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Temperature and impurity dependence of the electrical resistivity in dilute gallium-based alloys. II. *a*-axis crystals

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Measurements of the temperature-dependent part of the electrical resistivity $\Delta\rho(T)$, have been carried out on a series of single-crystal gallium-zinc alloys. The impurity concentration ranges from nominally pure (<0.1 wt. ppm) to 2500 wt. ppm, and the temperature range covered extends from 1.2 to 7.0 K. At fixed temperature, when plotted against the logarithm of the residual resistivity, the quantity $\Delta\rho$ exhibits an unusual maximum at all temperatures above 4.0 K. The data are compared to previous results on gallium-based alloys oriented for current flow along the *b* and *c* axes. A phenomenological model which includes phonon drag is developed to fit the results.

I. INTRODUCTION

Experimental data on the resistivity of dilute alloys at low temperatures provide ample evidence of deviations from the simple additivity of the residual and temperature-dependent contributions to the resistivity which is embodied in Matthiessen's rule. Although numerous theoretical explanations[1] of the effect have been put forward, it appears that most experimental data can be adequately explained by what is referred to as the "isotropization" model.[2] This model assumes that the deviations from Matthiessen's rule (DMR) arise from the gradual transformation of the off-balance electron distribution from a highly anisotropic function in reciprocal space (as is to be expected for resistance dominated by inelastic electron-phonon scattering) to a smoothly isotropic function as the concentration of impurities is increased and elastic impurity scattering becomes more of a dominant contributor to the resistance. The theory thus predicts the eventual saturation of the resistivity at the "dirty" limit as the impurity scattering is increased. It should be noted that the several variants of the isotropization theory of electron scattering are analogous to the purely phenomenological two-band model.[3]

Conclusive experimental evidence of the existence of the dirty limit in the resistivity of dilute alloys has not yet been presented, and so the present study was undertaken with the intention of making measurements on a series of samples with closely controlled physical properties. The repeatability of the results from samples with the same nominal impurity content has always been problematical in DMR studies. Presumably, one major cause of this problem is nonhomogeneity of the alloys on a microscopic scale. For this reason, the base metal for alloying was chosen to be gallium since uniform single-crystal alloys can quite readily be grown, with resistivity values that are repeatable to within approximately 1% for the same solute concentration. In addition, the extremely high purity of gallium enables changes in resistivity to be detectable even with the addition of only 1 ppm of impurity solute. Thus, this metal would appear to be well suited for such a study on this count also, because a relatively large range of variation in residual resistivity can be covered and the alloy can still be considered to be dilute.

In a previous paper[4] (hereafter referred to as I), measurements of the temperature dependence of the resistivity of a series of dilute gallium-based single-crystal alloys were reported for current flow along the *b* [010] axis. In that case, the DMR were observed to increase monotonically with increasing impurity concentration. Allowing for sufficient latitude in the fit, it was demonstrated that the *b*-axis data can be readily accounted for by the isotropization model, even though no hint of a dirty-limit saturation is observed. Other experimental evidence has been presented in the case of aluminum[5] to lend support to this model. In that case, the slope of the magnetoresistance versus temperature is plotted for samples of varying impurity concentration in both high and zero magnetic fields. The fact that the curves tend to converge in a high field led the authors to conclude that the effect of the field is to eliminate the DMR because of the isotropization it imposes on the scattering, and that therefore the isotropization model adequately explains the deviations. The above-mentioned results do not, however, represent convincing evidence of this hypothesis. The relatively large magnitude of the magnetoresistance derivative in the pure metal is not taken into account and it is highly likely that the actual DMR are small enough that any variation with impurity concentration is buried in the scatter of the data. One piece of evidence that appears inconsistent with the isotropization model is the observation that a normalization has been found for the metals in group III*B* (Ref. 4) which reduces their DMR to a universal curve regardless of Fermi surface shape or relative anisotropy. Such behavior would indicate that some universal scattering mechanism, which is independent of the detailed electronic band structure, is responsible for the observed DMR. Another result[6] which cannot be readily explained by isotropization is the observed similarity of the temperature dependences in samples which exhibit impurity-scattering-dominant and boundary-scattering-dominant DMR. Thus, there are still a number of unanswered questions regarding the source of the deviations in these metals.

