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## Twin-boundary dynamics and properties of high- $T_c$ superconductors

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We present evidence for collective twin-boundary oscillations in the high- $T_c$  oxide superconductors and propose that their coupling with electrons enhances  $T_c$ . These special modes, which we call "dyadons," have a highly anisotropic and low-frequency spectrum, causing the specific heat to change from a  $T^3$  temperature dependence to  $T^2$  at lower temperatures, consistent with anomalies below  $\sim 12$  K in doped La<sub>2</sub>CuO<sub>4</sub> and in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>. The low-frequency dyadons also account for the extended linear temperature dependence of the normal-state resistivity. Due to the unusual anisotropy of both dyadon and electron spectra, vertex corrections generate a strong electron-dyadon coupling at lower temperatures.

The recent discoveries of ceramic-type superconductors with a high transition temperature  $T_c$  led to intensive studies of these materials in search for a clue to the mechanism of high  $T_c$ .<sup>1,2</sup> In particular, a tetragonal-toorthorhombic (TO) transition at a temperature  $T_d$  above or near  $T_c$  seems to be a key feature.<sup>3-9</sup> A positive correlation is found between  $T_c$  and the volume fraction of the orthorhombic phase of YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub>.<sup>5,6</sup> Furthermore, in the La<sub>2-x</sub>Sr<sub>x</sub>CuO<sub>4- $\delta$ </sub> compounds,  $T_d$  decreases from 533 K at x = 0 to ~40 K at x = 0.19 (Refs. 3 and 4) where  $T_c$ shows a maximum. A similar correlation of  $T_c$  with structural transitions is well known in the A15 and other compounds.<sup>2</sup>

We consider the formation of twin boundaries (TB's) as the most dramatic manifestation of the TO transition and its martenistic nature. TB's exist well below  $T_d$  and in a sense maintain the anisotropy and lattice softening which occur at  $T_d$ . In particular, YBaCu<sub>3</sub>O<sub>7-8</sub> shows twinned orthorhombic bands with 10-100 parallel TB's in a grain and TB spacing of  $l = 10^2 - 10^3$  Å.<sup>6-9</sup>

A direct correlation of TB's and  $T_c$  was in fact demonstrated in Sn, In, Re, Tl, and Nb by mechanically generating TB's.<sup>10,11</sup> In particular, a single TB enhances  $T_c = 3.72$  K of Sn by 0.04 K, while a high density of TB's yields  $T_c = 7$  K.<sup>10</sup> These phenomena were analyzed<sup>12</sup> in terms of superconductivity localized and enhanced near a static TB by an unknown mechanism. The proximity effect can then considerably enhance the bulk  $T_c$  if  $l \leq \xi_0$ , where  $\xi_0$  is the superconducting coherence length. However, this mechanism seems ineffective in the high- $T_c$ compounds where  $\xi_0 = 30$  Å  $\ll l.^{13}$ 

We have recently reported<sup>14</sup> a study of the TB lattice dynamics with some unusual results. First, the detachment of very low-frequency collective modes from the transverse phonon branch; softening of the latter is associated with the martensitic transition at  $T_d$ . Second, due to the long-range nature of the elastic fields, these modes affect the entire transformed phase and therefore can couple to electrons in a bulk fashion. These modes might be called collective twin-boundary oscillations; we have chosen "dyadons" for short.

In the present work we first evaluate the contribution of dyadons to the specific heat and show that they can account for the anomalies observed<sup>15</sup> at  $\sim 12$  K in La<sub>1.85</sub>Ba<sub>0.15</sub>CuO<sub>4</sub> and La<sub>1.8</sub>Sr<sub>0.2</sub>CuO<sub>4</sub> and <sup>16,17</sup> below  $\sim 10$  K in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub>. Given the dyadon low frequencies, the electron-dyadon coupling also naturally explains the normal-state resistivity which is linear with temperature<sup>4,17-19</sup> to as low as  $\sim 40$  K. Finally, we examine vertex corrections caused by the electron-dyadon coupling. Due to the unusual anisotropy of both dyadon and electron spectra, Migdal's theorem is not applicable and the vertex function is strongly enhanced.

