$ behavior at small angles to $1/q^2$ behavior at larger angles. Exact calculations, which take into account all the ladder diagrams and the maximally crossed diagrams,[13-15] have verified this general line shape and confirmed the results obtained by the "diffusion" approach.[12] Recently, a precise measurement[16] of the line shape of the backscattered peak has revealed the crossover from the $1/q$ dependence to the $1/q^2$ dependence. Moreover, the generalized theory[17,18] which also includes polarization effects was found[16] to be in excellent agreement with the precise measurements of the BSP. The theory of the backscattered peak was recently extended to finite slabs[18-20] and to two-dimensional systems.[21]

In this Rapid Communication, we extended the theory of the backscattering of light to strong-disorder systems near the optical Anderson transition. We develop a scaling theory for the backscattered peak both above and below the transition, finding that the line shape of the peak is modified near the transition. Our theory of the line shape of the BSP may serve as a new tool to probe the features of the optical Anderson transition, which is the experimental goal in this new field. The line shape of the BSP above and below the Anderson transition is directly related to the behavior of the optical diffusion constant. Thus, instead of measuring the diffusion constant, we suggest that the BSP includes all the information needed about the transition.

The optical Anderson transition occurs when the Ioffe-Regel condition is reached[22-25]

$$l \simeq \lambda . \tag{2}$$

The scaling theory[26] of the Anderson transition states that the diffusion constant tends continuously to zero as the disorder is increased. Near the Anderson transition, the diffusion constant is scale dependent $D = D(L,\xi)$, where ξ is the correlation length[27] above the transition and the localization length below it.[2,3] For $L < \xi$, the diffusion constant is identical[26,27] on *both* sides of the transition, and given by

$$D = D_0(l/L),\ L < \xi , \tag{3}$$

where D_0 is the diffusion constant without the interference effects, $\frac{1}{3}Cl$. The crossover in the length dependence of D from weak disorder to strong disorder has recently been studied.[25] For $L < \xi$, D is no longer symmetric on both sides of the transition. Here, for $L > \xi$, D is given by[26]

$$D = \begin{cases} D_0 l/\xi & \text{above the transition} \\ 0 & \text{below the transition} . \end{cases} \tag{4}$$

This general scaling behavior of D can be converted[28,29] into a generalized diffusion constant $D(q,\omega)$. As will be shown below, for the theory of the BSP we need $D(q,\omega \to 0)$, which is given by

$$D(q,\omega\to 0) = \begin{cases} D_0, & q > l^{-1} \\ D_0 lq, & \xi^{-1} < q < l^{-1} \\ D_0 l/\xi, & q < \xi^{-1} , \end{cases} \tag{5}$$

above the transition. Below the transition, we find the *same* behavior for $q > \xi^{-1}$. However, for $q < \xi^{-1}$, we find that $D = 0$. As seen from Eq. (1), the backscattering intensity is essentially the two-dimensional Fourier transform of the random-walk probability

$$Q(\bar{r},\bar{r}') = P(\bar{r}-\bar{r}') - P(\bar{r}-\bar{r}'^{*}) ,$$

where r' is the starting point of the photon random walk and $\bar{r}'^{*}$ is the image of $\bar{r}'$ in the mirror plane.[12,13] $P(\bar{r})$ is