In an attempt to improve the accuracy and completeness of the DMR data for gallium-based alloys, the present study on a-axis [100] crystals was carried out. This axis was chosen in particular because, in contrast to the other crystalline orientations in gallium, it is observed that relatively large concentrations of solute can be included without perceptibly affecting the single-crystal growth. Hence it was anticipated that a wider range of variation in impurity concentration could be included in the series of samples under investigation. The temperature dependence of the resistivity was measured over a range of 1.2–7.2 K for ten samples with concentrations of zinc impurity ranging from nominally pure to 2500 ppm by weight. The residual resisitivity is observed to be a linear function of concentration over this entire span. For relatively pure samples, the a-axis results exhibit behavior that can be readily compared with previous data on gallium in agreement with the phenomenological model described above.

The results of the measurements on highly impure samples however, exhibit behavior which is quite different from that of the other metals mentioned above and in particular the other crystalline orientations of gallium. When plotted in the usual manner where the temperature-dependent part of the resistivity $\Delta\rho=\rho(T)-\rho_0$, is plotted against $\log_{10}\rho_0$, the logarithm of the residual resistivity, the usual monotonic increase is not observed. Instead, the curves pass through a maximum and then fall off at higher concentrations (higher values of ρ_0) to values that are lower than that of the extrapolated ideal contribution. These results will be discussed in terms of a possible phenomenological model which can be used to explain the data presented.

II. EXPERIMENTAL DETAILS

The resistivity of ten gallium single-crystal wires oriented for current flow along the a axis was measured in this study. The residual resistivity ranged from 0.2 to 300 nΩ cm. Each wire was grown from a supercooled liquid alloy that was injected into a clear Plexiglas$^{\text{TM}}$ mold and seeded by standard techniques. Alloys of different concentrations were prepared by diluting a carefully prepared and accurately assayed 2500 wt ppm parent stock with pure gallium. All mixing operations were carried out in an inert-gas environment to avoid oxidation and contamination. Each wire specimen was grown in the same mold and had the following dimensions: $0.125\times0.125\times6$ cm^3. Voltage probes were grown as an integral part of the specimen. X-ray diffraction was used to verify the desired crystal orientation by means of the Laue back-reflection technique. Crystals which were found to be more than 1° out of alignment were not used in this study.

Samples were mounted inside a helium cryostat of standard design on a special sample holder which has a thermal link to the liquid-helium bath in an otherwise adiabatic surrounding. The sample holder consists of a rectangular single-crystal gallium plate which is oriented so that its longitudinal axis lies along the a axis. Thermal contact with the liquid-helium bath is provided by a copper link. Temperature regulation is achieved by means of a heater connected in a feedback loop to a temperature sensor mounted on the backing plate itself. Resistivity measurements are made by means of a superconducting Lindeck bridge with feedback. A commerically built radio-frequency superconducting quantum interference device (rf SQUID) is used as a null detector in the circuit. The use of such a sensitive device allows a relative precision on the order of 0.01% to be achieved in the resistance measurements, in spite of the fact that specimen current was limited to 10 mA or less. This limitation was imposed in order to minimize magnetomorphic effects which might arise from the self-field of the measuring current. A more detailed discussion of the measurement technique appears in I.

III. THEORETICAL BACKGROUND

Previous data on DMR in dilute alloys at low temperature are readily compared with simple models which all exhibit monotonically increasing behavior with increasing residual resistivity. In fact, this common feature of almost all theoretical models of DMR has in the past led to difficulty in choosing between two or more equally plausible explanations for the effect. As mentioned earlier, however, the data for the a axis of gallium are unusual in that the dependence upon $\log_{10}\rho_0$ is not monotonic. Of all the many proposed explanations for DMR, there are few which predict a decrease in the temperature-dependent part of the resistivity as the residual resistance ρ_0 is increased in the dilute regime. In fact, the idea that increased impurity scattering can lead to a reduction in the dynamic-scattering rate is a rather subtle physical idea indeed. It is conceivable that the dilute-regime assumption is not valid beyond a certain impurity-concentration limit. However, the attendant Fermi surface distortion would be required to occur at the level of several hundreds of parts per million, and so this is not a plausible explanation. Assuming no distortion of the Fermi surface at these low solute concentrations, the one well-known mechanism where reduced dynamic resistance results from an increase in a secondary scattering process is phonon drag. We therefore will examine the theory of phonon drag as it applies to the electrical resistivity.

