Processes yielding high superconducting temperatures

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It is pointed out that any microscopic description of the new high-T_c superconductors should take into account a number of important points concerning strong couplings, whatever their nature: absence of the MacMillan limit, absence of a Migdal theorem, and importance of the Brovman-Kagan type of vertices with different singularities depending on the dimensionality. As a consequence, the applicability of standard techniques such as the Eliashberg theory, in particular, may be questioned in high-T_c superconductors.

The recent breakthrough achieved with superconducting temperatures T_c higher than 30 K in the Ba-La-Cu-O family[1] has highly stimulated both experimental and theoretical work. On the theory side, a number of processes have already been proposed[2] as possibly responsible for such high T_c. In the present note, we wish to emphasize a number of remarks that may be neglected in usual BCS-type weak coupling superconductivity, but which become crucial if a strong coupling process is (partly at least) at the source of such high T_c. The remarks that follow are quite general in the sense that they apply to any case where a boson-mediated electron-electron attraction arises, whatever is the boson nature, as long as it corresponds to a strong coupling.

As far as the electron-phonon coupling is concerned, the BCS pairing theory has, in the past, been extended to treat the so-called strong electron-phonon coupling in Ref. 3, but still within the domain of applicability of the Migdal theorem.[4] The Migdal theorem allows one to perform a perturbation calculation to lowest orders in the coupling, due to the presence of a small parameter m/M (which may be viewed equivalently as the ratio of the sound velocity to the Fermi one c/v_F, or the Debye to the Fermi energies ω_D/E_F). The existence of the Migdal theorem allows one then to neglect electron-phonon vertex corrections to the accuracy $(m/M)^{1/2}$. The Migdal theorem was supposed to hold in Ref. 5, which provided a formula for T_c within a given class of materials. This formula was proposed to reach its maximum value for the $\lambda=2$ limit (where λ is the electron-phonon coupling constant) with T_c of the form

$$T_c=\omega_0\exp\left(-\frac{\alpha+\beta\lambda}{\gamma+\delta\lambda}\right)$$

($\alpha,\beta,\gamma,\delta$ only depend on the Coulomb pseudopotential and pure numbers, and ω_0 is the cut-off frequency). However, it was later shown[6] that the $\lambda=2$ limit is spurious since the MacMillan equation is correct only for $\lambda<1.5$, but is in error for large λ. Reference 6 showed that for such large λ what actually governs the magnitude of T_c is the prefactor ω_0, which is proportional to $\lambda^{1/2}$ for $\lambda>10$. In such a case, one may wonder whether Migdal's argument is still sufficient to avoid considering higher-order corrections. It was recently argued[7] that in heavy-fermion systems[8] λ should be large, electron-phonon vertex corrections cannot be neglected, and the Migdal theorem is violated. This would be true as well if, as proposed in Ref. 9, the mediating interaction in high-T_c superconductors arises from resonance between an electronic excitation and an optic phonon of the same energy.

As far as nonphononic electron-electron attractions are concerned, for instance in the paramagnon problem, it has been proved long ago[10] that there is no Migdal theorem for paramagnon-mediated interaction in strongly correlated, nearly magnetic fermion systems, which may become triplet superconductors, as is the case for superfluid ^{3}He,[11] or triplet or anisotropic singlet superconductors in possibly some heavy-fermion compounds.[12] The argument for the absence of a Migdal theorem in the paramagnon problem[10] may be summarized as follows: There is no small parameter because the ratio of energies to be considered is $I/E_F=\bar{I}\sim 1$, where I is the strong Hubbard-type contact repulsion between opposite spins of the order of the Fermi energy E_F of the free fermions when the system is close to a magnetic instability. Equivalently, there is only one mass m involved (note that it is the bare quantities which have to be compared to start with). Still for superconductivity arising from boson exchange, other than phonons, it was later shown[13] that second-order corrections, beyond the Migdal approximation, change T_c drastically when the boson energy is an appreciable fraction of E_F.

Thus, we suggest that, in any theory studying boson-mediated electron-electron attraction yielding high-T_c values, whatever the nature of the boson is (phonon or nonphonon-type), one should first check whether a Migdal-type theorem applies or not. If the calculation of the first-order electron-boson vertex yields a correction V/E_F (where V is the electron-boson interaction) which is smaller than 1 but still appreciable, then such a correction (as well as other equivalent ones) must be incorporated to calculate T_c. If the result is of order 1 ($V/E_F\sim 1$), then one has to face a strong coupling problem for which perturbation theory breaks down and any standard (perturbative) treatment would be irrelevant.