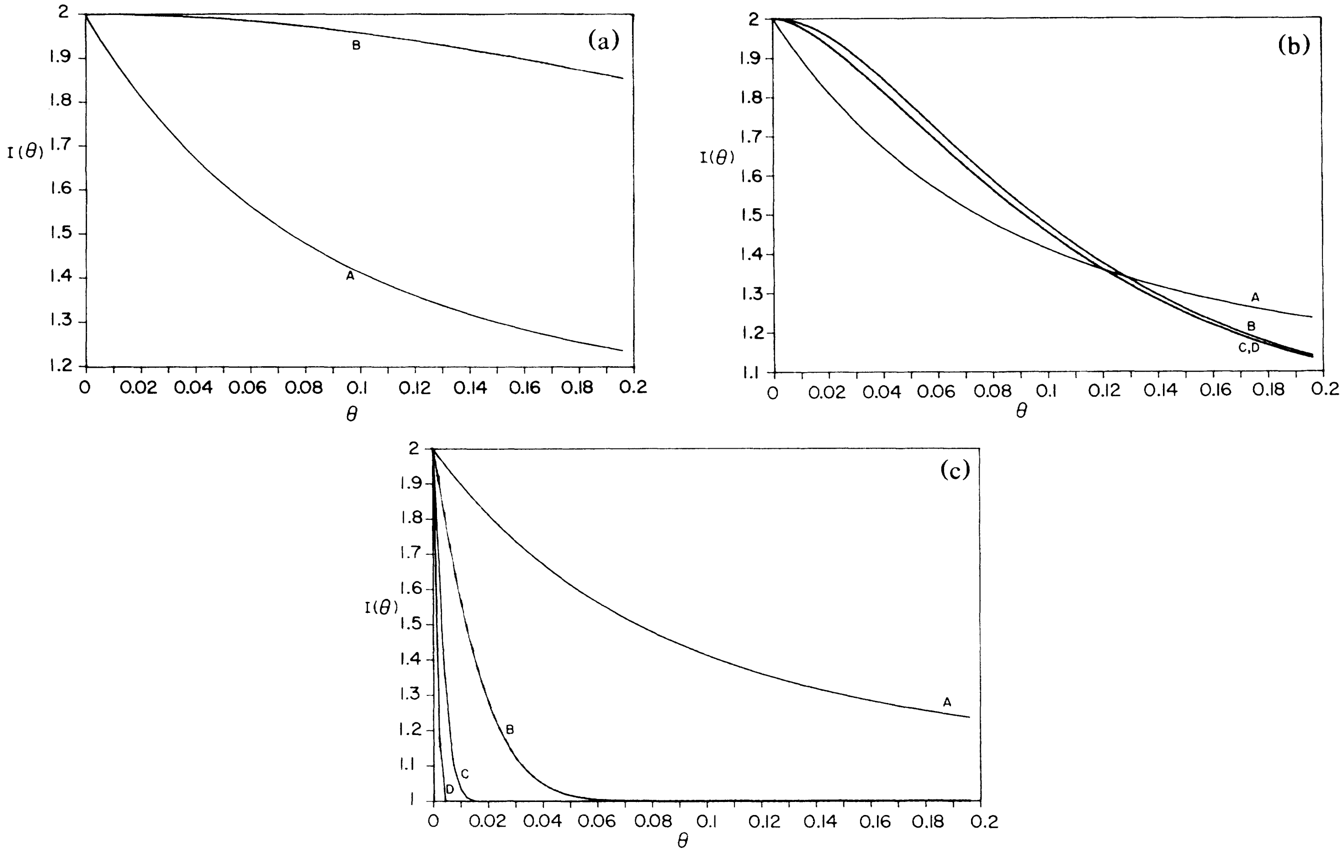

FIG. 1. The curves in (a)–(c) which are denoted by A correspond to the line shapes of $I_0(q)$ as a function of angle $\theta=(\lambda/2\pi)q$. (a) $I_1(q)$ as a function of θ, curve B. (b) $I_2(q)$ as a function of θ, the curves B, C, and D represent different values of ξ; $\xi=10l$, $100l$, $1000l$. (c) $I_3(q)$ as a function of θ, the curves B, C, and D represent different values of ξ; $\xi=10l$, $100l$, $1000l$.

given by

$$P(r)=(2\pi)^{-3}\int d^3q\, e^{i\vec{q}\cdot\vec{r}}n(\mathbf{q},\omega=0) \ , \tag{6}$$

where $n(q,\omega)$ is the time-and-space Fourier transform of the $n(\vec{r},t)$, the solution of the diffusion equation. From Eq. (6), it is clear that we need $D(q,\omega=0)$ which enters in $n(q,\omega=0)$ as defined by (5). From (5) and (6), it follows tht the BSP can be divided into *three* regions,

$$I(q)=I_1(q)+I_2(q)+I_3(q) \ , \tag{7}$$

corresponding to the three regions, respectively, in q that are given in (5). Each of the three contributions to $I(q)$ behaves entirely different. The quantity $I_1(q)$ is related to the region $q>l^{-1}$ where $D=D_0$. Consequently, it is expected that the asymptotic behavior of $I_1(q)$ will be the same as that for $I(q)$ far from the transition [where $I_1(q)$ is the only contribution]. Indeed, we find, for $q>l^{-1}$, $I_1(q)\sim 1/q^2$. The behavior of $I_2(q)$ is different because $I_2(q)$ is calculated with $D=D_0lq$. We find that the asymptotic behavior of $I_2(q)$ is $1/q^3$, which is consistent with a recent diagrammatic calculation.[15] The contribution of $I_3(q)$ is most important because it is the source of the difference in the BSP above and below the Anderson transition. Below the transition, $D=0$ and $I_3(q)$ does not contribute to $I(q)$.