Neglecting quantum corrections, the variational entropy flow method of solving the coupled Boltzmann equations as developed by Ziman[7] can be used to obtain a closed expression for the effect of phonon drag on the resistivity. As might be expected, the effect is negative in its contribution to the resistivity, and in fact can reach an ideal limiting value which exactly cancels the ordinary ideal resistivity due to normal electron-phonon scattering, thus yielding a net resistivity of zero. This cancellation can occur because, at least under the simplifying assumptions which Ziman makes (no umklapp scattering), the two contributions to the electrical resistivity exhibit identical temperature variations at low temperatures. The phenomenon is of course much more apparent in the thermoelectric transport coefficients where it contributes a term that has a different temperature dependence from that of the electron diffusion contribution.

Ziman derives an expression which includes the effects of phonon drag on the electrical resistivity that is given by

$$\rho=\rho_0+\rho_l\left[1-\frac{P_{1L}^2}{P_{11}P_{LL}}\right]=\rho_0+\rho_l+\rho_g\ , \tag{1}$$

where with current flow in the direction of the unit vector $\hat{\mathbf{u}}$ the factors in the second equality represent scattering integrals of the form:

$$P_{11}=\frac{1}{kT}\int\int\int[(\mathbf{k}-\mathbf{k}')\cdot\hat{\mathbf{u}}]^2P_{k,k',q}d\mathbf{k}\,d\mathbf{q}\,d\mathbf{k}'\ , \tag{2}$$

$$P_{1L}=\frac{1}{kT}\int\int\int[(\mathbf{k}-\mathbf{k}')\cdot\hat{\mathbf{u}}](\mathbf{q}\cdot\hat{\mathbf{u}})P_{k,k',q}d\mathbf{k}\,d\mathbf{q}\,d\mathbf{k}'\ , \tag{3}$$

$$P_{LL}=\frac{1}{kT}\int\int\int(\mathbf{q}\cdot\hat{\mathbf{u}})^2P_{k,k',q}d\mathbf{k}\,d\mathbf{q}\,d\mathbf{k}'\ . \tag{4}$$

The quantity ρ_l is the lattice resistivity calculated in the absence of phonon drag, i.e., the equilibrium lattice resistivity, and $P_{k,k',q}$ is the probability of scattering of an electron in state $\mathbf{k}$ to state $\mathbf{k}'$ by a phonon of wave vector $\mathbf{q}$. In Ziman's terminology, ρ_l can be expressed as

$$\rho_l=P_{11}/J_1^2\ ,$$

where J_1 is a band-structure constant.

These mixed electron-lattice transport integrals also occur in the expressions for the phonon drag or lattice thermopower, S_g, and for the lattice thermal conductivity, κ_g, where

$$S_g=\frac{U_LP_{1L}}{TJ_1P_{LL}}\ , \tag{5}$$

$$\kappa_g=\frac{U_L^2}{TP_{LL}}\ , \tag{6}$$

and U_L is the heat current carried by the lattice.

On forming the ratio S_g^2/κ_g, it can be seen that an expression proportional to the phonon-drag contribution to the electrical resistivity is obtained:

$$S_g^2/\kappa_g=\frac{P_{1L}^2}{J_1^2P_{LL}T}=\rho_g/T\ . \tag{7}$$

Although greatly oversimplied, this theory does allow an estimate of the phonon-drag contribution to the electrical resistivity to be made in terms of other transport coefficients.

In assessing the possible effect of phonon drag on the DMR, it is necessary to adopt a simple model for the DMR. We assume that the equilibrium lattice resistivity as it appears in Eq. (1) is modified by the substitution of the lattice resistivity with DMR, ρ_l' for ρ_l everywhere. If a simple two-band model for the DMR is assumed, then the monotonic variation of ρ_l' with increasing ρ_0 can be approximated by $\rho_l'\approx\rho_i+af(\rho_0)$, where $f(\rho_0)$ is a function that increases monotonically in the region well below the dirty limit, but which ultimately saturates at high ρ_0.

Kaveh and Wiser[8] have used similar considerations in calculating the effects of phonon drag on the pure lattice resistivity of potassium. These authors argue that the electron-lattice transport integrals in Eqs. (2)–(4) can each be split into two terms at low T: a normal contribution and an umklapp contribution. Furthermore, since the normal contributions involve the same integral over the Fermi surface, they make identical contributions to P_{11}, P_{1L}, and P_{LL}.