Before proceeding, some disclaimers: First, dyadons are not proposed as the fundamental mechanism of high  $T_c$ , rather they enhance electron-phonon or electronelectron interactions.<sup>1,2</sup> Second, we do not rely on proximity effect<sup>12</sup> since dyadons couple to electrons in the bulk. Third, the effect is not a conventional soft-phonon mechanism<sup>2</sup> since vertex corrections arising from anisotropy are essential.

Consider a periodic array of TB's parallel to the (110) plane; this structure can be described by an elastic displacement field u(s) in the [110] direction with period 2*l* and *s* is in the [110] direction.<sup>20,21</sup> When the TB thickness is much smaller than *l* we can define a collective coordinate  $S_n$  for the position along [110] of the *n*th TB. The normal coordinates  $\eta_n$  for small oscillations, i.e., dya8896

dons, <sup>14</sup> are given by the nonlocal transformation  $\eta_n = 2 \times \sum_{j=1}^{n-1} (-)^j \delta_j + \delta_0 - \delta u$ , where  $S_n = nl + \delta_n$  and  $\delta u$  is an overall shift of u(s). The restoring force for the TB motion is a long-range elastic force mediated through the interface with another phase. The latter could be the tetragonal parent phase, a differently oriented twin band or an altogether difference phase. However, the kinetic mass is a *bulk* one, involving the motion of the whole twinned product phase. The resulting frequencies  $\omega(q_1)$  for wave vectors  $q_1$  in the [110] direction are, therefore, very low with the dispersion relation <sup>14</sup>

$$\omega^{2}(q_{1}) = \omega_{d}^{2} \sin^{2}(\frac{1}{2}q_{1}l) \sum_{p=-\infty}^{\infty} \left[ \left| \frac{q_{1}l}{2\pi} - p \right|^{-1} - \left| \frac{1}{2} - p \right|^{-1} \right].$$
(1)

Here  $\omega_d = [4\alpha/\pi\rho lL_2]^{1/2}$  with  $\alpha$  an effective elastic constant,  $\rho$  the mass density, and  $L_2$  the width of the twin band in the  $[1\bar{1}0]$  direction;  $\omega_d \approx \Theta_D a/\sqrt{lL_2}$  where  $\Theta_D$  is the Debye temperature and a the lattice constant. For  $\Theta_D \approx 400 \text{ K}$ ,  $^{15-17} a = 3.8 \text{ Å}$ ,  $l \approx 10^2 \text{ Å}$ , and  $L_2 \approx 10^4 \text{ Å}$ ,  $^{6-9}$  we estimate  $\omega_d \approx 10^{11} \text{ s}^{-1} \approx 1 \text{ K}$ .

When the TB position  $\delta_n$  depends on coordinates perpendicular to s, it describes a TB bending. We expect that now the elastic energies scale with the kinetic mass so that the transverse dispersion is linear. For wave vectors  $q_{2,q_3}$ in the [110] and [001] directions respectively, the threedimensional dyadon dispersion is then

$$\omega_a^2 = \omega^2(q_1) + v_2^2 q_2^2 + v_3^2 q_3^2 , \qquad (2)$$

where  $v_2, v_3$  may be of the order of sound velocities.

Equation (1) yields  $\omega(q_1) \sim \sqrt{q_1}$  as  $q_1 \rightarrow 0$  which is a manifestation of the long-range force. As  $q_1 \rightarrow \pi/l$ ,  $\omega(q_1) \sim |q_1 - \pi/l|$  and  $\omega(\pi/l) = 0$  corresponds to the uniform translation mode of the twin lattice; impurities or discreteness effects for very sharp TB's may pin this mode and then  $\omega(\pi/l) \neq 0$ . For example, for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7- $\delta$ </sub> with  $\delta = 0$  oxygen ordering can be related to the TO transition and a TB separates distinct directions of oxygen ordering.<sup>9,22</sup> We note that thermal fluctuations of a TB are [using (2)]  $\sim 1\%$  of the lattice constant at room temperature.