In the case where the above vertex corrections have to be retained in higher orders in the perturbation theory, with $V/E_F<1$, there is another important question to consider which involves the "multitail" ring diagrams of

Ref. 14 (see Fig. 1 of either reference in Ref. 14). These are closed electron loops with an arbitrary number of "tails" attached to them (static or dynamic). The tails were phonons in the three-dimensional case of Ref. 14 and were associated with the indirect interaction between the ions via the conduction electrons. They enter as well in any property of the electron-phonon problem which may be expanded in a series of the electron-ion interaction. Reference 14 showed that the multitail diagrams with static external fields are highly singular for a certain combination of the incoming and outgoing momenta of the external fields; the first singularity is the well-known Kohn's one, where the number of tails is equal to two. The degree of the singularity was shown to increase with the number of tails. Thus, Ref. 14 was able to account for a number of unexplained anomalies in the phonon spectrum of some high-precision measurements on Al.

Instead of phonons as the external tails, one may generalize the arguments to any other boson-type fluctuations. Reference 15 extended the three-dimensional calculation of Ref. 14 to the two-dimensional paramagnon problem, where the external tails are frequency-dependent paramagnons. Reference 15 thus showed that, *stricto sensu*, and if $I/E_F \sim 1$, perturbation theory breaks down for the calculation, in particular for the spin susceptibility of a two-dimensional, nearly magnetic fermion system, precisely because of the singularities arising from the Brovman-Kagan multitail diagrams. It was also shown that the singularities are stronger, for a given number of tails, in two dimensions than in three dimensions.

If, in the high-T_c superconductors, a strong electron-phonon-type of coupling is relevant for their superconductivity, inelastic neutron scattering measurements providing the phonon spectrum would be illuminating;[16] then the presence (if any) of anomalous structures would be the signature of such singularities and would tell how reliable perturbation theory may be for those systems. Needless to say, if strong electron-phonon couplings are involved, Matthiessen's rule most likely breaks down too: Phonons and electrons would have to be treated on the same footing and self-consistently.

On the other hand, if it is confirmed that the high-T_c superconductors exhibit a strong quasi-two-dimensional character, or at least, for those which do so, the above singularities, as just mentioned, will be stronger than in three dimensions. As usual, lowering the dimensionality emphasizes any singularity. For instance, it has already been suggested[17] that triplet pairing superconductivity, via two-dimensional paramagnon-mediated interaction, exhibits a higher λ-type value than in three dimensions. Then the problem concerning Migdal's theorem is all the more important as was already warned in Ref. 17. [For the paramagnon case, however, possible arguments analogous to those invoked in three dimensions for ^3He (see Ref. 18) may be useful in two dimensions as well.]

The above remarks hold independent of a possible anisotropy of the electron-boson coupling which if accounted for would obviously render a meaningful microscopic description more complicated. In the BCS description of phonon-mediated electron-electron attraction, the simplified attraction used is a constant independent of the angle θ between the two momenta of the pair. Then the spatial wave function of the pair is symmetric, the spin one is antisymmetric, and the pairing is singlet. But if the attraction strongly depends on θ, one may have more complicated spin situations,[19] such as those encountered in superfluid ^3He,[11] or in heavy-fermion materials.[12] More generally, if the electron-boson interaction exhibits a spatial anisotropy, possibly due to the quasi-two-dimensional character of some of the high-T_c superconductors, the spatial and spin dependencies of the pair system may become very difficult to handle in view of the remarks underlined in the present paper.[20]

A last remark: Although nonphononic-mediated interactions should be accompanied by the absence of the isotope effect,[21] this would not be proof of the nonphononic nature of the interactions.[22]

To conclude, we suggest that if high-T_c superconductivity is due to a strong electron-boson coupling of any kind, standard formalisms like the Eliashberg theory may be inappropriate. One should first examine whether a Migdal-type theorem can be derived before proceeding with usual techniques.