This separation of the integrals into two distinct parts is the operational equivalent of the two-band model,[9] and thus can be used to construct a simple phenomenological expression for $\Delta\rho$ if the two terms in ρ_l' can be identified with the "ideal" normal and the "mixed" normal/umklapp contributions, respectively. Following Kaveh and Wiser, and using previous experimental results on gallium alloys oriented along the other two crystal axes, we assume

$$P_1/J_1^2\approx\rho_i+aT^3f(\rho_0)=\zeta T^5+aT^3f(\rho_0)\ , \tag{8}$$

$$P_{1L}/J_1^2\approx\rho_i+\alpha(T)T^3f(\rho_0)=\zeta T^5+\alpha(T)T^3f(\rho_0)\ , \tag{9}$$

$$P_{LL}/J_1^2\approx\rho_i+\beta(T)T^3f(\rho_0)=\zeta T^5+\beta(T)T^3f(\rho_0)\ , \tag{10}$$

where ρ_i is assumed to follow the Bloch model; and $\alpha(T)$ and $\beta(T)$ are assumed to be temperature-dependent functions which depend on the details of the band structure in their explicit behavior.

Substitution of the assumed dependences into the Ziman expression [Eq. (1)] and expansion of the denominator in terms of $\beta(T)f(\rho_0)/\zeta T^2$ yields the following expression for the temperature-dependent part of the resistivity:

$$\begin{aligned}\Delta\rho\equiv\rho(T)-\rho_0=&[a-2\alpha(T)+\beta(T)]T^3f(\rho_0)\\&-\frac{1}{\zeta}[\alpha(T)-\beta(T)]^2Tf^2(\rho_0)\\&+\frac{\beta(T)}{\zeta^2}[\alpha(T)-\beta(T)]^2\frac{1}{T}f^3(\rho_0)\\&+O(T^{-3})\ .\end{aligned} \tag{11}$$

A consistent feature of the resulting dependence upon $f(\rho_0)$, no matter what the behavior of ρ_l', is the higher power of the DMR functional form contributed by the first negative-phonon-drag term. This feature would ensure that the phonon-drag contribution, when present, will ultimately cause the DMR to pass through a maximum as a function of ρ_0. All higher-order terms may be neglected as the temperature increases because the expansion introduces increasing powers of T^{-1}.

IV. RESULTS AND DISCUSSION

The residual resistivity of each of the ten single-crystal alloy samples is presented in Table I along with nominal concentration. Overall, a range of over three decades in the value of ρ_0 was covered in this study. As illustrated in Fig. 1, the residual resistivity is a linear function of concentration with a coefficient of 0.13 (nΩ cm)/wt ppm. This linear dependence provides evidence that the solute is uniformly dissolved in the host metal and that the spurious results[10] obtained with other solutes in other orientations of these crystals are not in evidence.

It is useful to compare the residual resistivity of $\rho_0 = 0.220$ nΩ cm obtained for the nominally pure sample with previous results on the *b* axis. It is, of course, important to take into account the size effect on the residual resistivity in making the comparison. Boughton[11] has carried out a size-effect study on *b*-axis single crystals, and has obtained a residual resistivity of 0.06 nΩ cm for a sample of the same 0.125-cm diameter. This result gives a residual-resistivity anisotropy ratio of 3.76 for the *a*- to *b*-axis resistivity, respectively. Although the residual-resistivity values for *a*-axis gallium quoted by Bass in a recent compendium[12] are quite a bit higher, the anisotropy ratio is comparable to Olsen-Bar and Powell's results[13] on material with a residual-resistivity ratio of 8000. The material used in this investigation is close to an order of magnitude purer. The anisotropy ratio of "pure" gallium quoted by Powell *et al.*[14] is 3.1 at 293 K, and 3.4 at 83 K for these two orientations. It is clear that the residual anisotropy ratio in the size-effect regime is somewhat higher than in the elastic-phonon-scattering regime. A slight disagreement of this sort is not surprising in view of the fact that a more localized portion of the Fermi surface is being sampled[15] in the boundary-scattering regime.