The spectrum [(1) and (2)] implies an unusual specific heat. There are four distinct temperature regimes. (a)  $T \leq v\pi/L_2 \simeq \Theta_D a/L_2$  where v is a typical sound velocity. Since acoustic phonons with wave vectors  $|q_1| \leq \pi/L_2$ cannot be confined in the product phase, we expect a normal specific heat  $c_v \sim T^3$  for  $T \leq 0.1$  K. (b)  $v\pi/L_2 \leq T \ll \omega_d$  where the  $(q_1)^{1/2}$  dispersion implies a  $c_v \sim T^4$  contribution for  $T \ll 1$  K. (c)  $\omega_d \leq T \leq v\pi/l$  $l \simeq \Theta_D a/l$  where all the frequencies  $\omega(q_1)$  are excited and the specific heat is effectively two-dimensional. Thus, for  $1 \leq K \leq T \leq 10$  K we expect  $c_v \sim T^2$ . (d)  $T \geq v\pi/l$  where ordinary phonons dominate and  $c_v \sim T^3$ .

Numerical evaluation of the contribution of dyadons using Eqs. (1) and (2) to the specific heat is shown in Fig. 1. For comparison we also show the specific heat of isotropic acoustic phonons assuming a victory  $v = v_2 = v_3$  $= (\alpha/\rho)^{1/2}$ , and  $L_2/l = 100$  (dashed line). These two specific-heat terms are not simply additive; the dyadon de-

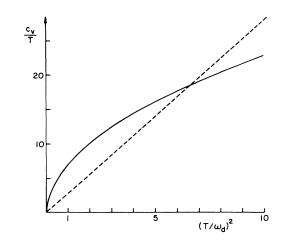


FIG. 1. Specific heat  $c_v$  due to dyadon excitations (full line) and due to normal acoustic modes (dashed line) with  $v = v_2 = v_3$ and  $L_2/l = 100$ ;  $c_v/T$  is in units of  $\omega_d/(2\pi v_2 v_3 l)$ .

grees of freedom are formed at the expense of lowfrequency acoustic phonons; hence the two lines in Fig. 1 are limiting forms for either regimes b and c (full line) or regime d (dashed line).

Specific-heat data on La<sub>1.85</sub>Ba<sub>0.15</sub>CuO<sub>4</sub> and La<sub>1.8</sub>Sr<sub>0.2</sub>-CuO<sub>4</sub> (Ref. 15) show a deviation from a  $T^3$  law at  $\sim 12$ K. We suggest that the bend at  $\sim 12$  K is the crossover to a  $T^2$  law at lower temperatures. Thus, the extrapolation to T=0 and the apparent large  $c_v \sim T$  term<sup>15</sup> should be reexamined by experiments at lower temperatures. Data on YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7</sub> shows<sup>16,17</sup> a peak of magnetic origin at  $\sim 2$  K. A magnetic field H partially freezes the magnetic moments and the peak disappears; the data<sup>16,17</sup> are then consistent with Fig. 1 with  $\omega_d \approx 1$  K. Sample dependence<sup>16,17</sup> of this anomaly indicates different distributions of TB arrays. Further experiments on the  $c_v(H)$  dependence should clarify the relative roles of spins and TB's.