Before discussing further our final results as presented in Figs. 1(a)–1(c) and Figs. 2(a) and 2(b), we point out that $I_2(q)$ contains a constant background that if treated improperly leads to an artificial logarithmic divergence when $\xi\to\infty$. Consequently, such logarithmic divergence would also appear in the total reflectance[30] R; this of course is unphysical since, for a semi-infinite space geometry, R must always be unity. This artificial logarithmic divergence arises[31] only if one makes the same assumption as in the weak-disorder limit that the emitted photons are similar to the entering photons leaving the sample at a distance $z=a$ $(a=1.7l)$ from the boundary $z=0$. This is incorrect when the diffusion is anomalous as described by Eq. (5). This is best seen in the time domain where it is easy to show[28] that Eq. (5) corresponds to a time-dependent diffusion constant given above the transition by

$$D=\begin{cases} D_0, & t<\tau \\ D_0(t/\tau)^{-1/3}, & \tau<t<\xi^2/D_0 \\ D_0(l/\xi), & t>\xi^2/D_0 \ , \end{cases} \tag{8}$$

with the same behavior below the transition except that $D=0$ for $t>\xi^2/D_0$. The diffusion equation with a mirror image as a solution as introduced by Akkermans, Wolf, and Maynard[12] *is* still valid. The solution of the present diffusion equation is of course different and is given exactly by

$$n(R,z,t)=(6\pi D_0\tau^{1/3}t^{2/3})^{-3/2}\{\exp[R^2+(z-a)^2/6D_0\tau^{1/3}t^{2/3}]-\exp[R^2+(z+a)^2/6D_0\tau^{1/3}t^{2/3}]\} \ . \tag{9}$$

The reflectance is then given by $R=F(t=0)-F(t=\infty)$, where

$$F(t)=\int n(\bar{R},z,t)d^2R\,dz \ ,$$

which leads to the expected result $R=1$. The problem arises when we calculate the kernel $Q(\bar{r},\bar{r}')$ in Eq. (1) where we need the *exit* probability $p(R)=\int n(R,z,t)dt$ from a given plane $z=z_0$. If one were to assume (incorrectly) that all photons exit the system once they arrive again at $z=a$, then $P(R)=\int n(r,z_0=a,t)dt$ would diverge logarithmically. This assumption is correct only when D is time independent. Near the optical transition where (8) is valid, large photon trajectories which correspond to large times are characterized by a diffusion constant which continuously decreases. Thus, the photon will exit the material at a plane z_0 *closer* to the boundary. This plane depends on the time spent in the material and the correct exit plane is given by $z=z_0(t)$ where $z_0(t)=a(t/\tau)^{-1/3}$. The exit probability is now $P(R)=\int n(R,z_0(t),t)dt$ and is *finite* without the artificial logarithmic divergence.[32] We have calculated the three contributions of $I(q)$ in Eq. (7) in the time-domain approach by using (8) and in the q domain by using Eq. (5) and (6) and have obtained identical results. In the q domain approach each mode q has a different diffusion constant and the exit plane z_0 will be q dependent (analogous to the time-domain approach). Thus, in Eq. (6), $P(\bar{R},\bar{z})$ must be obtained from

$$P(\bar{R},\bar{z})=\int \exp[i\bar{q}\cdot(\bar{R}+\bar{z})]n(\bar{q},\omega=0)d^3q \ ,$$

where we use $z=lqa$ instead of $z=a$ which applies for a constant D. This removes the artificial logarithmic singularity as discussed above. We see that the boundary does *not* lead to new conceptual difficulties in the scaling theory for the optical Anderson transition. We still obtain a coherent backscattered peak even *below* the transition once the localization length satisfies $\xi\gg l$. The anomalously diffusive trajectories lead to constructive interference and to a BSP. However, the line shape of the BSP is modified in this region as compared to the weak-disorder limit $l\gg\lambda$.