The temperature dependence of each of the alloys is shown in Fig. 2 where the temperature-dependent part of the resistivity $\Delta\rho$ is plotted versus the cube of the temperature. It is readily apparent that the resistivity of the purer samples obeys a cubic-power-law temperature dependence below about 4.2 K, in agreement with previous results on the other two orientations of gallium.[4,15] It was demonstrated in I that a common normalization can be applied to the DMR data for the following column III*B* metals: *b*-axis Ga, Al, and In, as well as to *c*-axis Ga.[15] This procedure very closely produces the same slope for the DMR data when plotted against $\log_{10}\rho_0$. The normalization factor is based on the assumption that the low-temperature DMR in these metals are caused by some "universal" mechanism that is insensitive to band-structure details. The magnitude of the DMR is assumed to scale with the average phonon density at low temperatures and is represented by the factor $\Delta\rho' = \rho_\Theta (T/\Theta)^3$, where Θ is the Grüneisen temperature and ρ_Θ is the metal's resistivity at $T = \Theta$.

The comparative behavior of the normalized DMR for the *a* axis and the other two orientations of gallium is shown in Fig. 3, where $\Delta\rho(4.2\text{ K})/\Delta\rho'$ is plotted against $\log_{10}\rho_0$. In spite of the fact that error bars are relatively large for the highly impure samples, it is clearly evident that the common behavior exhibited by *b*- and *c*-axis alloys (and other III*B* metals as well) is not obeyed by the *a*-axis results. Rather, the DMR appear to pass through a maximum value as a function of $\log_{10}\rho_0$ in the vicinity of $\rho_0 \approx 25$ nΩ cm. For residual resistivities below this point, the *a*-axis data correlate well with the results for the other two orientations, giving a slope of 1.3 ± 0.2 per decade. One is thus led to the conclusion that the common mechanism for producing DMR is operative in the low-residual-resistivity regime. In the high-residual-resisitivity region however, some unique mechanism is apparently overriding the "common" behavior in the *a*-axis samples.

This changeover is also accompanied by a more rapidly increasing temperature dependence of $\Delta\rho$ exhibited by the high-ρ_0 samples. The limiting exponent for temperatures above 6.3 K has been obtained by a least-squares

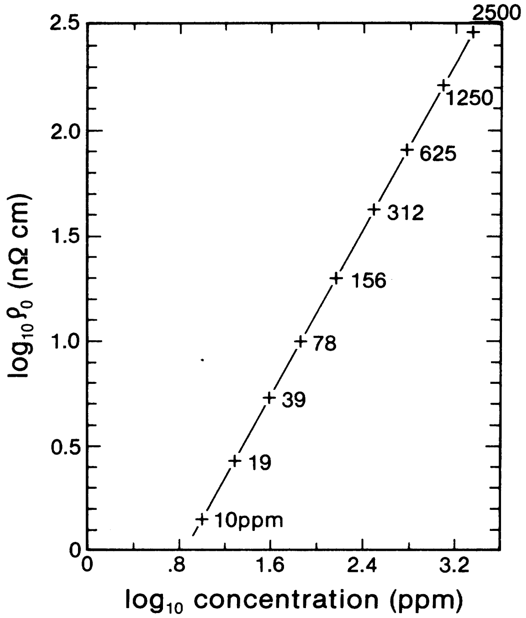

FIG. 1. Plot of the logarithm of residual resistivity (referenced to 1 nΩ cm) vs the logarithm of zinc solute concentration (in wt ppm)

TABLE I. Concentration and residual resistivity of the samples.

Specimen	Nominal impurity (concentration wt ppm)	Residual resistivity (nΩ cm)
1	pure (<0.1)	0.220
2	10	1.404
3	19	2.652
4	39	5.292
5	78	9.970
6	156	19.511
7	312	42.943
8	625	79.123
9	1250	166.65
10	2500	303.56

log-log fit of $\Delta\rho$ against temperature, and the results are presented in Fig. 4. It is clear that the limiting power-law exponent increases with impurity concentration, approaching the idealized Bloch law value of 5 in this limit. Once again, the onset of the rise in temperature exponent is sudden, and appears for ρ_0 values above 25 nΩ cm. The low-ρ_0 regime samples all exhibit a limiting temperature exponent near $n=4.2$ or smaller. This value is consistent with the results on the most impure b-axis sample in I, where the limiting temperature exponent is 4.1±0.1. Thus another feature of the onset of the new mechanism is a sudden increase in the limiting temperature exponent to a limit which not only exceeds the b-axis data, but also exceeds the value of 4.5 obtained by other workers for pure gallium at low temperatures.[13,16] To illustrate the temperature dependence of