We turn now to address the electron-dyadon coupling, first via the normal-state resistivity and then its effects on  $T_c$ . The Fourier transforms of u(s) and  $\eta_n$  are related by <sup>14</sup>

$$u(q_1) = \bar{u}(q_1) + i(q_1l)^{-1} [\exp(iq_1l) - 1] \eta(q_1) + O(\eta^2) ,$$

where  $\bar{u}(q_1)$  corresponds to the static TB lattice. Thus,  $\eta(q_1)$  produces a significant modulation in u(s) if  $|q_1| \leq \pi/l$ . Electrons, which would ordinarily couple to a phonon displacement  $u(\mathbf{q})$  with a coupling constant  $g_0(\mathbf{q})$ , will then couple to dyadons with a coupling

$$g_d(\mathbf{q}) = ig_0(\mathbf{q})(e^{iq_1l} - 1)/(q_1l) .$$
(3)

We define a dimensionless coupling by a *two-dimensional* average

$$\lambda_d(q_1) = (2/\pi v_F) \sum_{q_2,q_3} |g_d(q)|^2 / \omega_q ,$$

where  $v_F$  is a Fermi velocity in the  $q_1$  direction. We are mainly interested in  $|q_1| < \pi/l$ , where  $\lambda_d(q_1) \simeq \lambda_d(0) \equiv \lambda_d$  is weakly dependent on  $q_1$  [Eq. (3)]. The

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shear displacement u is parallel to  $q_2$  and  $g_0(q) \sim q_2$ ; so although the very low-frequency modes precisely at  $q_2=q_3=0$  do not couple, those in the neighborhood can enhance  $\lambda_d$  to  $\lambda_d > 1$ , i.e., larger than the corresponding  $\lambda_0$  (Ref. 23) for the coupling to the ordinary phonons.

The linear temperature dependence of the resistivity  $\rho(T)$  (Refs. 4 and 7-19) is a natural outcome of the low-frequency dyadon spectrum. Note that  $\rho(T) \sim T$  down to  $\sim 40$  K (Refs. 4 and 19) in doped La<sub>2</sub>CuO<sub>4</sub> for which a normal Debye spectrum with  $\Theta_D \simeq 400$  K (Ref. 15-17) cannot account. Analogous to the electron-phonon formalism<sup>23</sup> the electron-dyadon contribution to  $\rho(T)$  for  $T \gtrsim \omega_q$  is  $\rho(T) = 2\pi m^* f \lambda_d T/(e^2 n)$  where  $m^*$  and n are the electron effective mass and density, respectively. The factor f accounts for the reduced phase space  $|q_1| < \pi/l$  in which  $\lambda_d$  is effective, i.e.,  $f \simeq a/l$ . Using data <sup>17-19</sup> for samples with the lowest intercept  $\rho_0$  of  $\rho(T)$  at T = 0 we estimate  $f \lambda_d \simeq m/m^* < 1$ . We can view the small coupling  $f \lambda_d$  as a three-dimensional average; since  $f \ll 1$  the two-dimensional average is large,  $\lambda_d \gg 1$ .

The slope  $d\rho/dT$  exhibits large sample-dependent variations consistent with expected variations in  $f \approx a/l$ . Curiously,  $d\rho/dT$  increases with  $\rho_0$  which may indicate that an increase in a/l implies an increased  $\rho_0$  via static disorder in the TB lattice.

The electron-dyadon coupling provides a long-range attractive coupling between electrons. It is well known that in both one-dimensional<sup>24</sup> and quasi-one-dimensional metals<sup>25</sup> a long-range attractive force favors superconductivity relative to other types of instabilities, provided the mediating phonons have a sufficiently high frequency. For isotropic soft phonons however the leading effect cancels,<sup>26</sup> so that scenario does not apply directly to dyadons.

The unusual feature in the present system is the anisotropy. Dyadons are highly two dimensional in the (110) plane while electrons are quasi two dimensional in the (001) plane, or even one dimensional since the Fermi surface may have flat regions<sup>1</sup> parallel to {110}. Furthermore, the static part of the TB lattice enhances this one dimensionality by folding the Fermi surface into a narrow slab of width  $2\pi/l$  in the [110] direction. At temperatures below  $\sim v_F/l$  we expect the electrons to become susceptible to the strong coupling  $\lambda_d$ .