[1] J. G. Bednorz and K. A. Müller, Z. Phys. B **64**, 189 (1986); S. Uchida *et al.*, Jpn. J. Appl. Phys. Lett. **26**, L1 (1987); C. W. Chu *et al.*, Phys. Rev. Lett. **58**, 405 (1987); R. J. Cava *et al.*, *ibid.* **58**, 408 (1987); M. K. Wu *et al.*, *ibid.* **58**, 908 (1987); P. H. Hor *et al.*, *ibid.* **58**, 911 (1987).

[2] J. E. Hirsch and D. J. Scalapino, Phys. Rev. Lett. **56**, 2732 (1986); J. Labbé and J. Bok, Europhys. Lett. **3**, 1225 (1987); P. W. Anderson, Science **235**, 1196 (1987); B. K. Chakraverty, J. Phys. (Paris) **42**, 1351 (1981); A. Alexandrev and J. Ranninger, Phys. Rev. B **24**, 1164 (1981); P. Prelovsek *et al.* (unpublished); M. Cyrot (unpublished); Y. Hasegawa *et al.* (unpublished); S. Barisi *et al.* (unpublished); D. J. Jorgensen *et al.*, Phys. Rev. Lett. **58**, 1024 (1987); C. Varma *et al.* (unpublished).

[3] D. J. Scalapino *et al.*, Phys. Rev. **148**, 263 (1966).

[4] A. B. Migdal, Zh. Eksp. Teor. Fiz. **34**, 1438 (1958) [Sov. Phys. JETP **7**, 996 (1958)]; see, also, D. J. Scalapino, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1960), pp. 473–477.

[5] W. L. MacMillan, Phys. Rev. **167**, 905 (1968).

[6] P. B. Allen and R. C. Dynes, Phys. Rev. B **12**, 905 (1975).

[7] P. Entel and M. Matschke (unpublished).

[8] For a review, see G. Stewart, Rev. Mod. Phys. **56**, 755 (1984).

[9] Z. Schlesinger *et al.* (unpublished); C. L. Fu and A. J. Freeman (unpublished); W. Weber, Phys. Rev. Lett. **58**, 1371 (1987).

[10] J. A. Hertz, K. Levin, and M. T. Béal-Monod, Solid State Commun. **18**, 803 (1976); in that paper the first electron-paramagnon vertex correction was actually calculated and shown to be of order one.

[11] See the review in K. Levin and O. Valls, Phys. Rep. **98**, 1 (1983); and Phys. Rev. B **20**, 1101 (1973).

[12] M. T. Béal-Monod, C. Boubonnais, and V. J. Emery, Phys. Rev. B **34**, 7716 (1986); D. J. Scalapino, E. Loh, Jr., and J. E.

Hirsch, *ibid.* **34**, 8190 (1986); K. Miyake, S. Schmitt-Rink, and C. Varma, *ibid.* **34**, 6554 (1986).

[13]M. Grabowski and L. J. Sham, Phys. Rev. B **29**, 6132 (1984).

[14]E. G. Brovman and Yu. Kagan, Zh. Eskp. Teor. Fiz. **63**, 1937 (1973) [Sov. Phys. JETP **36**, 1025 (1973)]; see also *Lattice Dynamics,* edited by A. A. Maradudin and G. K. Horton (North-Holland, Amsterdam, 1974), Chap. 4.

[15]A. Theumann and M. T. Béal-Monod, Phys. Rev. B **29**, 2567 (1984).

[16]Preliminary data by A. P. Ramirez *et al.* (unpublished) would be most interesting to pursue.

[17]M. T. Béal-Monod, Europhys. Lett. **4**, 467 (1987).

[18]In liquid ^{3}He it has been assumed (Ref. 11 above) that the neglect of vertex corrections may be partially compensated by choosing the phenomenological fermion-fermion interaction I such as to fit an experimentally determined quantity.

[19]See, for instance, P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966).

[20]C. Noguera and P. Garoche (unpublished) also emphasize the inadequacy of standard theories for high-T_c superconductors, based on arguments different from ours but complementary.

[21]See J. R. Schrieffer, *Theory of Superconductivity* (Benjamin, New York, 1964).

[22]No isotope effect was found in the Y compounds by B. Batlogg *et al.*, Phys. Rev. Lett. **58**, 2333 (1987), and by L. C. Bourne, *ibid.* **58**, 2337 (1987), while an isotope effect was observed in the La compounds [K. A. Müller (private communication)].