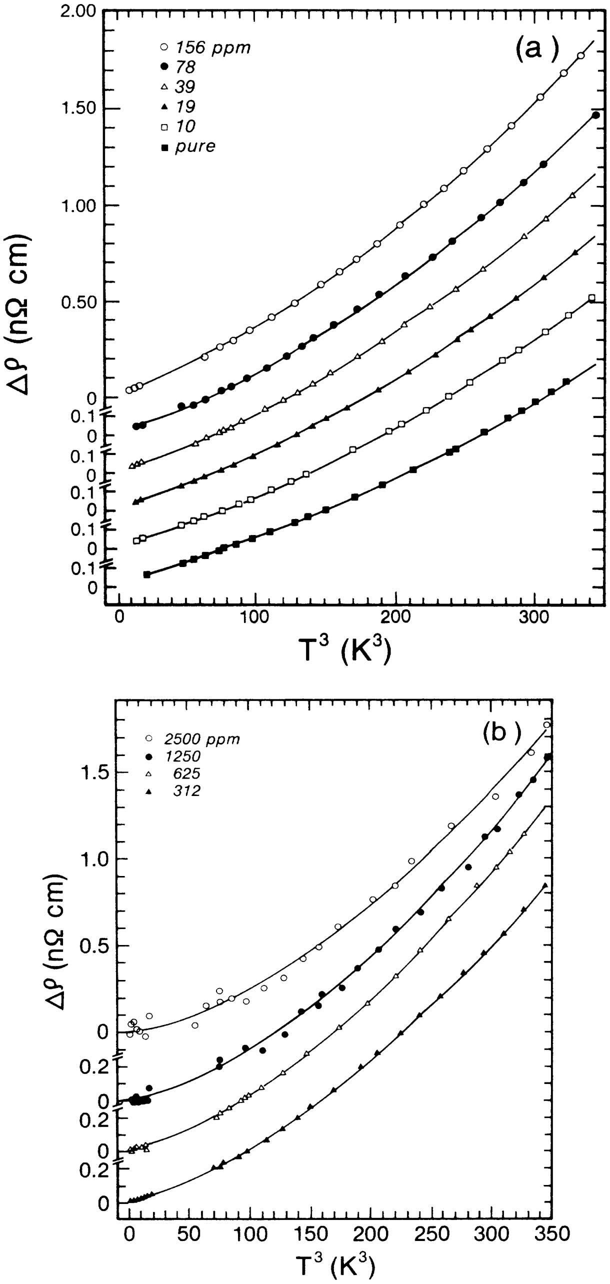

FIG. 2. (a) Temperature-dependent part of the resistivity $\Delta\rho$ vs T^3 for samples 1–6. (b) Temperature-dependent part of the resistivity $\Delta\rho$ vs T^3 for samples 7–10.

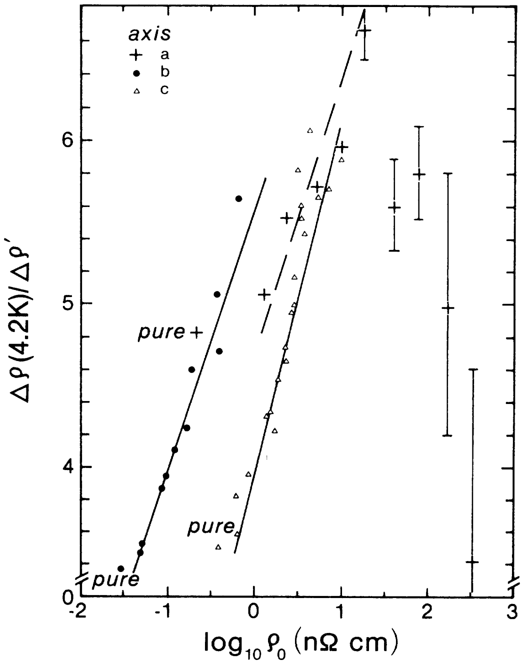

FIG. 3. Normalized temperature-dependent part of resistivity $\Delta\rho(4.2\text{ K})/\Delta\rho'$ vs logarithm of residual resistivity for three orientations of gallium.