To demonstrate the effect we assume a Fermi surface parallel to (110) and show that the vertex function  $\Gamma(\mathbf{k},n;\mathbf{q}m)$  (Fig. 2) can diverge when  $T \simeq v_F/l$ , i.e., Migdal's theorem<sup>27</sup> is not applicable. Figure 2 represents an integral equation for the renormalization of *any* coupling  $g_e$  to electrons due to exchange of dyadons with coupling  $\lambda_d$  when  $|k_1 - k_1'| < \pi/l$  or exchange of the rest of the acoustic branch with ordinary coupling  $\lambda_0$  when  $|k_1 - k_1'| > \pi/l$ . Using the Matsubara representation<sup>27</sup> of  $\Gamma$  and keeping just the n'=n terms we find a divergence for  $q_1=0$ ; the highest temperature where this divergence can occur is for  $\omega_n = -\omega_{n+m} = \pi T$ , i.e., n=0, m=-1. The main  $k_1'$  dependence is in the electron propagators which for  $|k_1'| < \pi/l$  yield

$$\sum_{k_1'} (\pi^2 T^2 + v_F^2 k_1'^2)^{-1} \sim \tan^{-1}(v_F/Tl),$$

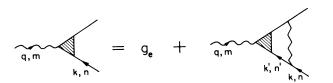


FIG. 2. Vertex renormalization of a coupling  $g_e$  due to any interaction (wavy line) with electrons (straight lines). The zigzag line is a dyadon or acoustic phonon propagator;  $\mathbf{q}, \mathbf{k}, \mathbf{k}'$  are wave vectors and m, n, n' correspond to the frequencies  $v_m, \omega_n, \omega_n'$ , where  $v_m = 2\pi Tm$  and  $\omega_n = \pi T(2n+1)$ .

while the  $k'_2k'_3$  summation yields the two-dimensional coupling  $\lambda_d$ . The  $|k'_1| > \pi/l$  sum involves  $\lambda_0$  with the final result

$$\Gamma = g_e / [1 - \lambda_0 - (2/\pi)(\lambda_d - \lambda_0) \tan^{-1}(v_F / Tl)] .$$
 (4)

This result shows the crossover from the weak coupling  $\lambda_0$  at  $T \gtrsim v_F/l$  to the strong coupling  $\lambda_d$  at lower temperatures. In particular for  $\lambda_d > 1 > \lambda_0$ , (4) diverges at  $T \equiv T_0 \simeq v_F/l$ . Note that for  $l \rightarrow \infty$ , (4) shows a temperature-independent correction which is usually neglected in one-dimensional theories.<sup>24</sup> Thus, the divergence in (4) is specific to the dyadon anisotropy and the appearance of a new energy scale  $v_F/l$ .

Since  $\lambda_d$  depends on a/l there may be an optimal  $\lambda_d$  for which  $T_0$  is maximal. If  $T_c$  is enhanced by dyadons, a distribution of grains with various a/l yields the  $T_c$  of the optimal grains, as long as they form a percolating cluster. Thus, small variations in the distribution [which *can* affect  $c_v$  and  $\rho(T)$ ] do not change  $T_c$ ; large variations do change  $T_c$ , as in the case with rapid quenching<sup>5</sup> where  $T_c$  drops by  $\sim \frac{1}{2}$ .

We note that the coupling  $g_e$  in Fig. 2 and Eq. (4) is not specified. At present, not having computed  $T_c$ , we do not know whether dyadons could be more effective in enhancing the conventional electron-phonon mechanism or other nonconventional mechanisms.<sup>1,2</sup>

In conclusion, we have shown that dyadons can account for the specific-heat anomaly at  $\sim 12$  K, for the observed  $\rho \sim T$  relation and lead to enhanced electron interactions. Further experiments on the low-temperature specific heat and on the effect of the twin-band geometry on  $T_c$ ,  $d\rho/dT$ and the specific heat are essential for illuminating the role of dyadons in the high- $T_c$  superconductors.

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