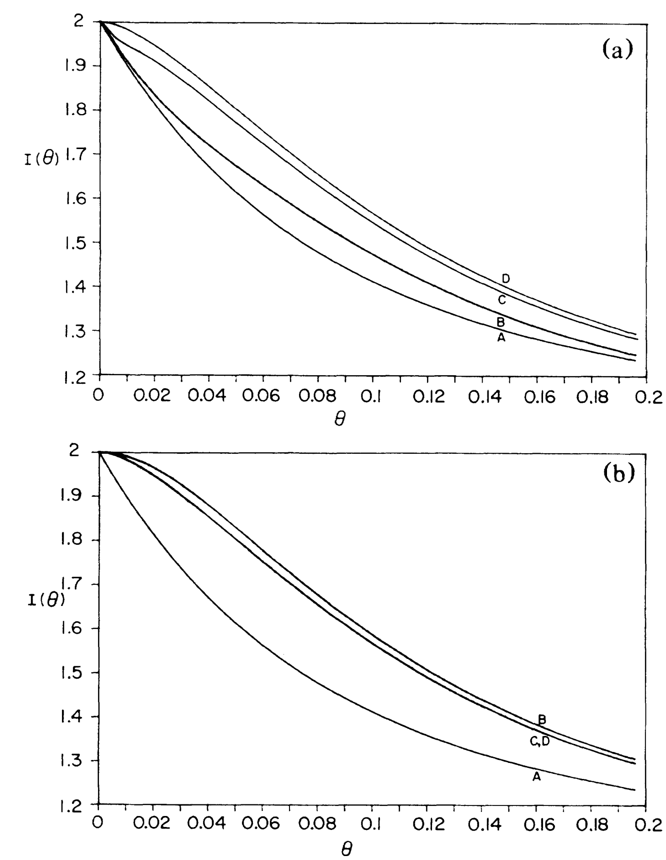

FIG. 2. The curves in (a) and (b) which are denoted by A correspond to the line shapes of the BSP of $I_0(q)$ as a function of θ. (a) $I(q)$ as a function of θ *above* the transition with curves B, C, and D correspond to different values of the correlation length ξ; $\xi=10l$, $100l$, $1000l$. (b) $I(q)$ as a function of θ below the transition with curves B, C, and D corresponding to different values of the localization length ξ; $\xi=10l$, $100l$, $1000l$.

We now analyze the difference in the line shape for the BSP near the optical Anderson transition. In Figs. 1(a)–1(c), we plot the three contributions to the BSP and compare them to the BSP $I_0(q)$ which is obtained if we use the weak-scattering approximation $D=D_0$ instead of (5) or (8). Figure 1(a) corresponds to $I_1(q)$ which results from small loops and therefore its half-width is much wider than $I_0(q)$. In Fig. 1(b), we plot $I_2(q)$ which corresponds to the anomalous behavior of $D(q,\omega)$ in the region $\xi^{-1}<q<l^{-1}$. $I_2(q)$ is rounder for small angles in contrast with the triangular shape of $I_0(q)$. $I_2(q)$ becomes broader as ξ becomes smaller. For large angles, $I_2(q)$ crosses $I_0(q)$ and eventually falls more rapidly due to the *small* values of the diffusion constant $D=D_0ql$. In Fig. 1(c), we plot $I_3(q)$ which is seen to be very narrow with a half-width of order λ/ξ. Thus, as ξ becomes larger, $I_3(q)$ becomes very narrow. In Figs. 2(a) and 2(b), we plot the total BSP $I(q)$ as a function of θ $[q=(2\pi/\lambda)\theta]$ both above and below the transition. For $\xi\rightarrow\infty$, both line shapes of $I(q)$ above and below the transition *coincide* and show a much *broader* line shape than expected from the weak-disorder limit given by $I_0(q)$. In particular, we find that for $\xi>100l$, the line shapes of the BSP below and above the transition become very similar with a *rounded* (nontriangular) shape on both sides of the transition. The asymptotic behavior of $I(q)$ is dominated by the contribution of $I_1(q)$ which falls off as $1/q^2$. Consequently, we find that the best fit to our calculated values of $I(q)$ is given by $I(q)\sim 1/q^{1.9\pm 0.1}$. As ξ becomes shorter, the contribution of $I_3(q)$ becomes more important. This leads to an *asymmetry* in the line shape of the BSP below and above transition. For example, as seen from Figs. 2(a) and 2(b), for $\xi = 10l$ the shape of the BSP below the transition is *very* rounded, whereas above the transition it is almost triangular. Far below the transition for extremely strong disorder, as ξ becomes of the order of interparticle distances, we expect a totally flat curve with no BSP. We hope that the above predictions will enable the experimentalist to locate the onset of the optical Anderson transition which is the next goal in this new and rapidly growing field.

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[30]We are grateful to E. Akkermans for pointing this out to us.

[31]For the sake of simplicity, we discuss the solution of this problem for the simplified expression for the BSP given in Ref. 12, which assumes that for $Q(\bar{r},\bar{r}')$, $\bar{r}$ and $\bar{r}'$ are located on the same plane $z=a$. The required generalization to the form of Eq. (1) is straightforward.

[32]We also obtain the correct behavior for $F(t)$ for large t by using $F(t) = \int P(\bar{R},z_0(t),t)d^2R$, which decays as $t^{-4/3}$.