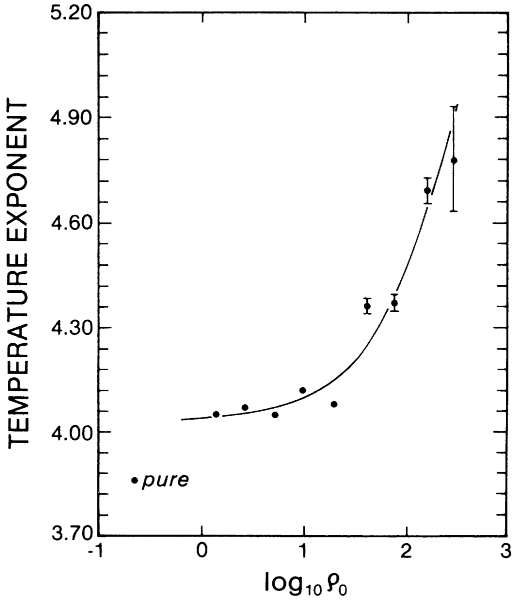

FIG. 4. Temperature exponent of $\Delta\rho(T)$ vs logarithm of residual resistivity.

DMR as a function of impurity concentration, the quantity $\Delta\rho$ is plotted versus $\log_{10}\rho_0$ at $T=4.0$, 5.0, 6.0, 6.6, and 7.0 K, in Fig. 5. The data are characterized by a maximum that increases in relative prominence with increasing temperature. The location of the maximum along the $\log_{10}\rho_0$ axis, however, does not appear to be very sensitive to temperature, suggesting that a fundamental role is played by the amount of impurity scattering present.

Using the results of the analysis in Eq. (11) as a guide, the results for all five temperatures were fitted to a function of the form

$$\rho_{\mathrm{DMR}}=\Delta\rho(T)-\Delta\rho(4.0\ \mathrm{K})=A(T)[\log_{10}(\rho_0)]^n - B(T)[\log_{10}(\rho_0)]^{2n} \ . \qquad (12)$$

The best fit to the data assuming $f(\rho_0)\approx[\log_{10}(\rho_0)]^n$ is obtained for $n=2$ rather than 1 as assumed earlier. The coefficients A and B as obtained by a least-squares-fit to the data, as shown in Fig. 6, are tabulated as functions of temperature in Table II. It has been assumed that, to within the accuracy of the data, the values of $\Delta\rho$ at 4.0 K closely approach the value of the equilibrium lattice resistivity, and these have therefore been subtracted from $\Delta\rho(T)$ in order to obtain the quantity ρ_{DMR} which is indicated above.

Both coefficients A and B are found to vary linearly with temperature over this range. The phenomenological model represented by Eq. (11) can be used to describe these results if the following assumptions about the "two-band" contributions to each term are made:

$$\alpha(T)=(2.1-380/T^2)\times10^{-4}\ \mathrm{n\Omega\,cm\,K^{-3}}\ ,$$
$$\beta(T)=(-1.0-380/T^2)\times10^{-4}\ \mathrm{n\Omega\,cm\,K^{-3}}\ . \qquad (13)$$

The values for ζ and a, respectively, have been determined from the size-induced DMR results of Boughton[17] on b-axis gallium crystals, corrected for the difference in crystal orientation; and from the relation obtained for ordinary DMR (Ref. 4) in this metal which can be expressed as

$$a=\rho_\Theta/\Theta^3\ , \qquad (14)$$

where ρ_Θ is the a-axis resistivity at the Grüneisen temperature Θ. The values used are

$$\zeta=5.4\times10^{-5}\ \mathrm{n\Omega\,cm\,K^{-5}}$$

and

$$a=5.3\times10^{-4}\ \mathrm{n\Omega\,cm\,K^{-3}}\ .$$

The essential features of the model reflect an identical and dominant T^{-2} behavior for both $\alpha(T)$ and $\beta(T)$, as well as an effective cancellation of both of the equilibrium lattice terms represented in P_{11}/J_1^2. This result contrasts with the calculated dependences obtained by Kaveh and Wiser on potassium, where $\alpha(T)$ is sensibly constant and $\beta(T)$ decreases slightly as the temperature decreases. There is of course a much different Fermi surface topology to consider in the case of gallium. Ample evidence[18] exists to assume widespread contact with the zone boundaries and therefore that a much different umklapp contribution should result, with no topological cutoff in the allowed phonon wave vector in the electron-phonon scattering process.

A favorable comparison of these results with the relation in Eq. (7) would, at a minimum, ensure the self-consistency of the phenomenological model. The data of Mahajan *et al.*[19] on the phonon-drag thermopower, and of Gorter and Nordermeer[20] on the lattice thermal conductivity have been used to compare with the results and

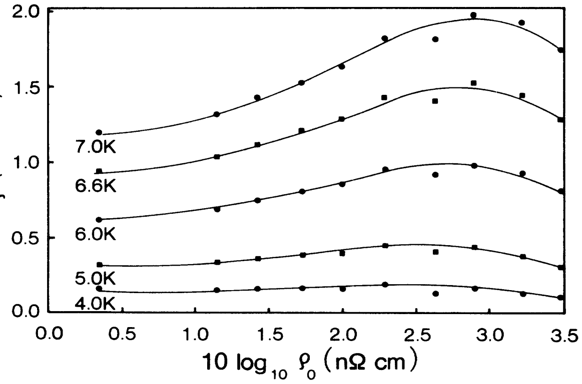

FIG. 5. Temperature-dependent part of the resistivity $\Delta\rho$ vs logarithm of residual resistivity (referenced to 0.1 nΩ cm) at $T=4.0$, 5.0, 6.0, 6.6, and 7.0 K.

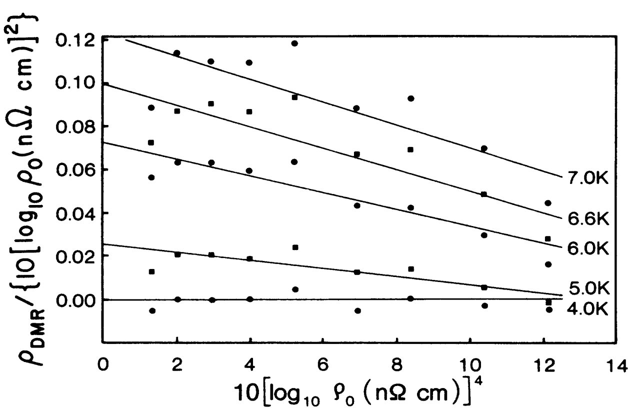

FIG. 6. Plot of $\rho_{\mathrm{DMR}}/[\log_{10}(\rho_0)]^2$ vs $[\log_{10}(\rho_0)]^4$ at $T=4.0$, 5.0, 6.0, 6.6, and 7.0 K (ρ_0 referenced to 0.1 nΩ cm).

TABLE II. Parameters A and B resulting from least-squares fit to $\rho_{\mathrm{DMR}}=A[\log_{10}(\rho_0)]^2+B[\log_{10}(\rho_0)]^4$.

Temperature (K)	A (nΩ cm)	B (nΩ cm)
4.0	0.000±0.002	−0.0002±0.0003
5.0	0.024±0.004	−0.0017±0.0005
6.0	0.073±0.005	−0.0041±0.0007
6.6	0.100±0.008	−0.005±0.001
7.0	0.12±0.01	−0.005±0.001

yield a value that is several orders of magnitude too small, at $\rho_g = 3.2\times 10^{-8}T^7$ nΩ cm. This wide discrepancy between the simple lattice thermoelectric model and the present data is not unexpected since it represents the same argument that has always been applied in dismissing phonon drag as an important contribution to the temperature-dependent impurity resistivity. The lattice thermal and thermoelectric data were obtained from measurements on the pure material rather than on an alloy, and evidence does exist to indicate that alloying, even in dilute amounts, produces significant changes in the lattice thermopower. The recognition by Kaveh and Wiser that phonon drag can play an important role in *cancelling* the equilibrium lattice resistivity must be considered. There is no question but that the DMR cause a change in the equilibrium resistivity and the electron distribution, but also since phonons are involved in the DMR mechanism, their distribution function can be significantly altered. This aspect of the problem is what the above phenomenological model has attempted to account for. The final test of its validity awaits the accurate calculation of the factors $\alpha(T)$ and $\beta(T)$ for a model complex metal which has scattering and topological parameters comparable to those of gallium. We can nevertheless conclude that the onset of the observed change in the variation of the DMR at a rather low impurity concentration is a manifestation of a new DMR regime for which the cause is different from whatever the common source of low-temperature DMR is in the IIIB group of metals.

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